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Conteúdo

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A CLASS OF KIRCHHOFF-TYPE PROBLEM IN HYPERBOLIC SPACE $\mathcal{H}^N$ INVOLVING CRITICAL SOBOLEV EXPONENT

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Abstract

In this work a class of the critical Kirchhoff-type problems in Hyperbolic space is studied. Because of the Kirchhoff term the nonlinearity u^q became "concave" for $2 < q < 4$, bringing some difficulty to prove the boundedness of the Palais Smale sequence. To overcome this we used scaled functional which was employed by Jeanjean [2] and Jeanjean and Le Coz [3], where the Pohozaev manifold is considered to be a constrained manifold. See also [4] and [5]. In addition to the definition of the Pohozaev manifold, another difficulty is to overcome the singularities on the unit sphere. The result is obtained by using variational methods.

1 Introduction

In this paper we are concerned with the following Kirchhoff-type problem

$$-\left(a + b\int_{\mathbf{B}^3} |\nabla_{\mathbf{B}^3} u|^2 dV_{\mathbf{B}^3}\right)\Delta_{\mathbf{B}^3} u \quad = \quad \lambda|u|^{q-2}u + |u|^4 u \quad \text{in} \quad H^1(\mathbf{B}^3), \tag{1}$$

in Hyperbolic space $\mathbf{B}^3$, where a, b, λ are positive constants, $2 < q < 4$, $H^1(\mathbf{B}^3)$ is the usual Sobolev space on the disc model of the Hyperbolic space $\mathbf{B}^3$, and $\Delta_{\mathbf{B}^3}$ denotes the Laplace Beltrami operator on $\mathbf{B}^3$. This problems, when the non-linearity behaves as a polynomial function of degree $2^* = \frac{2N}{N-2}$, in $\mathbb{R}^N (N \geq 3)$, was studied in a remarkable paper due to Brezis Nirenberg [1].

2 Main Result

Theorem 2.1. *Suppose $2 < q < 4$. Then, for $\lambda > 0$ sufficiently large, the problem (1) has a nontrivial solution $u \in H^1(\mathbf{B}^3)$.*

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NONLOCAL KIRCHHOFF PROBLEMS WITH EXPONENTIAL CRITICAL NONLINEARITIES

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Abstract

The work deals with existence of solutions for a class of nonlinear elliptic equations, involving a nonlocal Kirchhoff term and possibly Trudinger–Moser critical growth nonlinearities, of the type

$$-M(||u||^2)\left(Ł_K u + \int_{\mathbb{R}} V(x)|u|^2 \mathrm{d}x\right) = P(x)f(u) \quad \text{in } \mathbb{R}, \tag{1}$$

where

$$\|u\| = \left(\int_{\mathbb{R}} V(x)|u|^2 \mathrm{d}x + \iint_{\mathbb{R}^2} |u(x)-u(y)|^2 K(x-y)\mathrm{d}x\mathrm{d}y\right)^{1/2},$$
$$Ł_K u(x) = \frac{1}{2}\int_{\mathbb{R}} \big[u(x+y)+u(x-y)-2u(x)\big]K(x-y)\mathrm{d}y, \tag{2}$$

and $K : \mathbb{R}\setminus\{0\} \to \mathbb{R}^+$ is a measurable positive kernel, which was used in [2], verifying

(K_1) *$mK \in L^1(\mathbb{R})$ with $m(x) = \min\{1, |x|^2\}$,*

(K_2) *There exists $\theta > 0$ such that $K(x) \geq \theta|x|^{-(2)}$ for any $x \in \mathbb{R}\setminus\{0\}$.*

Thus, when K reduces to the prototype $K(x) = |x|^{-2}$, then $-Ł_K$ becomes $(-\Delta)^{1/2}$.

The Kirchhoff function $M : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is assumed to be continuous in $\mathbb{R}_0^+$ and to satisfy

(M_1) *there exists $\gamma \in [1,\infty)$ such that $tM(t) \leq \gamma\mathbb{M}(t)$ for any $t \in \mathbb{R}_0^+$, where $\mathbb{M}(t) = \int_0^t M(\tau)\mathrm{d}\tau$,*

(M_2) *for any $\tau > 0$ there exists $m = m(\tau) > 0$ such that $M(t) \geq m$ for all $t \geq \tau$.* Condition (M_2) first appears in [3].

The lack of compactness of the associated energy functional due to the unboundedness of the domain and to the Moser Trudinger embedding has to be overcome via new techniques.

The assumptions required on V and P are taken from [2] and can be summarized in these three conditions.

(I) (sign of V and P) *The potentials V and P are continuous and strictly positive in $\mathbb{R}$;*

(II) (decay of P) *If $\{A_n\}_n$ is a sequence of Borel sets of $\mathbb{R}$, with $|A_n| \leq R$ for all $n \in \mathbb{N}$ and some $R > 0$, then*

$$\lim_{r\to\infty} \int_{A_n \cap B_r^c(0)} P(x)\mathrm{d}x = 0, \quad \textit{uniformly with respect to } n \in \mathbb{N}, \tag{3}$$

where $B_R^c(0)$ is the complement of the closed interval $B_R = [-R, R]$.

(III) (interrelation between V and P) *The potential P is in $L^\infty(\mathbb{R})$ and there exists $C_0 > 0$ such that $V(x) \geq C_0$ for all $x \in \mathbb{R}$.*

We will assume on f the following conditions.

(f_1) (behavior at zero) *$f : \mathbb{R} \to \mathbb{R}_0^+$ is differentiable, with $f = 0$ on $\mathbb{R}^-$ and*

$$\lim_{t\to 0^+} \frac{f(t)}{t^{2\gamma-1}} = 0,$$

where $\gamma \geq 1$ is the number given in condition (M_1).

(f_2) (critical growth) *there exists* $\omega \in (0, \pi]$ *and* $\alpha_0 \in (0, \omega]$

$$\lim_{t\to\infty} \frac{f(t)}{e^{\alpha t^2} - 1} = 0 \quad \textit{for all } \alpha > \alpha_0,$$
$$\limsup_{t\to\infty} \frac{f(t)}{e^{\alpha t^2} - 1} = \infty \quad \textit{for all } \alpha < \alpha_0.$$

(f_3) (super–quadraticity) $t^{1-2\gamma} f(t)$ *is nondecreasing in* $\mathbb{R}^+$ *and there are* $q > 2\gamma$ *and* $C_q > 0$ *with*

$$F(t) \geq C_q t^q \quad \textit{for all } t \in \mathbb{R}_0^+.$$

(AR) (Ambrosetti–Rabinowitz) *there exists* $\theta > 2\gamma$ *such that*

$$\theta F(t) \leq t f(t) \quad \textit{for all } t \in \mathbb{R}_0^+.$$

1 Main Results

Theorem 1.1. *Assume that* (I), (II), (III), (M_1)–(M_2), (f_1), (f_2), (f_3) *and* (AR) *hold. Then* (1) *has a nontrivial nonnegative solution* $u \in H^{1/2}_{V,K}(\mathbb{R})$, *provided that the constant* C_q *in condition* $(f_3)'$ *is sufficiently large.*

Proof See [6].

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ACKNOWLEDGMENTS: O. H. Miyagaki was partially supported by INCTmat/MCT/Brazil and CNPq/Brazil. P. Pucci is member of the *Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni* (GNAMPA) of the *Istituto Nazionale di Alta Matematica* (INdAM). The manuscript was realized within the auspices of the INdAM – GNAMPA Projects 2018 *Problemi non lineari alle derivate parziali* (Prot_U-UFMBAZ-2018-000384). P. Pucci was partly supported by the Italian MIUR project *Variational methods, with applications to problems in mathematical physics and geometry* (2015KB9WPT_009).

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CRITICAL SCHRÖDINGER EQUATION COUPLED WITH BORN-INFELD TYPE EQUATIONS

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Abstract

In this talk we consider a quasilinear Schrödinger-Poisson system with a subcritical nonlinearity f, depending on the two parameters $\lambda, \varepsilon > 0$. We prove existence and behaviour of the solutions with respect to the parameters.

1 Introduction

In the mathematical literature many papers deal with the nonlinear Schrödinger equation coupled with the electrostatic field. These equations are variational in nature, hence the system which describes the phenomenon appear as the Euler-Lagrange equation of some Lagrangian.

The best way to describe the electromagnetic field seems to be by using the Born-Infeld Lagrangian, introduced in the seminal paper [2]. The advantage of working with such a Lagrangian is that it is relativistic invariant which is natural when dealing with electromagnetic phenomena. Explicitly the Lagrangian is

$$\mathcal{L}_{\text{B-I}} = \frac{1}{8\pi\varepsilon^4}\left(1 - \sqrt{1 - 2\varepsilon^4(|\nabla\phi + \partial_t \mathbf{A}|^2 - |\nabla \times \mathbf{A}|^2)}\right)$$

where $\phi, \mathbf{A}$ are the gauge potentials.

Of course dealing with such a Lagrangian implies some mathematical difficulties: in the simplest case, the equation of the electrostatic field generated by a density charge ρ is

$$\nabla \cdot \left(\frac{\nabla\phi}{\sqrt{1 - |\nabla\phi|^2}}\right) = \rho \quad \text{in } \mathbb{R}^3$$

which is not easy to work with.

Note that the first order approximation in ε of $\mathcal{L}_{\text{B-I}}$ is exactly the familiar Maxwell Lagrangian

$$\mathcal{L}_{\text{Max}} = \frac{1}{8\pi}\left(|\nabla\phi + \partial_t \mathbf{A}|^2 - |\nabla \times \mathbf{A}|^2\right)$$

which gives rise to the classical Maxwell equations and, in the electrostatic case, to the well known and more accessible Poisson equation

$$-\Delta\phi = \rho \quad \text{in } \mathbb{R}^3.$$

Here we are interested instead in considering the second order approximation in ε of $\mathcal{L}_{\text{B-I}}$, namely

$$\mathcal{L} = \frac{1}{8\pi}\left(|\nabla\phi + \partial_t \mathbf{A}|^2 - |\nabla \times \mathbf{A}|^2\right) + \frac{\varepsilon^4}{16\pi}\left(|\nabla\phi + \partial_t \mathbf{A}|^2 - |\nabla \times \mathbf{A}|^2\right)^2.$$

Now the equation for the electrostatic field is the quasilinear equation

$$-\Delta\phi - \varepsilon^4\Delta_4\phi = \rho \quad \text{in } \mathbb{R}^3,$$

and the coupling (according to the Abelian Gauge Theories) with the Schrödinger equation led to the system

$$\begin{cases} -\Delta u + u + \phi u = g(x,u) & \text{in } \mathbb{R}^3, \\ -\Delta \phi - \varepsilon^4 \Delta_4 \phi = u^2 & \text{in } \mathbb{R}^3. \end{cases} \tag{$P_{\lambda,\varepsilon}$}$$

The main difficulty is to deal with the second equation, which although has a unique solution for every u, an explicit formula and nice properties are not known. To overcome this fact we use a truncation in the energy functional in front of this "bad term" which permits to apply Mountain Pass arguments and prove the existence of solutions.

2 Our result

Let $\lambda > 0$ and $\varepsilon > 0$ parameters, $2^* = 6$ the critical Sobolev exponent, $f : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ continuous and such that

1. $f(x,t) = 0$ for $t \leq 0$,
2. $\lim_{t\to 0} \dfrac{f(x,t)}{t} = 0$, uniformly on $x \in \mathbb{R}^3$,
3. there exists $q \in (2, 2^*)$ verifying $\displaystyle\lim_{t\to+\infty} \frac{f(x,t)}{t^{q-1}} = 0$ uniformly on $x \in \mathbb{R}^3$,
4. there exists $\theta \in (4, 2^*)$ such that $0 < \theta F(x,t) = \theta \int_0^t f(x,s)ds \leq t f(x,t)$ for all $x \in \mathbb{R}^3$ and $t > 0$.

Theorem 2.1. *Under the above assumptions, there exists $\lambda^* > 0$, such that for all $\lambda \geq \lambda^*$ and $\varepsilon > 0$, problem*

$$\begin{cases} -\Delta u + u + \phi u = \lambda f(x,u) + |u|^{2^*-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi - \varepsilon^4 \Delta_4 \phi = u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{$P_{\lambda,\varepsilon}$}$$

admits nonnegative solutions $(u_{\lambda,\varepsilon}, \phi_{\lambda,\varepsilon}) \in H^1(\mathbb{R}^3) \times \Big(D^{1,2}(\mathbb{R}^3) \cap D^{1,4}(\mathbb{R}^3)\Big)$.

For every fixed $\overline{\varepsilon} > 0$ we have:

$$\lim_{\lambda\to+\infty} \|u_{\lambda,\overline{\varepsilon}}\|_{H^1} = 0, \quad \lim_{\lambda\to+\infty} \|\phi_{\lambda,\overline{\varepsilon}}\|_{D^{1,2}\cap D^{1,4}} = 0, \quad \lim_{\lambda\to+\infty} |\phi_{\lambda,\overline{\varepsilon}}|_{L^\infty} = 0.$$

For every fixed $\overline{\lambda} \geq \lambda^*$ we have:

$$\lim_{\varepsilon\to 0^+} \|u_{\overline{\lambda},\varepsilon} - u_{\overline{\lambda},0}\|_{H^1} = 0, \quad \lim_{\varepsilon\to 0^+} \|\phi_{\overline{\lambda},\varepsilon} - \phi_{\overline{\lambda},0}\|_{D^{1,2}} = 0,$$

where $(u_{\overline{\lambda},0}, \phi_{\overline{\lambda},0}) \in H^1(\mathbb{R}^3) \times D^{1,2}(\mathbb{R}^3)$ is a positive solution of the Schrödinger-Poisson system

$$\begin{cases} -\Delta u + u + \phi u = \overline{\lambda} f(x,u) + |u|^{2^*-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2 & \text{in } \mathbb{R}^3. \end{cases}$$

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THE BIFURCATION DIAGRAM OF A KIRCHHOFF-TYPE EQUATION

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Abstract

We study a superlinear and subcritical Kirchhoff type equation which is variational and depends upon a real parameter λ. The nonlocal term forces some of the fiber maps associated with the energy functional to have two critical points. This suggest multiplicity of solutions and indeed we show the existence of a local minimum and a mountain pass type solution. We characterize the first parameter λ_0^* for which the local minimum has non-negative energy when $\lambda \geq \lambda_0^*$. Moreover we characterize the extremal parameter λ^* for which if $\lambda > \lambda^*$, then the only solution to the Kirchhoff equation is the zero function. In fact, λ^* can be characterized in terms of the best constant of Sobolev embeddings. We also study the asymptotic behavior of the solutions when $\lambda \downarrow 0$.

1 Introduction

In this work we study the following Kirchhoff type equation

$$\begin{cases} -\left(a+\lambda\displaystyle\int |\nabla u|^2\right)\Delta u = |u|^{\gamma-2}u & \text{in } \Omega, \\ u=0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $a > 0$, $\lambda > 0$ is a parameter, Δ is the Laplacian operator and $\Omega \subset \mathbb{R}^3$ is a bounded regular domain.

Kirchhoff type equations have been extensively studied in the literature. It was proposed by Kirchhoff as an model to study some physical problems related to elastic string vibrations and since then it has been studied by many author, see for example the works of Lions [1], Alves et al. [1], Wu et al. [2], Zhang and Perera [5] and the references therein. Physically speaking if one wants to study string or membrane vibrations, one is led to the equation (2.3), where u represents the displacement of the membrane, $|u|^{p-2}u$ is an external force, a and λ are related to some intrinsic properties of the membrane. In particular, λ is related to the Young modulus of the material and it measures its stiffness.

Our main interest here is to analyze equation (2.3) with respect to the parameter λ (stiffness) and provide a description of the bifurcation diagram. To this end, we will use the fibering method of Pohozaev to analyse how the Nehari set change with respect to the parameter λ and then apply this analysis to study bifurcation properties of the problem (2.3) (see also Chen et al. [2] and Zhang et al. [6]).

2 Main Results

Let $H_0^1(\Omega)$ denote the standard Sobolev space and $\Phi_\lambda : H_0^1(\Omega) \to \mathbb{R}$ the energy functional associated with (2.3), that is

$$\Phi_\lambda(u) = \frac{a}{2}\int |\nabla u|^2 + \frac{\lambda}{4}\left(\int |\nabla u|^2\right)^2 - \frac{1}{\gamma}\int |u|^\gamma. \tag{1}$$

We observe that Φ_λ is a C^1 functional. By definition a solution to equation (2.3) is a critical point of Φ_λ. Our main result is:

Theorem 2.1. *Suppose $\gamma \in (2, 4)$. Then there exist parameters $0 < \lambda_0^* < \lambda^*$ and $\varepsilon > 0$ such that:*

1) *For each* $\lambda \in (0, \lambda^*]$ *problem* (2.3) *has a positive solution* u_λ *which is a global minimizer for* Φ_λ *when* $\lambda \in (0, \lambda_0^*]$, *while* u_λ *is a local minimizer for* Φ_λ *when* $\lambda \in (\lambda_0^*, \lambda^*)$. *Moreover* $\Phi''_\lambda(u_\lambda)(u_\lambda, u_\lambda) > 0$ *for* $\lambda \in (0, \lambda^*)$ *and* $\Phi''_{\lambda^*}(u_{\lambda^*})(u_{\lambda^*}, u_{\lambda^*}) = 0$.

2) *For each* $\lambda \in (0, \lambda_0^* + \varepsilon)$ *problem* (2.3) *has a positive solution* w_λ *which is a mountain pass critical point for* Φ_λ.

3) *If* $\lambda \in (0, \lambda_0^*)$ *then* $\Phi_\lambda(u_\lambda) < 0$ *while* $\Phi_{\lambda_0^*}(u_{\lambda_0^*}) = 0$ *and if* $\lambda \in (\lambda_0^*, \lambda^*]$ *then* $\Phi_\lambda(u_\lambda) > 0$.

4) $\Phi_\lambda(w_\lambda) > 0$ *and* $\Phi_\lambda(w_\lambda) > \Phi_\lambda(u_\lambda)$ *for each* $\lambda \in (0, \lambda_0^* + \varepsilon)$.

5) *If* $\lambda > \lambda^*$ *then the only solution* $u \in H_0^1(\Omega)$ *to the problem (2.3) is the zero function* $u = 0$.

Proof See [4]. □

Concerning the asymptotic behavior of the solutions when $\lambda \downarrow 0$ we prove the following

Theorem 2.2. *There holds*

i) $\Phi_\lambda(u_\lambda) \to -\infty$ *and* $\|u_\lambda\| \to \infty$ *as* $\lambda \downarrow 0$.

ii) $w_\lambda \to w_0$ *in* $H_0^1(\Omega)$ *where* $w_0 \in H_0^1(\Omega)$ *is a mountain pass critical point associated to the equation* $-a\Delta w = |w|^{p-2}w$.

Proof See [4]. □

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CONTRIBUTIONS TO THE STUDY OF ASYMMETRIC SEMILINEAR ELLIPTIC PROBLEMS

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Abstract

We are here concerned with the critical semilinear elliptic problem

$$\begin{cases} -\Delta u = -\mu|u|^{q-2}u + \lambda u + u_+^{2^*-1} \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is a smooth bounded domain, $\mu > 0$, $1 < q < 2$, $2^* = 2N/(N-2)$, and λ is a real parameter. Since the problem (1) has appeared in the works [1, 2, 3, 4, 4], many others have investigated it in more general situations.

We provide new contributions to the problem above by either allowing λ to lie between two consecutive eigenvalues of the Laplacian operator or even to be an eigenvalue for μ small enough. The main results are contained in the references [5, 3].

1 The main results

In these notes we study the semilinear elliptic problem

$$\begin{cases} -\Delta u = -\mu|u|^{q-2}u + \lambda u + (u^+)^{p-1} \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subseteq \mathbb{R}^N$ is a bounded domain with regular boundary $\partial\Omega$, $N \geq 3$, $1 < q < 2 < p \leq 2^*$, $\lambda \in \mathbb{R}$, μ is a positive parameter and $u^+ = \max\{u, 0\}$.

The above problem is the combination of three well-known ones. The concave-convex problem introduced by Ambrosetti, Brezis and Cerami in [1], which has resulted in a large number of related ones as by replacing the superlinear term by a function g,

$$-\Delta u = \mu|u|^{q-2}u + g(u) \text{ in } \Omega. \tag{2}$$

Many authors have been considering it for asymmetric and asymptotically linear g or asymmetric g that is superlinear at infinity, $g_+ = \lim_{t\to+\infty} g(t)/t = \infty$, combining the concave-convex problems with the asymmetric ones. Here asymmetric means that g satisfies an Ambrosetti-Prodi type condition (i.e. $\lim_{t\to-\infty} g(t)/t < \lambda_k < \lim_{t\to+\infty} g(t)/t$). It is worth mentioning that crossing eigenvalues, in particular the first one, is closely related to existence and multiplicity of solutions. Notice that the nonlinearity $g(t) = \lambda t + (t^+)^{p-1}$, with $\lambda > \lambda_1$, has not taken in account so far the Ambrosetti-Prodi problem studied by Ruf and Srikanth in [4] and De Figueiredo and Yang in [2],

$$-\Delta u = \lambda u + (u^+)^p + f(x) \text{ in } \Omega, \tag{3}$$

which assumes an important role in our study.

For the critical case, the main motivation is the pioneering Brezis-Nirenberg work [2], where the following was considered

$$\begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\lambda < \lambda_1$. They noticed that it had a breaking point for the convergence of PS-sequences at the value $\frac{S^{N/2}}{N}$, so that they constructed minimax levels for the energy functional below this value. Such ideas have been permeating many later works as well as ours. One of them was the Capozzi, Fortunato and Palmieri work [3]. They basically allow λ to be between two eigenvalues. They showed that the problem above has a nontrivial solution for all $\lambda > 0$ when $N \geq 5$ and for λ different from eigenvalues of the Laplacian when $N = 4$.

We are denoting by $0 < \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_j \leq \cdots$ the eigenvalues of $(-\Delta, H_0^1(\Omega))$. In what follows, we state the results obtained.

Theorem 1.1. *Let $N \geq 3$ and $\lambda_k < \lambda < \lambda_{k+1}$. If $2 < p < 2^*$, then, for λ small enough, (P) has at least three nontrivial solutions.*

Theorem 1.2. *Let $N \geq 4$ and $\lambda_k < \lambda < \lambda_{k+1}$. If $p = 2^*$, for λ small enough, (P) has at least three nontrivial solutions.*

The positive and negative solutions of the above theorems are derived from the Mountain-Pass theorem. In order to apply the linking theorem to obtain the third one, we resort to a trick of modifying the eigenfunctions via cut-off functions. To handle the resonant case, we have developed in the most our recent work [4] an appropriate linking theorem which reassures again the existence of solutions.

Theorem 1.3. *Let $N \geq 4$ and $\lambda_1 < \lambda$. If $p = 2^*$, for λ small enough, (P) has at least three nontrivial solutions.*

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XIII ENAMA - Novembro 2019 *19–20*

POSITIVE SOLUTIONS FOR WEAKLY COUPLED NONLINEAR SCHRÖDINGER SYSTEMS

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Abstract

This article is concerned with the application of variational methods in the study of positive solutions for a system of weakly coupled nonlinear Schrödinger equations in the Euclidian space. The results on multiplicity of positive solutions are established under the hypothesis that the coupling is either sublinear or superlinear with respect to one of the variables. Conditions for the existence or non existence of a positive least energy solution are also considered.

1 Introduction

In this work we apply variational methods to study the existence of positive solutions for the following weakly coupled nonlinear Schrödinger system

$$\begin{cases} -\Delta u + \lambda_1 u = |u|^{p-2}u + \frac{2\beta\alpha}{\alpha+\mu}|u|^{\alpha-2}u|v|^{\mu}, & \text{in } \mathbb{R}^N, \\ -\Delta v + \lambda_2 v = |v|^{q-2}v + \frac{2\beta\mu}{\alpha+\mu}|v|^{\mu-2}v|u|^{\alpha}, & \text{in } \mathbb{R}^N, \end{cases} \tag{1}$$

with $N \geq 2$, $\beta,\ \lambda_1,\ \lambda_2 > 0$, $\alpha, \mu > 1$, $2 < p,\ q,\ \alpha+\mu < 2^*$, where $2^* = \infty$ if $N = 2$ and $2^* = 2N/(N-2)$ if $N \geq 3$.

Existence of positive least energy solution will also be established. To establish such results we used variational methods, more specifically, we consider the associated functional restricted to nehari manifold and apply local and global minimization arguments combined with minimax methods. Our primary motivation to study System (1) were the articles [1, 2, 3, 4]. In particular, we emphasize the articles due to Ambrosetti-Colorado [1, 2].

2 Main Results

In our first result the existence of a positive solution is obtained independently of the coupling being sublinear, linear or superlinear with respect to any one of the variables.

Theorem 2.1. *There exist $\beta_0, \beta_1 > 0$ such that System (1) has a positive solution for every $\beta \in [0, \beta_0)$ and a positive least energy solution for every $\beta \in (\beta_1, +\infty)$.*

In the case where the coupling is doubly partially sublinear, we are able to verify that System (1) has a positive least energy solution for every $\beta > 0$. Furthermore we may establish the existence of a third positive solutions for System (1) whenever $\beta > 0$ is sufficiently small.

Theorem 2.2. *Suppose the coupling is doubly partially sublinear. Then System (1) has a positive least energy solution for every $\beta > 0$. Furthermore there is $\beta_0 > 0$ such that System (1) has at least three positive solutions for every $0 < \beta < \beta_0$.*

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XIII ENAMA - Novembro 2019 *21–22*

A LIMITING FREE BOUNDARY PROBLEM FOR A DEGENERATE OPERATOR IN ORLICZ-SOBOLEV SPACES

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Abstract

A free boundary optimization problem involving the Φ-Laplacian in Orlicz-Sobolev spaces is considered for the case where Φ does not satisfy the natural conditions introduced by Lieberman. A minimizer u_Φ having non-degeneracy at the free boundary is proved to exist and some important consequences are established, namely, the Lipschitz regularity of u_Φ along the free boundary, the locally uniform positive density of positivity set of u_Φ and that the free boundary is porous with porosity $\delta > 0$ and has finite $(N-\delta)$-Hausdorff measure.

1 Introduction

In the present work, we are interested in a degenerate case. For a given smooth bounded domain Ω in $\mathbb{R}^N$, $N \geq 2$, and a positive real parameter λ, we consider the minimization problem

$$\min\left\{J(u)\,:\, u \in W^{1,\Phi}(\Omega), |\nabla u| \in K_\Phi(\Omega), u = f \text{ on } \partial\Omega\right\}, \tag{1}$$

for a prescribed function $f \in C(\overline{\Omega})$ with $|\nabla f| \in K_\Phi(\Omega)$ and $f \geq 0$, where

$$J(u) = \int_\Omega [\Phi(|\nabla u|) + \lambda\chi_{\{u>0\}}]\,dx,$$

$\Phi(t) = \exp(t^2) - 1$ and $K_\Phi(\Omega)$ is the Orlicz class. We observe that $\Phi(t) = \exp(t^2) - 1$ satisfies

$$1 \leq \frac{\phi'(t)t}{\phi(t)}, \quad \forall\, t > 0,$$

where $\phi(t) = \Phi'(t)$. However,

$$\lim_{t\to+\infty} \frac{\phi'(t)t}{\phi(t)} = +\infty,$$

enabling us to call (1) as a degenerate minimization problem. We observe that the fact that $\Phi(t) = \exp(t^2) - 1$ does not satisfy Δ_2-condition implies that the Banach space $W^{1,\Phi}(\Omega)$ is neither reflexive nor separable, as a result, the use of minimizing sequences to find solutions to (1) breaks down. To overcome this difficulty, for each $k \in \mathbb{N}$, we consider the truncated function G_k defined for $t \in \mathbb{R}$ by

$$G_k(t) = \sum_{n=1}^{k} \frac{1}{n!}|t|^{2n}, \tag{2}$$

with the purpose of transferring the information obtained with regard to G_k to Φ. Set $g_k(t) = G_k'(t)$, $t \geq 0$. The function g_k satisfies

$$\delta_0 \leq \frac{tg_k'(t)}{g_k(t)} \leq g_0, \quad t > 0, \tag{3}$$

for $\delta_0 = 1$ and $g_0 = 2k-1$. Since $f \in W^{1,\Phi}(\Omega)$, with $|\nabla f| \in K_\Phi(\Omega)$, and the immersion $W^{1,\Phi}(\Omega)$ is continuous in $W^{1,G_k}(\Omega)$ for every k, which in turn is embedding in $C^{0,\alpha}(\overline{\Omega})$ for some $\alpha \in (0,1)$ for k sufficiently large, the function $f \in W^{1,G_k}(\Omega) \cap C^{0,\alpha}(\overline{\Omega})$ for k sufficiently large. By [2], there is a minimizer u_k of the problem

$$\min\left\{\int_\Omega (G_k(|\nabla u|) + \lambda\chi_{\{u>0\}})dx \,:\, u \in W^{1,G_k}(\Omega),\ u = f \text{ on } \partial\Omega\right\}. \tag{4}$$

We begin by proving that this sequence of minimizers u_k converges (passing to a subsequence if necessary) to a solution for the problem

$$\min\left\{J(u) \,:\, u \in W^{1,\Phi}(\Omega),\ u = f \text{ on } \partial\Omega, |\nabla u| \in K_\Phi(\Omega)\right\}, \tag{5}$$

where

$$J(u) = \int_\Omega (\Phi(|\nabla u|) + \lambda\chi_{\{u>0\}})\, dx.$$

2 Main Results

The first result in this paper is about the existence of a minimizer, which somehow resembles [3] in having a limiting free boundary problem involving the infinity Laplacian operator after taking $k \to \infty$.

Theorem 2.1. *Let $u_k \in W^{1,G_k}(\Omega)$ be a minimizer of (4). Then, there is a subsequence (still denoted by u_k) such that $u_k \to u_\Phi$, as $k \to \infty$, uniformly on $\overline{\Omega}$, where $u_\Phi \in W^{1,\Phi}(\Omega)$ is a solution to problem (5). The function u_Φ is a weak solution, and also in the viscosity sense, to the equation $\Delta u_\Phi + 2\Delta_\infty u_\Phi = 0$ in $\{u_\Phi > 0\}$.*

Motivated by the the above-mentioned results of [2], the question naturally arises whether some these properties are satisfied by minimizer u_Φ. We begin by proving some geometric properties of u_Φ along the free boundary.

Theorem 2.2. *Let $u_\Phi \in W^{1,\Phi}(\Omega)$ be the solution to (5) given Theorem 2.1, $D \subset\subset \Omega$ be any set and $B_r(x) \subset D \cap \{u_\Phi > 0\}$ be a ball touching the free boundary $\partial\{u_\Phi > 0\}$ for $r > 0$ is sufficiently small. Then,*

1. ***Non-degeneracy.*** *There are positive constants c and C depending only on N, λ and f such that*

$$cr \le u_\Phi(x) \le Cr.$$

2. ***Harnack inequality in a touching ball.*** *There is a positive constant C depending only on r and $M := \sup_{\overline{\Omega}} f$ such that*

$$\sup_{B_{\sigma r}(x)} u_\Phi \le C \inf_{B_{\sigma r}(x)} u_\Phi,$$

for any $\sigma \in (0,1)$.

In order to proof Theorem 2.2, we revisit the Lieberman's proof in [1] of a Harnack inequality for G_k-harmonic functions for G_k given by (2). The point that requires extra care is the verification of the independence of the respective constants from k, which constituted much of the work.

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XIII ENAMA - Novembro 2019 *23–24*

EQUIVALENT CONDITIONS FOR EXISTENCE OF THREE SOLUTIONS FOR A PROBLEM WITH DISCONTINUOUS AND STRONGLY-SINGULAR TERMS

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Abstract

In this paper, we are concerned with a Kirchhoff problem in the presence of a strongly-singular term perturbed by a discontinuous nonlinearity of the Heaviside type in the setting of Orlicz-Sobolev space. The presence of both strongly-singular and non-continuous terms bring up difficulties in associating a differentiable functional to the problem with finite energy in the whole space $W_0^{1,\Phi}(\Omega)$. To overcome this obstacle, we established an optimal condition for the existence of $W_0^{1,\Phi}(\Omega)$-solutions to a strongly-singular problem, which allows us to constrain the energy functional to a subset of $W_0^{1,\Phi}(\Omega)$ to apply techniques of convex analysis and generalized gradient in Clarke sense.

1 Introduction

In this work, we are concerned in presenting equivalent conditions for the existence of three solutions for the quasilinear problem

$$(Q_{\lambda,\mu})\ \begin{cases} -M\left(\displaystyle\int \Phi(|\nabla u|)dx\right)\Delta_\Phi u = \mu b(x)u^{-\delta} + \lambda f(x,u) \text{ in } \Omega, \\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

which are linked to an optimal compatibility condition between (b,δ) for existence of solution to the strongly-singular problem

$$(S)\ \begin{cases} -\Delta_\Phi u = b(x)u^{-\delta} \text{ in } \Omega, \\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega \end{cases}$$

with the boundary condition still in the sense of the trace.

We suppose that $\Phi(t) = \int_0^{|t|} a(s)sds,\ s \in \mathbb{R}$;

(ϕ_0): $a \in C^1((0,\infty),(0,\infty))$ and $\phi(t) := a(t)t$ is an increasing odd homeomorphisms from $\mathbb{R}$ onto $\mathbb{R}$;

(ϕ_1): $0 < a_- := \inf\limits_{t>0} \dfrac{t\phi'(t)}{\phi(t)} \leq \sup\limits_{t>0} \dfrac{t\phi'(t)}{\phi(t)} := a_+ < \infty$; and $a_+ + 1 < \inf\limits_{t>0} \dfrac{t\phi_*(t)}{\Phi_*(t)}$, where $\Phi_*^{-1} := \int_0^t \frac{\Phi^{-1}(s)}{s^{1+1/N}}ds$.

(M): $M : [0,\infty) \to [0,\infty)$ is a continuous function satisfying
$M(t) \geq m_0 t^{\alpha-1}$ for all $t \geq 0$ and for some $\alpha > 0$ such that $\Phi_\alpha \prec\prec \Phi_*$, that is, $\lim\limits_{t\to\infty} \dfrac{\Phi_\alpha(\tau t)}{\Phi_*(t)} = 0$ for all $\tau > 0$, where $\Phi_\alpha(t) := \Phi(t^\alpha)$.

(f_0): $f(x,\cdot) \in \mathbf{C}(\mathbb{R} - \{\tilde{a}\})$ for some $\tilde{a} > 0$ where $-\infty < \lim_{s\to\tilde{a}^-} f(x,s) < \lim_{s\to\tilde{a}^+} f(x,s) < \infty,\ x \in \Omega$,

(f_1): there exist constants $a_1,\ a_2$ and a_3 such that

$$|\eta| \leq a_1 + a_2\widetilde{H}^{-1} \circ H(a_3|t|),\ \forall\ \eta \in \partial F(x,t),\ t \in \mathbb{R},\ x \in \overline{\Omega},$$

where $H(t)=\int_0^{|t|}h(s)ds$ is a N-function satisfying $H\in\Delta_2$, $H\prec\prec\Phi_*$ and $\frac{th(t)}{H(t)}\leq h_+$ for all $t\geq t_0$ with $1<h_+\leq\frac{\phi_-^*}{2}+1$.

(f_2): $\displaystyle\lim_{t\to 0^+}\frac{\sup_{\overline{\Omega}}F(x,t)}{t^{\alpha\phi_+}}=0$ and $\displaystyle\lim_{t\to\infty}\frac{\sup_{\overline{\Omega}}F(x,t)}{t^{\alpha\phi_-}}=0.$

2 Main Results

Theorem 2.1. *Assume f satisfies (f_0), (f_1) and $0<b\in L^1(\Omega)$ holds. If $u\in W_0^{1,\Phi}(\Omega)$ is such that:*

i) *u is either a local minimum or a local maximum of I, then $|[u=\tilde{a}]|=0$,*

ii) *u is a critical point of I and $b\in L^2_{\mathrm{loc}}(\Omega)$, then $|[\nabla u=0]|=0$. In particular, $|[u=c]|=0$ for each $c>0$.*

Moreover, if u satisfies i) or ii) above, then:

(iii) *u is a solution of Problem $(Q_{\lambda,\mu})$,*

(iv) *there exists $C>0$ such that $u(x)\geq Cd(x)$ for $x\in\overline{\Omega}$, where d stands for the distance function to the boundary $\partial\Omega$,*

(v) *u solves $(Q_{\lambda,\mu})$ almost everywhere in Ω if in addition $bd^{-\delta}\in L^{\tilde{H}}(\Omega)$.*

Theorem 2.2. *Assume $\delta>1$, $b\in L^1(\Omega)\cap L^2_{\mathrm{loc}}(\Omega)$, $(\phi_0)-(\phi_2),(f_0)-(f_2)$ and (M) hold. They are equivalents:*

i) *$\exists\ 0<u_0\in W_0^{1,\Phi}(\Omega)$ such that $\int_\Omega bu_0^{1-\delta}dx<\infty$;*

ii) *(S) has a (unique) weak solution $u\in W_0^{1,\Phi}(\Omega)$ such that*

$$u(x)\geq Cd(.)\ a.e.\ \overline{\Omega},$$

where $C>0$ independent of u;

iii) *for each $\lambda>\lambda^*$, $\exists\ \mu_\lambda>0$ such that for $\mu\in(0,\mu_\lambda]$, $(Q_{\lambda,\mu})$ has at least three solutions (two local minima and one a MPCP of the functional I), where*

$$\lambda^*=\inf\left\{\frac{\hat{M}\left(\int_\Omega\Phi(|\nabla u|)\right)}{\int_\Omega F(x,u)dx}\ :\ u\in W_0^{1,\Phi}(\Omega),\ \int_\Omega F(x,u)dx>0\right\}. \tag{1}$$

Moreover, for each of such solutions the $|[u=\tilde{a}]|=0$. Besides this, u is a almost every solution of $(Q_{\lambda,\mu})$. If in addition $bd^{-\delta}\in L^{\tilde{H}}(\Omega)$ and if either

a) *M is non-decreasing and $f(x,t)=f(x),\ 0<t<1,\ x\in\Omega$,*

b) *or M is such that a Comparison Principle holds to Problem $(Q_{0,\mu})$ and $\alpha\phi_->1$,*

then there exists $\tilde{a}^\star>0$ such that for each $\tilde{a}\in(0,\tilde{a}^\star)$ we have

$$|\{x\in\Omega\ :\ u(x)>\tilde{a}\ \text{and}\ u\ \text{is a solution of}\ (Q_{\lambda,\mu})\}|>0.$$

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XIII ENAMA - Novembro 2019 *25–26*

RADIAL SOLUTION FOR HÉNON EQUATION WITH NONLINEARITIES INVOLVING SOBOLEV CRITICAL GROWTH

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Abstract

Our goal is to study the following class of Hénon type problems

$$\begin{cases} -\Delta u = \lambda |x|^{\mu} u + |x|^{\alpha}|u|^{2^*_\alpha - 2}u & \text{in} \quad B_1, \\ \quad u = 0 & \text{on} \quad \partial B_1, \end{cases}$$

where B_1 is the ball centered at the origin of $\mathbb{R}^N$ $(N \geq 3)$ and $\mu \geq \alpha \geq 0$. Under appropriate hypotheses on the constant λ, we prove existence of at least one radial solution for this problem using variational methods.

1 Introduction

We search for one non-trivial radially symmetric solutions of the Dirichlet problem involving a Hénon-type equation of the form

$$\begin{cases} -\Delta u = \lambda |x|^{\mu} u + |x|^{\alpha}|u|^{2^*_\alpha - 2}u & \text{in} \quad B_1, \\ \quad u = 0 & \text{on} \quad \partial B_1, \end{cases} \tag{1}$$

where $\lambda > 0$, $\mu \geq \alpha \geq 0$, B_1 is a unity ball centered at the origin of $\mathbb{R}^N$ $(N \geq 3)$, where $2^*_\alpha = \dfrac{2(N+\alpha)}{N-2}$.

When $\alpha = \mu = 0$, the pioneiring work is due to Brézis and Nirenberg in [2], where they got positive solutions when $\lambda < \lambda_1$. When $\alpha, \mu > 0$, these classes of problems are called in the literature by Hénon type problems. Actually, Hénon in [4] introduced the problem (1) with $\lambda = 0$, as a model of clusters of stars for the case that N = 1. Since then, many authors have been worked with this type of the equation in several point of view. The pioneeiring paper is due to Ni [7], where he established a compact embedding result, namely, the embedding $H^1_{0,\text{rad}}(B_1) \subset L^p(B_1, |x|^{\alpha})$ is compact for all $p \in [1, 2^*_\alpha)$, where $2^*_\alpha = \frac{2(N+\alpha)}{N-2}$, in order to get radial solutions. Here $H^1_{0,\text{rad}}(B_1) = \{u \in H^1_{0,\text{rad}}(B_1) : u \text{ is radial, that is, } u(x) = u(|x|), \forall x \in B_1\}$.

For Hénon problem involving usual Sobolev exponents we would like to cite [6, 5, 8, 9], and in their references. Up to our knowledgement, there are few works treating problem (1) with $\lambda \neq 0$ involving the Sobolev critical exponent given by Ni, that is, 2^*_α. In [1] is studied by a nonhomegeneous perturbations, when $\lambda > 0$ is smaller than the first eigenvalue, while in [3] is studied some concentration phenomena for linear perturbation, when λ is small enough. In [6], Long and Yang established an existence of nontrivial solution result for problema (1) with $\mu = 0$, when $\lambda \neq \lambda_k$, for all k, and $N \geq 7$. Also, they proved that $(\lambda_k, 0)$ is a bifurcation point for problem (1), for all k. The aim here is to complement or extend above results, for instance, treating all λ positive.

2 Main Results

We divide our results in three theorems. The first one deals with the non-trivial solution of the problem when $\lambda > 0$ and $N > 4 + \mu$. The second also concerns the non-trivial solution, when the dimension in which we are working is equal to $4 + \mu$, in this case we need to consider $\lambda \neq \lambda^*_j$ for all $j \in \mathbb{N} = \{1, 2, 3, ...\}$. In the third, for $N < 4 + \mu$, we also look for a non-trivial solution. For this matter, in order to recover the compactness of the functional associated with Problem (1), we need to have λ large, with $\lambda \neq \lambda_j$.

Theorem 2.1. *For $0 < \lambda < \lambda_1^*$ or $\lambda_k^* \leq \lambda < \lambda_{k+1}^*$, the problem (1) possesses a non-trivial radial solution when $N > 4 + \mu$.*

Theorem 2.2. *For $0 < \lambda < \lambda_1^*$ or $\lambda_k^* < \lambda < \lambda_{k+1}^*$, the problem (1) possesses a non-trivial radial solution when $N = 4 + \mu$.*

Theorem 2.3. *For $\lambda > 0$ sufficiently large and $\lambda \neq \lambda_j^*$ for all $j \in \mathbb{N}$, (1) possesses a non-trivial radial solution when $N < 4 + \mu$.*

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ON THE BEHAVIOR OF LEAST ENERGY SOLUTIONS OF A FRACTIONAL $(P,Q(P))$-LAPLACIAN PROBLEM AS $P \to \infty$

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Abstract

Inspired by [1] and [2] we study the asymptotic behavior, as $p \to \infty$, of positive least energy solutions of a problem involving the fractional operator $(-\Delta_p)^\alpha + \left(-\Delta_{q(p)}\right)^\beta$ and the Dirac delta distribution.

1 Introduction

In the first part of this work we prove the existence of a positive least energy solution for the problem

$$\begin{cases} \left[(-\Delta_p)^\alpha + (-\Delta_q)^\beta\right] u = \mu \|u\|_\infty^{p-2} u(x_u)\delta_{x_u} & \text{in} \quad \Omega \\ u = 0 & \text{in} \quad \mathbb{R}^N \setminus \Omega \\ |u(x_u)| = \|u\|_\infty , \end{cases} \tag{1}$$

where: Ω is a bounded, smooth domain of $\mathbb{R}^N$, δ_{x_u} is the Dirac delta distribution supported at x_u, the parameters α, β, p, q satisfy one of the following conditions

$$(0 < \alpha < \beta < 1 \quad \text{and} \quad N/\alpha < p < q) \quad \text{or} \quad (0 < \beta < \alpha < 1 \quad \text{and} \quad N/\beta < q < p), \tag{2}$$

and

$$\mu > \lambda_{\alpha,p} := \inf\left\{ \frac{[u]_{\alpha,p}^p}{\|u\|_\infty^p} : u \in W_0^{\alpha,p}(\Omega) \setminus \{0\} \right\}.$$

Here, $W_0^{s,m}(\Omega)$ denotes the Sobolev space of fractional order $s \in (0,1)$ and exponent $m > 1$ endowed with the Gagliardo (semi)norm $[.]_{s,m}$. That is,

$$W_0^{s,m}(\Omega) := \left\{ u \in L^m(\mathbb{R}^N) : u = 0 \text{ in } \mathbb{R}^N \setminus \Omega \quad \text{and} \quad [u]_{s,m} := \left(\int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^m}{|x-y|^{N+sm}} \mathrm{d}x\mathrm{d}y \right)^{\frac{1}{m}} < \infty \right\}.$$

The energy functional associated with (1) is defined by

$$E_p(u) := \frac{1}{p}[u]_{\alpha,p}^p + \frac{1}{q}[u]_{\beta,q}^q - \frac{\mu_p}{p}\|u\|_\infty^p \quad \forall\, u \in X(\Omega),$$

where

$$X(\Omega) := \begin{cases} \left(W_0^{\beta,q}(\Omega), [\cdot]_{\beta,q}\right) & \text{if} \quad 0 < \alpha < \beta < 1 \quad \text{and} \quad N/\alpha < p < q \\ \left(W_0^{\alpha,p}(\Omega), [\cdot]_{\alpha,p}\right) & \text{if} \quad 0 < \beta < \alpha < 1 \quad \text{and} \quad N/\beta < q < p. \end{cases}$$

Assuming the above conditions on α, β, p, q and μ we show the existence of at least one positive weak solution u_p that minimizes the energy functional either on $W_0^{\beta,q}(\Omega) \setminus \{0\}$, when the first condition in (2) holds, or on the following Nehari-type set

$$\mathcal{N}_\mu := \left\{ u \in W_0^{\alpha,p}(\Omega) \setminus \{0\} : [u]_{\alpha,p}^p + [u]_{\beta,q}^q = \mu \|u\|_\infty^p \right\},$$

when the second condition in (2) holds. Both type of minimizers are referred in this work as *least energy solutions* of (1).

In the second part of this work, we study the asymptotic behavior, as $p \to \infty$, of the positive least energy solution u_p obtained when q and μ are assumed to be functions of p (i.e. $q = q(p)$ and $\mu = \mu_p$) satisfying, respectively,

$$\lim_{p\to\infty} \frac{q(p)}{p} = Q \in \begin{cases} (0,1) & \text{if} \quad 0 < \beta < \alpha < 1 \\ (1,\infty) & \text{if} \quad 0 < \alpha < \beta < 1 \end{cases} \quad \text{and} \quad \lim_{p\to\infty} \sqrt[p]{\mu_p} > R^{-\alpha}, \tag{3}$$

with R denoting the inradius of Ω (i.e. the radius of the largest ball inscribed in Ω).

Our main result is stated as follows, where

$$C_0^{0,\beta}(\overline{\Omega}) := \left\{ u \in C_0(\overline{\Omega}) : \, |u|_\beta < \infty \right\},$$

with $|\cdot|_\beta$ denoting the β-Hölder seminorm, defined by

$$|u|_\beta := \sup \left\{ \frac{|u(x) - u(y)|}{|x-y|^\beta} : x, y \in \overline{\Omega} \quad \text{and} \quad x \neq y \right\}.$$

Theorem 1.1. *Suppose* (3) *and take* $p_n \to \infty$. *There exist* $x_\infty \in \Omega$ *and* $u_\infty \in C_0^{0,\beta}(\overline{\Omega})$ *such that, up to a subsequence,* $x_{u_{p_n}} \to x_\infty$ *and* $u_{p_n} \to u_\infty$ *uniformly in* $\overline{\Omega}$. *Moreover:*

(i) $0 < u_\infty(x) \le \left(\Lambda R^\beta\right)^{\frac{1}{Q-1}} (\operatorname{dist}(x, \partial\Omega))^\beta \quad \forall\, x \in \Omega,$

(ii) $\operatorname{dist}(x_\infty, \partial\Omega) = R,$

(iii) $u_\infty(x_\infty) = \|u_\infty\|_\infty = R^\beta (\Lambda R^\beta)^{\frac{1}{Q-1}},$

(iv) $|u_\infty|_\beta = (\Lambda R^\beta)^{\frac{1}{Q-1}},$

(v) $\dfrac{|u_\infty|_\beta}{\|u_\infty\|_\infty} = R^{-\beta} = \min \left\{ \dfrac{|v|_\beta}{\|v\|_\infty} : v \in C_0^{0,\beta}(\overline{\Omega}) \setminus \{0\} \right\},$

(vi) u_∞ is a viscosity solution of

$$\max\left\{ \mathcal{L}_\alpha^+ u, \left(\mathcal{L}_\beta^+ u\right)^Q \right\} = \max\left\{ -\mathcal{L}_\alpha^- u, \left(-\mathcal{L}_\beta^- u\right)^Q \right\} \quad \text{in}\, \Omega \setminus \{x_\infty\}.$$

In the above equation the operators are defined according to the following notation, where $0 < s < 1$:

$$\left(\mathcal{L}_s^+ u\right)(x) := \sup_{y \in \mathbb{R}^N \setminus \{x\}} \frac{u(y) - u(x)}{|y-x|^s} \quad \text{and} \quad \left(\mathcal{L}_s^- u\right)(x) := \inf_{y \in \mathbb{R}^N \setminus \{x\}} \frac{u(y) - u(x)}{|y-x|^s}.$$

Acknowledgements. This work was partially supported by CNPq/Brazil (306815/2017-6 and 422806/2018-8) and Fapemig/Brazil (CEX-PPM-00137-18).

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QUASILINEAR PROBLEMS UNDER LOCAL LANDESMAN-LAZER CONDITION

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Abstract

This work presents results on the existence and multiplicity of solutions for quasilinear problems in bounded domains involving the p-Laplacian operator under local versions of the Landesman-Lazer condition. The main results do not require any growth restriction at infinity on the nonlinear term which may change sign. The existence of solutions is established by combining variational methods, truncation arguments and approximation techniques based on a compactness result for the inverse of the p-Laplacian operator. These results also establish the intervals of the projection of the solution on the direction of the first eigenfunction of the p-Laplacian operator. This fact is used to provide the existence of multiple solutions when the local Landesman-Lazer condition is satisfied on disjoint intervals.

1 Introduction

This work deals with the study of weak solutions for a class of nonlinear problems involving the p-Laplacian operator. More specifically, we are concerned with the quasilinear problem

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u + \mu h_\mu(x,u) \text{ in } \Omega, \\ \qquad u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded regular domain in $\mathbb{R}^N$, $p > 1$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, $\lambda > 0$, $\mu \neq 0$ are real parameters and $h_\mu : \Omega \times \mathbb{R} \to \mathbb{R}$ is a family of Carathéodory functions depending on μ.

Our main objective is to provide local hypotheses on the family of functions h_μ that guarantee the existence and multiplicity of solutions for problem (1) when the parameters μ and λ are close, respectively, to zero and λ_1, the principal eigenvalue of the operator $-\Delta_p$ with zero boundary conditions.

2 Main Results

We say that the family of functions h_μ satisfies the local Landesman-Lazer condition (H_μ^+), respectively (H_μ^-), on the interval (t_1, t_2) if there exists a Carathéodory function $h_0 : \Omega \times \mathbb{R} \to \mathbb{R}$ such that $h_\mu(x,s) \to h_0(x,s_0)$, as $(\mu, s) \to (0, s_0)$, for every $s_0 \in \mathbb{R}$, a.e. in Ω, and

$$(LL^+) \quad \int_\Omega h_0(x, t_1\varphi_1)\varphi_1 dx > 0 > \int_\Omega h_0(x, t_2\varphi_1)\varphi_1 dx,$$

respectively

$$(LL^-) \quad \int_\Omega h_0(x, t_1\varphi_1)\varphi_1 dx < 0 < \int_\Omega h_0(x, t_2\varphi_1)\varphi_1 dx.$$

We suppose that the family of functions h_μ is uniformly locally $L^\sigma(\Omega)$-bounded:

(H_1) Given $S > 0$, there are $\mu_1 > 0$ and $\eta_S \in L^\sigma(\Omega)$, $\sigma > \max\{N/p, 1\}$, such that

$$|h_\mu(x,s)| \leq \eta_S(x), \text{ for every } |s| \leq S, \text{ a.e. in } \Omega, \text{ for every } \mu \in (0,\mu_1).$$

Taking $X = \{v \in W_0^{1,p}(\Omega); \int_\Omega |\nabla\varphi_1|^{p-2}\nabla\varphi_1 \cdot \nabla v dx = 0\}$, our first result provide the existence of a solution for problem (1) which is a local minimum of a functional associated with an appropriate truncated problem.

Theorem 2.1. *If h_μ satisfies (H_1) and (H_μ^+) on the interval (t_1,t_2), then there exist positive constants μ^* and ν^* such that, for every $\mu \in (0,\mu^*)$ and $|\lambda - \lambda_1| < \mu\nu^*$, problem (1) has a weak solution $u_\mu = t\varphi_1 + v$, with $t \in (t_1,t_2)$ and $v \in X$.*

As a direct consequence of Theorem 2.1, we may establish a multiplicity result for (1) when (H_μ^+) is satisfied on disjoint open intervals $(t_{2j-1}, t_{2j}), 1 \leq j \leq k$. Note that this implies (H_μ^-) is satisfied on each interval $(t_{2j}, t_{2j+1}), 1 \leq j \leq k-1$. It is worthwhile mentioning that when dealing with the hypothesis (H_μ^-) for $p \neq 2$, unlike in [4], we may not rely on the Lyapunov-Schmidt reduction method since problem (1) involves the quasilinear p-Laplacian operator. In the next result, we obtain the existence of another solution of problem (1) applying the Mountain Pass Theorem. We note that one of the most important difficulties we face when applying minimax methods is exactly to establish the region where the minimax critical point is located.

Theorem 2.2. *If h_μ satisfies $h_\mu(x,0) \geq 0$ a.e. in Ω, (H_1), (H_μ^-) on the interval (t_1,t_2), with $t_1 > 0$, and (H_μ^+) on the interval (t_2,t_3), then there exist positive constants μ^*, ν^* such that, for every $\mu \in (0,\mu^*)$ and $|\lambda - \lambda_1| < \mu\nu^*$, problem (1) has two nonnegative nonzero weak solutions $u_\mu^i = \tau_i\varphi_1 + v_i$, with $v_i \in X, i = 1,2$, and $\tau_1 \in (t_1,t_3)$, $\tau_2 \in (t_2,t_3)$.*

As an application of the above result we may establish the existence of k nonnegative nontrivial solutions for problem (1) when the hypotheses (H_μ^-) and (H_μ^+) are satisfied on consecutive open intervals. For example, supposing

$(H_\mu)_k$ h_μ satisfies $h_\mu(x,s) \to h_0(x,s_0)$, as $(\mu,s) \to (0,s_0)$, for every $s_0 \in \mathbb{R}$, a.e. in Ω, and there exist $k \in \mathbb{N}$ and $0 < t_1 < t_2 < \cdots < t_k < t_{k+1}$ such that

$$\Big[\int_\Omega h_0(x,t_j\varphi_1)\varphi_1 dx\Big]\Big[\int_\Omega h_0(x,t_{j+1}\varphi_1)\varphi_1 dx\Big] < 0, \; 1 \leq j \leq k, \text{ and } \int_\Omega h_0(x,t_{k+1}\varphi_1)\varphi_1 dx < 0,$$

we may state:

Corollary 2.1. *If h_μ satisfies $h_\mu(x,0) \geq 0$, a.e. in Ω, (H_1) and $(H_\mu)_k$, then there exist positive constants μ^* and ν^* such that, for every $\mu \in (0,\mu^*)$ and $|\lambda - \lambda_1| < \mu\nu^*$, problem (1) has k nonnegative nonzero solutions $\{u_\mu^1, \cdots, u_\mu^k\}$.*

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EXISTENCE OF SOLUTIONS FOR A GENERALIZED CONCAVE-CONVEX PROBLEM OF KIRCHHOFF TYPE

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Abstract

In this work we prove a result on the existence of weak solutions for a elliptic problem

$$\sum_{i=1}^{i=2} M_i(\int_{\Omega_i} |\nabla u|^{p_i}\, dx)\Delta_{p_i} u \chi_{\Omega_i} = f(x,u)|u|_{s(x)}^{t(x)}, \text{ with } i=1,2,\ p_1=2, p_2=p \text{ and } \overline{\Omega} = \overline{\Omega_1 \bigcup \Omega_2}.$$

We establish that this problem shows a convex concave nature for certain exponent γ of the nonlocal source with $1 < \gamma < p-1$. We obtain our result by applying Galerkin's approximation and the theory of the variable exponent Sobolev spaces.

1 Introduction

We are concerned with the existence of solutions to the following system of nonlinear elliptic system

$$\begin{gathered} -M_1(\int_{\Omega_2} |\nabla u|^2\, dx)\Delta u = f(u)|u|_{s(x)}^{t(x)} \quad in \quad \Omega_1 \\ -M_2(\int_{\Omega_2} |\nabla u|^p\, dx)\Delta_p u = f(u)|u|_{s(x)}^{t(x)} \quad in \quad \Omega_2 \\ M_1(\int_{\Omega_1} |\nabla u|^2\, dx)\frac{\partial u}{\partial \nu} = M_2(\int_{\Omega_2} |\nabla u|^p\, dx)\frac{\partial u}{\partial \nu} \quad , \quad u|_{\Omega_1} = v|_{\Omega_2} \quad on \quad \Gamma \\ u = 0 \quad on \quad \partial\Omega \end{gathered}$$

where Ω is a bounded domain in $\mathbb{R}^n$, $N \geq 1$, which is split into two subdomains $\overline{\Omega} = \overline{\Omega_1 \bigcup \Omega_2}$, $\Omega_1 \bigcap \Omega_2 = \emptyset$ (we assume that Ω_1 and Ω_2 are Lipschitz), $s, t, f \in C(\overline{\Omega})$ for any $x \in \overline{\Omega}$, $p > 2$; $M_i : [0, +\infty[\longrightarrow [m_{0i}, +\infty[$, $i = 1, 2$ are continuous functions. We confine ourselves to the case where $M_1 = M_2 \equiv M$ with $m_{01} = m_{02} = m_0 > 0$ for simplicity. Notice that the results of this work remain valid for $M_1 \neq M_2$.
For the case $M_1 = M_2 = 1$, $f(s) = \lambda u^q$, $2 < q+1 < p$ and $t(x) = 0$ the problem (1) can be rewritten involving the $p(x)$-Laplacian, that is

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u) = f(u) & x \in \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

with a discontinuous exponent

$$p(x) = \begin{cases} 2 & \text{if} \quad x \in \Omega_1, \\ p > 2 & \text{if} \quad x \in \Omega_2. \end{cases}.$$

Problems that involve the $p(x)$-Laplacian with a discontinuous variable exponent, which is assumed to be constant in disjoint pieces of the domain Ω, are recently used to model organic semiconductors (i.e., carbon-based materials conducting an electrical current). In these models $p(x)$ describes a jump function that characterizes Ohmic and non-Ohmic contacts of the device material,see [2]. The study of Kirchhoff type problems has been receiving considerable attention in more recent years, see [1] and references therein. This work is devoted to the study of operators with a power nonlinearity on the right hand side that has a concave-convex nature with respect to the Kirchhoff type operators. That is, convex (superlinear) for the Laplacian and concave (sublinear) for the p-Laplacian, see [3]. Motivated by the above works and especially [3], we consider (1) to study the existence of weak solutions.

2 Main Results

We are ready to state and prove the main result of the present paper

Theorem 2.1. *Suppose that the following conditions hold*

M) $M : [0, +\infty[\longrightarrow [m_0, +\infty[$ *, is a continuous function .*

(f_0) $f : \Omega \times \mathbb{R} \to \mathbb{R}$ *is a continuous function satisfying the following condition*

$$|f(s)| \leq c_1 |s|^{\alpha(x)-1}), \quad \forall x \in \Omega, s \in \mathbb{R},$$

for some $\alpha \in C_+(\Omega)$ *such that* $1 < \alpha^+ < p$ *for* $x \in \overline{\Omega}$, $\alpha^+ = \max_{x \in \overline{\Omega}} \alpha(x)$ *and* $c_1 > 0$ *.*

(f_1) $f(t)t \leq a|t|^{\alpha(x)}$, $\forall (x,t) \in \Omega \times \mathbb{R}$, *where* $a > 0$

(h) $t \in C(\overline{\Omega})$, $s \in C_+(\overline{\Omega})$ *with*

$t^+ + \alpha^+ < p$, $t^+ = \max_{x \in \overline{\Omega}} t(x)$, $s^+ < p$.

Then (1) has a weak solution.

Proof: We apply the Galerkin method and a well known variant of Brouwer's fixed point theorem., in the setting of the Sobolev spaces with variable exponents.

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A NOTE ON HAMILTONIAN SYSTEMS WITH CRITICAL POLYNOMIAL-EXPONENTIAL GROWTH

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Abstract

In this paper we deal with the following class of Hamiltonian systems given by $-\Delta u = v^p$, $-\Delta v = f(x,u)$ in Ω and $u = v = 0$ on $\partial\Omega$, where $\Omega \subset \mathbb{R}^N$ with $N \geq 3$ is bounded with smooth boundary, $p = 2/(N-2)$ and f has either subcritical or critical behavior of Trudinger-Moser type. We prove existence of nontrivial solution for this system using variational methods on an equivalent higher order elliptic problem.

1 Introduction

In this work, we use reduction by inversion to study a hamiltonian system of the form

$$\begin{cases} -\Delta u = v^p & \text{in} \quad \Omega, \\ -\Delta v = f(x,u) & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ with $N \geq 3$ is bounded with smooth boundary, $p = 2/(N-2)$ and f has a critical or subcritical behavior of Trudinger-Moser type. We detail the assumptions on f in the next section. Here and throughout this paper, $s^k = |s|^k \text{sgn}(s)$, which are the odd extensions of the power functions.

Putting $v = (-\Delta u)^{1/p}$, we apply variational methods to prove existence of a nontrivial solution for a class of elliptic problems of fourth-order given by:

$$\begin{cases} -\Delta \left[(-\Delta u)^{1/p}\right] = f(x,u) & \text{in} \quad \Omega, \\ u = \Delta u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

which is equivalent to system (1). This problem has been object of study of many researchers in the last decades. Supose that $f(x,s) \sim s^q$ uniformly in $x \in \Omega$, where $f \sim g$ means that $|f(s)| = O(|g(s)|)$ as $|s| \to \infty$. Then, it is well known since the 90' that one can treat these systems with variational methods if (p,q) lies below the so-called critical hyperbola given by $1/(p+1) + 1/(q+1) = 1 - 2/N$ with $p, q > 0$. For more details, see [1] and [4].

The novelty here is that we treat the case $p = 2/(N-2)$ which lies in an asymptote of this celebrated hyperbola. Notice that in case $0 < p \leq 2/(N-2)$, q is free to assume any positive value, so a natural question arises: what is the maximum growth condition allowed to $f(x,s)$ in order to treat this problem variationally? In 2004, de Figueiredo and Ruf [2] studied the case $0 < p < 2/(N-2)$ and were able to prove that there is a nontrivial solution to this system where no growth condition, other than the famous Ambrosetti-Rabinowitz assumption, is required to $f(x,s)$. Ruf [5], in 2006, proved that in this limiting case $p = 2/(N-2)$, exponential growth is necessary to the nonlinearity $f(x,s)$ and treated the case $N = 3$ using direct variational framework in a product space of appropriated Lorentz-Sobolev settings, but only for $f(x,s)$ having subcritical growth related to this exponential behavior. We aim to study the limiting case with $f(x,s)$ lying in the critical or subcritical range and with $N \geq 3$, which completes both mentioned results. We apply the reduction by inversion method and consider problem (2), which can be treated variationally in a proper second order Sobolev Space with Navier boundary conditions.

2 Main Results

Suppose that f is a continuous functional in $\Omega \times \mathbb{R}$ satisfying:

(f_1) subcritical: For all $\alpha > 0$, we have $\lim_{u \to +\infty} \dfrac{f(x,u)}{e^{\alpha u^{N/N-2}}} = 0$, uniformly in $x \in \overline{\Omega}$.

or critical: There exists α_0 such that $\lim_{u \to +\infty} \dfrac{f(x,u)}{e^{\alpha u^{N/N-2}}} = \begin{cases} 0, & \text{if } \alpha < \alpha_0 \\ +\infty & \text{if } \alpha > \alpha_0 \end{cases}$, uniformly in $x \in \overline{\Omega}$.

(f_2) $f(x,t) = o(t^{\frac{N-2}{2}})$ as $t \to 0$ uniformly in $x \in \Omega$.

(f_3) $\exists R > 0$ and $M > 0$ such that $\forall |t| \geq R, \forall x \in \Omega$, $0 < F(x,t) = \int_0^t f(x,\tau) d\tau \leq M|f(x,t)|$

(f_4) $\lim_{t \to +\infty} t f(x,t) e^{-\alpha_0 t^{N/N-2}} \geq \gamma_0 > \dfrac{2}{d^N \omega_N (N-2)} \left(\dfrac{\beta_0}{\alpha_0}\right)^{(N-2)/2}$, uniformly in $x \in \Omega$, where $d > 0$ is given by

$$d = \sup\{k > 0; \exists x_0 \in \Omega \text{ such that } B(x_0,k) \subset \Omega\}.$$

Our main results are the following

Theorem 2.1. *Under the hypothesis* $(f_0), (f_1$ *subcritical* $), (f_2)$ *and* (f_3)*, Problem* (2) *has a nontrivial solution.*

Theorem 2.2. *Suppose that conditions* (f_0), $(f_1$ *critical* $)$, (f_2), (f_3) *and* (f_4) *hold. Then, Problem* (2) *has a nontrivial solution.*

In the subcritical case, the problem is easier to handle and we give full proofs of existence of solution. For that, we rely on an Adams' inequality developed by Tarsi [6] for the case of homogeneous Navier boundary conditions. For the critical case, we use the results of Concentrarion Compacteness Principle obtained in [3] and follow their techniques to ensure that there exists also a nontrivial solution for the problem.

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XIII ENAMA - Novembro 2019 *35–36*

FRACTIONAL ELLIPTIC SYSTEM WITH NONCOERCIVE POTENTIALS

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Abstract

In this work we establish the existence of weak solution to the following class of fractional elliptic systems

$$\begin{cases} (-\Delta)^s u + a(x)u = F_u(x,u,v), & x \in \mathbb{R}^N, \\ (-\Delta)^s v + b(x)v = F_v(x,u,v), & x \in \mathbb{R}^N, \end{cases}$$

where $s \in (0,1)$, the potentials a, b are bounded from below and may change sign. The nonlinear term $F \in C^1(\mathbb{R}^N \times \mathbb{R}^2, \mathbb{R})$ can be asymptotically linear or superlinear at infinity. It interacts with the eigenvalues of the linearized problem.

1 Introduction

Recently, great attention has been paid on the study of fractional and non-local operators of elliptic type, both for the pure mathematical research and in view of concrete applications, since these operators arise in a quite natural way in many different contexts, such as the thin obstacle problem, optimization, finance, phase transitions, anomalous diffusion, crystal dislocation, semipermeable membranes, conservation laws, ultra-relativistic limits of quantum mechanics, see for instance [1, 2] and references therein.

In this work we deal with the following class of fractional elliptic systems of gradient type

$$\begin{cases} (-\Delta)^s u + a(x)u = F_u(x,u,v), & x \in \mathbb{R}^N, \\ (-\Delta)^s v + b(x)v = F_v(x,u,v), & x \in \mathbb{R}^N, \end{cases} \tag{P}$$

where $s \in (0,1)$, $N > 2s$, $(-\Delta)^s$ denotes the *fractional Laplace operator* which may be defined as

$$(-\Delta)^s u(x) := C(N,s) \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{u(x) - u(y)}{|x-y|^{N+2s}} \, \mathrm{d}y,$$

where $C(N,s) > 0$ is a normalizing constant which we omit for simplicity. Such class of systems arise in various branches of Mathematical Physics and nonlinear optics (see for instance [3]). Solutions of System (1) are related to standing wave solutions of the following two-component system of nonlinear equations.

We suppose that the potentials a and b satisfy:

(H_1) there exist $a_0, b_0 > 0$ such that $a(x) \geq -a_0, b(x) \geq -b_0$ for all $x \in \mathbb{R}^N$. Moreover, $a(x)b(x) \geq 0$, for all $x \in \mathbb{R}^N$;

(H_2) $\mu(\{x \in$ $mathbbR^N : a(x)b(x) < M\}) < \infty$, for every $M > 0$, where μ denotes the Lebesgue measure in $\mathbb{R}^N$;

(H_3) there hold

$$\inf_{u \in E_a} \left\{ \frac{[u]_s^2 + \int_{\mathbb{R}^N} a(x)u^2 \, \mathrm{d}x}{\int_{\mathbb{R}^N} u^2 \, \mathrm{d}x} \right\} > 0 \quad \text{and} \quad \inf_{v \in E_b} \left\{ \frac{[v]_s^2 + \int_{\mathbb{R}^N} b(x)v^2 \, \mathrm{d}x}{\int_{\mathbb{R}^N} v^2 \, \mathrm{d}x} \right\} > 0.$$

The basic assumptions on the nonlinearity F are the following:

(F_1) $F \in C^1(\mathbb{R}^N \times \mathbb{R}^2, \mathbb{R})$;

(F_2) there exist $c_1, c_2 > 0$, $2 \leq \sigma \leq 2_s^*$ and $\gamma \in L^t(\mathbb{R}^N)$, for some $t \in [2N/(N+2s), 2]$ such that

$$|\nabla F(x,z)| \leq c_1|z|^{\sigma-1} + c_2|z| + \gamma(x), \quad \forall\ (x,z)\ \in \mathbb{R}^N \times \mathbb{R}^2;$$

(F_3) there exist functions α, $\beta \in L^\infty(\mathbb{R}^N)$ and $c_3 \geq 0$ such that

$$|F(x,u,v)| \leq c_3|u||v| + \frac{\alpha(x)}{2}|u|^2 + \frac{\beta(x)}{2}|v|^2, \quad \forall\, x \in \mathbb{R}^N,\ (u,v) \in \mathbb{R}^2,$$

where α, β satisfy

$$\limsup_{|x|\to\infty} \alpha(x) = \alpha_\infty < \kappa_a, \quad \limsup_{|x|\to\infty} \beta(x) = \beta_\infty < \kappa_b.$$

We also consider the following hypotheses:

(F_∞) there exists $A_\infty \in L^\theta(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, $\theta > N/(2s)$, such that

$$\lim_{|(u,v)|\to+\infty} \frac{F(x,u,v) - A_\infty(x)uv}{|(u,v)|^2} = 0, \quad \text{uniformly for a.e. } x \in \mathbb{R}^N;$$

(NQ) there exists $\Gamma \in L^1(\mathcal{R}^N)$ such that

$$\begin{cases} \lim\limits_{\substack{|u|\to\infty \\ |v|\to\infty}} \nabla F(x,u,v)\cdot(u,v) - 2F(x,u,v) = \infty, & \text{for a.e. } x \in \mathbb{R}^N, \\ \nabla F(x,z)\cdot z - 2F(x,z) \geq \Gamma(x), & \forall\,(x,z) \in \mathbb{R}^N \times \mathbb{R}^2, \end{cases}$$

where $w \cdot z$ denotes the usual inner product between $w, z \in \mathbb{R}^2$.

2 The Main Result

The first result of this work can be stated as follows:

Theorem 2.1. *Suppose that $(H_1)-(H_3)$ hold. If F satisfies $(F_1)-(F_3)$, (F_∞) and (NQ), then System (P) has at least one solution.*

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EXISTENCE OF SOLUTIONS FOR A FRACTIONAL $P(X)$-KIRCHHOFF PROBLEM VIA TOPOLOGICAL METHODS

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Abstract

The purpose of this article is to obtain weak solutions for p(x)-fractional kirchhoff problem with a nonlocal source. Our result is obtained using a Fredholm-type result for a couple of nonlinear operators and the theory of the fractional Sobolev spaces with variable exponent and the fractional $p(x)$-Laplacian.

1 Introduction

In this paper we discuss the existence of weak solutions for the following nonlinear elliptic problem involving the the fractional $p(x)$-Laplacian

$$M(\iint\limits_{\Omega\times\Omega} \frac{|u(x)-u(y)|^{p(x,y)}}{\mu^{p(x,y)}\,|x-y|^{N+sp(x,y)}}dy)\mathcal{L}u = f(x,u)|u|^{t(x)}_{s(x)} \qquad \text{in } \Omega,$$
$$u = 0 \qquad\qquad \text{on } \partial\Omega, \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, and $N \geq 1$, $p, s, t \in C(\overline{\Omega})$ for any $x \in \overline{\Omega}$, $\mu > 0$; $M : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function, f is a Caratheodory function, the operator $\mathcal{L}$ is given by

$$\mathcal{L}u(x) = P.V.\int\limits_{\Omega} \frac{|u(x)-u(y)|^{p(x,y)-2}(u(x)-u(y))}{|x-y|^{N+sp(x,y)}}dy.$$

where is $P.V.$ is a commonly used abbreviation in the principal value sense , $0 < s < 1$ and $p : \overline{\Omega}\times\overline{\Omega} \to]1,\infty[$ is a continuous function with $s.p(x,y) < N$ for any $(x,y) \in \overline{\Omega}\times\overline{\Omega}$.
The study of differential and partial differential equations with variable exponent has been received considerable attention in recent years. This importance reflects directly into various range of applications. There are applications concerning elastic mechanics, thermorheological and electrorheological fluids, image restoration and mathematical biology, see [1]. Also, problems involving fractional Laplace operator has become an interesting topic since they are arises in many fields of sciences, notably the fields of physics, probability, and finance, see for instance [3]. Recently, the existence and multiplicity results of weak solutions for nonlocal fractional $p(.,.)$-Laplacian problem have been studied in [4] . Motivated by the above works and [4], we consider (1) to study the existence of weak solutions; we note that this problem has no variational structure and to solved it, our method is topological and it is based on a result of the Fredholm alternative type for a couple of nonlinear operator [2].

2 Main Results

We are ready to state and prove the main result of the present paper

Theorem 2.1. *Suppose that the following conditions hold*

M) the function $M : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ *is a continuous function and there is a constant* $m_0 > 0$ *such that*

$$M(t) \geq m_0 \qquad \textit{for all} \qquad t \geq 0.$$

(F) $f : \Omega \times \mathbb{R} \to \mathbb{R}$ *is a Caratheodory function satisfying the following conditions*

$$|f(x,s)| \leq c_1 + c_2|s|^{\alpha(x)-1}), \quad \forall x \in \Omega, s \in \mathbb{R},$$

for some $\alpha \in C_+(\Omega)$ *such that* $1 < \alpha(x) < p_s^*(x)$ *for* $x \in \overline{\Omega}$ *and* c_1, c_2 *are positive constants. Then (1) has a weak solution.*

Proof: We apply theorem 2.1 of [2], in the setting of the fractional Sobolev spaces with variable exponents.

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BEST HARDY-SOBOLEV CONSTANT AND ITS APPLICATION TO A FRACTIONAL P-LAPLACIAN EQUATION

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Abstract

In this work, we consider $x \in \mathbb{R}^N$, $s \in (0,1)$, $p \in (1,+\infty)$, $N > sp$, $\alpha \in (0, sp)$, and $\mu < \mu_H$. The Gagliardo seminorm is defined by $u \mapsto [u]_{s,p} = \left(\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} |u(x)-u(y)|^p/|x-y|^{N+sp}\,dx\,dy\right)^{1/p}$, and the best Hardy constant is defined by $\mu_H := \inf_{u\in D^{s,p}(\mathbb{R}^N)\setminus\{0\}} [u]_{sp}^p/||u||_{s,p}^p > 0$; finally, the Sobolev space is denoted by $D^{s,p}(\mathbb{R}^N) := \{u \in L^{p_s^*}(\mathbb{R}^N)\colon [u]_{sp} < \infty\}$. Our main goal is to prove that the best Hardy-Sobolev inequality, defined by $\frac{1}{K(\mu,\alpha)} = \inf_{\substack{u\in D^{s,p}(\mathbb{R}^N)\\ u\neq 0}} \left([u]_{s,p}^p - \mu\int_{\mathbb{R}_N} |u|^p/|x|^{ps}\,dx\right) \div \left(\int_{\mathbb{R}^N} |u|^{p_s^*(\alpha)}/|x|^\alpha\,dx\right)^{\frac{p}{p_s^*(\alpha)}}$, is attained by a nontrivial function $u \in D^{s,p}(\mathbb{R}^N)$. To do this, we use a refined version of the concentration-compactness principle.

1 Introduction and main result

The fractional p-Laplacian operator is a non-linear and non-local operator defined for differentiable functions $u\colon \mathbb{R}^N \to \mathbb{R}$ by

$$(-\Delta_p)^s u(x) := 2\lim_{\varepsilon\to 0^+}\int_{\mathbb{R}^N\setminus B_\varepsilon(x)} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+sp}}\,dy,$$

where $x \in \mathbb{R}^N$, $p \in (1,+\infty)$, $s \in (0,1)$ and $N > sp$.

Non-local problems involving the fractional p-Laplacian operator $(-\Delta_p)^s$ have received the attention of several authors in the last decade, mainly in the case $p = 2$ and in the cases where the nonlinearities have pure polynomial growth involving subcritical exponents (in the sense of the Sobolev embeddings). For example, consider the problem with multiple critical nonlinearities in the sense of the Sobolev embeddings and also a nonlinearity of the Hardy type, which consistently appears on the side of the nonlocal operator,

$$(-\Delta_p)^s u - \mu\frac{|u|^{p-2}u}{|x|^{ps}} = \frac{|u|^{p_s^*(\beta)-2}u}{|x|^\beta} + \frac{|u|^{p_s^*(\alpha)-2}u}{|x|^\alpha} \qquad (x \in \mathbb{R}^N) \tag{1}$$

where $s \in (0,1)$, $P \in (1,+\infty)$, $N > sp$, $\alpha \in (0, sp)$, $\beta \in (0, sp)$ with $\beta \neq \alpha$, $\mu < \mu_H$ (the constant μ_H is defined below) and $p_s^*(\alpha) = (p(N-\alpha)/(N-ps)$; in particular, if $\alpha = 0$ then $p_s^*(0) = p_s^* = Np/(N-p)$.

The choice of the space function where we look for the solutions to problems with variational structure such as problem (1) is an important step in its study. Let $\Omega \subset \mathbb{R}^N$ be an open, bounded subset with differentiable boundary. We consider tacitly that all the functions are Lebesgue integrable and we introduce the fractional Sobolev space $W_0^{s,p}(\Omega) := \{u \in L^1_{\text{loc}}(\mathbb{R}^N)\colon [u]_{s,p} < +\infty;\ u \equiv 0 \text{ a.e. } \mathbb{R}^N\setminus\Omega\}$ and the fractional homogeneous Sobolev space $D^{s,p}(\mathbb{R}^N) := \{u \in L^{p_s^*}(\mathbb{R}^N)\colon [u]_{s,p} < \infty\} \supset W_0^{s,p}(\Omega)$. In these definitions, the symbol $[u]_{s,p}$ stands for the Gagliardo seminorm, defined by

$$u \longmapsto [u]_{s,p} = \left(\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}}\,dx\,dy\right)^{1/p} \qquad (u \in C_0^\infty(\mathbb{R}^N)).$$

For $p \in (1, +\infty)$, the function spaces $W_0^{s,p}(\Omega)$ and $D^{s,p}(\mathbb{R}^N)$ are separable, reflexive Banach spaces with respect to the Gagliardo seminorm $[\,\cdot\,]_{s,p}$.

The variational structure of problem (1) can be established with the help of the following version of the Hardy-Sobolev inequality, which can be found in the paper by Chen, Mosconi and Squassina [5]. Let $s \in (0,1)$, $p \in (1,+\infty)$ and $\alpha \in [0, sp)$ with $sp < N$. Then there exists a positive constant $C \in \mathbb{R}_+$ such that

$$\left(\int_\Omega \frac{|u|^{p^*_\alpha}}{|x|^\alpha}\,dx\right)^{1/p^*_\alpha} \leqslant C\left(\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}}\,dx\,dy\right)^{1/p}$$

for every $u \in W_0^{s,p}(\Omega)$. The parameter $p^*_s(\alpha)$ is the critical fractional exponent of the Hardy-Sobolev embeddings $D^{s,p}(\mathbb{R}^N) \hookrightarrow L^p(\mathbb{R}^N; |x|^{-sp})$ where the Lebesgue space $L^p(\mathbb{R}^N; |x|^{-sp})$ is equipped with the norm $||u||_{L^p(\mathbb{R}^N;|x|^{-sp})} := \left(\int_{\mathbb{R}^N} \frac{|u|^p}{|x|^{sp}}\,dx\right)^{1/p}$. Indeed, the embeddings $W_0^{s,p}(\Omega) \hookrightarrow L^q(\Omega; |x|^\alpha)$ are continuous for $0 \leqslant \alpha \leqslant ps$ and for $1 \leqslant q \leqslant p^*_s(\alpha)$; and these embeddings are compact for $1 \leqslant q < p^*_s(\alpha)$. Moreover, the best constants of these embeddings are positive numbers, that is, $\mu_H := \inf_{\substack{u\in D^{s,p}(\mathbb{R}^N)\\ u\neq 0}} [u]^p_{s,p}/\|u\|^p_{L^p(\mathbb{R}^N;|x|^{-sp})} > 0$.

A crucial step to prove the existence of solution to problem (1) is to show that the following result, which has an independent interest.

Theorem 1.1. *The best Hardy-Sobolev constant, defined by*

$$\frac{1}{K(\mu,\alpha)} = \inf_{\substack{u\in D^{s,p}(\mathbb{R}^N)\\ u\neq 0}} \frac{[u]^p_{s,p} - \mu \displaystyle\int_{\mathbb{R}_N} \frac{|u|^p}{|x|^{ps}}\,dx}{\left(\displaystyle\int_{\mathbb{R}^N} \frac{|u|^{p^*_s(\alpha)}}{|x|^\alpha}\,dx\right)^{\frac{p}{p^*_s(\alpha)}}},$$

is attained by a nontrivial function $u \in D^{s,p}(\mathbb{R}^N)$.

To prove Theorem 2.2 we use a refined version of the concentration-compactness principle.

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XIII ENAMA - Novembro 2019 *41–42*

EQUAÇÕES DE SCHRÖDINGER QUASELINEARES COM UM PARÂMETRO POSITIVO: O CASO EXPONENCIAL

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Abstract

Neste trabalho, estabelecemos existência e propriedades qualitativas de soluções to tipo ondas estacionárias para uma classe de equações de Schrödinger quaselineares em $\mathbb{R}^2$. Tais equações envolvem não linearidades com crescimento crítico exponencial e um parâmetro positivo. Após uma mudança de variáveis, obtemos uma solução no espaço de Sobolev usual, quando o parâmetro é pequeno. Os resultados são obtidos por meio de métodos variacionais, combinados com estimativas do tipo Nash-Moser.

1 Introdução

Neste trabalho, estamos interessados em soluções positivas para equações de Schrödinger quaselineares da forma

$$-\Delta u + V(x)u + \frac{\kappa}{2}[\Delta(u^2)]u = h(u) \quad \text{em} \quad \mathbb{R}^2, \tag{1}$$

onde κ é um parâmetro positivo, $h : \mathbb{R} \to \mathbb{R}$ é uma função localmente Hölder contínua tendo crescimento crítico exponencial e o potencial $V : \mathbb{R}^2 \to \mathbb{R}$ é uma função contínua, com ínfimo positivo e que converge a uma constante no infinito.

Há poucos resultados conhecidos na literatura sobre existência de soluções para (1) quando $\kappa > 0$. Ambrosetti e Wang em [1] consideraram perturbações da equação quaselinear $-u'' + V(x)u + \kappa(u^2)''u = \lambda u^q$ em $\mathbb{R}$ onde $q > 1$, $\lambda > 0$ e κ é um parâmetro real. Usando métodos variacionais os autores provaram que existe $\kappa_0 > 0$ tal que para $\kappa < \kappa_0$ esta equação tem uma solução positiva $u \in H^1(\mathbb{R})$. Recentemente, Alves, Wang e Shen em [3] usaram uma mudança de variáveis, diferente da que vem sendo usada para κ negativo, para lidar com o problema (1) quando $\kappa > 0$ e $N \geq 3$. Considerando não linearidades do tipo $h(s) = |s|^{p-2}s$, $2 < p < 2^*$ e $h(s) = \left[1 - 1/(1+s^2)^3\right] s$, eles provaram a existência de soluções não triviais para (1) desde que $\kappa > 0$ seja pequeno. Usando esta mudança de variáveis há alguns resultados posteriores, ainda considerando dimensões $N \geq 3$ (veja [2, 6, 5]). Em [2], os autores consideraram potenciais que podem se anular no infinito e trataram de não linerаridades no caso superlinear. Em [6], o autor tratou de não linearidades da forma $h(s) = \lambda|s|^{p-2}s + |s|^{q-2}s$ com $p \geq 2^*$, $2 < q < 2^*$ e provou a existência de soluções positivas para (1) quando κ, $\lambda > 0$ são pequenos. Em [5], os autores estudaram (1) com uma não linearidade $h(s)$ assintoticamente linear ou superlinear no infinito. Além disso, eles provaram uma identidade de Pohozaev e obtiveram um resultado de não existência de solução.

No presente trabalho, para $\kappa > 0$ e $N = 2$, assumiremos as seguintes hipóteses sobre o potencial $V \in C(\mathbb{R}^2, \mathbb{R})$:

(V_1) $V_0 := \inf_{x \in \mathbb{R}^2} V(x) > 0$;

(V_2) existe uma constante $V_\infty > 0$ tal que $V(x) \leq V_\infty$ para todo $x \in \mathbb{R}^2$ e $\lim_{|x| \to \infty} V(x) = V_\infty$.

Com relação ao termo não linear $h \in C^{0,\gamma}_{loc}(\mathbb{R}^+, \mathbb{R})$, assumiremos que este satisfaz:
(c_{α_0}) (crescimento crítico exponencial) existe $\alpha_0 > 0$ tal que

$$\lim_{s \to +\infty} \frac{h(s)}{e^{\alpha s^2}} = \begin{cases} 0 & \text{if} \quad \alpha > \alpha_0, \\ +\infty & \text{if} \quad \alpha < \alpha_0. \end{cases}$$

Além disso, assumiremos as seguintes condições:

(h_1) $\lim\limits_{s\to 0^+} h(s)/s = 0$;

(h_2) (Ambrosetti–Rabinowitz) existe $\mu > 2$ tal que $0 < \mu H(s) := \mu \int_0^t h(s)\, s \le sh(s)$ para todo $s > 0$.

(h_3) existem $p > 2$ e $\xi > 0$ tais que $H(s) \ge \xi s^p$ para todo $s > 0$, onde

$$\xi > \left[\frac{3\alpha_0\mu(p-2)}{\mu-2}\right]^{(p-2)/p} \left[\frac{4(1+V_\infty)}{p}\right]^{p/2}.$$

2 Resultado Principal

Teorema 2.1. *Suponha que* $(V_1)-(V_2)$, (c_{α_0}), $(h_1)-(h_3)$ *são satisfeitas. Então existe* $\kappa_0 > 0$ *tal que* (1) *admite uma solução positiva* $u_\kappa \in H^1(\mathbb{R}^2) \cap L^\infty(\mathbb{R}^2)$ *para todo* $\kappa \in (0, \kappa_0)$.

Prova: Consideramos um problema auxiliar:

$$-\operatorname{div}\big(g^2(u)\nabla u\big) + g(u)g'(u)|\nabla u|^2 + V(x)u = h(u) \quad \text{em} \quad \mathbb{R}^2, \tag{1}$$

com $g(t) = \sqrt{1-\kappa t^2}$ para $|t| < 1/\sqrt{3\kappa}$. Buscamos uma solução para a equação (1) satisfazendo $\|u_\kappa\|_\infty < 1/\sqrt{3\kappa}$, de modo que u_κ é uma solução para (1). Para garantir que (1) tem uma solução não trivial com esta estimativa em L^∞, usamos uma mudança de variáveis, apresentada em [3], e aplicamos técnicas minimax juntamente com o método de iteração de Nash-Moser ao problema semilinear

$$-\Delta v + V(x)\frac{G^{-1}(v)}{g(G^{-1}(v))} = \frac{h(G^{-1}(v))}{g(G^{-1}(v))} \quad \text{em} \quad \mathbb{R}^2,$$

onde G é primitiva de g. Para detalhes da prova, nos referimos a [4]. □

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XIII ENAMA - Novembro 2019 *43–44*

INFINITELY MANY SMALL SOLUTIONS FOR A SUBLINEAR FRACTIONAL KIRCHHOFF-SCHRÖDINGER-POISSON SYSTEMS

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Abstract

We study the following class of Kirchhoff-Schrödinger-Poisson systems

$$\begin{cases} m([u]_\alpha^2)(-\Delta)^\alpha u + V(x)u + k(x)\phi u = f(x,u), & x \in \mathbb{R}^3, \\ (-\Delta)^\beta \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases}$$

where $[\,\cdot\,]_\alpha$ denotes the Gagliardo semi-norm, $(-\Delta)^\alpha$ denotes the fractional Laplacian operator with $\alpha, \beta \in (0,1]$, $4\alpha + 2\beta \geq 3$ and $m : [0,+\infty) \to [0,+\infty)$ is a Kirchhoff function satisfying suitable assumptions. The functions $V(x)$ and $k(x)$ are nonnegative and the nonlinear term $f(x,s)$ satisfies certain local conditions. By using a variational approach, we use a Kajikiya's version of the symmetric mountain pass lemma and Moser iteration method to prove the existence of infinitely many small solutions.

1 Introduction

In recent years, systems of the form

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(x,u), & x \in \mathbb{R}^3, \\ -\Delta \phi = u^2, & x \in \mathbb{R}^3, \end{cases} \tag{1}$$

have been studied by many researchers. In (1), the first equation is a nonlinear Schrödinger equation in which the potential ϕ satisfies a nonlinear Poisson equation. We call attention to the work of G. Bao [1], where it was studied the existence of infinitely many small solutions. There are some works concerned with the following class of nonlinear fractional Schrödinger-Poisson systems

$$\begin{cases} (-\Delta)^\alpha u + V(x)u + k(x)\phi u = f(x,u), & x \in \mathbb{R}^3, \\ (-\Delta)^\beta \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases} \tag{2}$$

where $\alpha, \beta \in (0,1]$. For instance, in [4], W. Liu has studied the case when $\alpha, \beta \in (0,1)$, $V(x) \equiv 1$, $f(x,u) = |u|^{p-1}u$, $k(x) = V(|x|)$ and $1 < p < (3+2\alpha)/(3-2\alpha)$. By considering a general nonlinear term, K. Li [3], studied the case when $k(x), V(x) \equiv 1$ and $\alpha, \beta \in (0,1]$ with $4\alpha + 2\beta > 3$. In a similar fashion, R. C. Duarte et al. [2] studied (2) under more general conditions, where it is assumed a positive potential $V(x)$ which is bounded away from zero, and an autonomous nonlinearity with 4-superlinear growth. To the best of our knowledge, there are few works concerned with this class of fractional Schrödinger-Poisson equations with the presence of Kirchhoff term and $\alpha \in (0,1]$.

Motivated by the above discussion, we study the following class of fractional Kirchhoff-Schrödinger-Poisson equations

$$\begin{cases} m([u]_\alpha^2)(-\Delta)^\alpha u + V(x)u + k(x)\phi u = f(x,u), & x \in \mathbb{R}^3, \\ (-\Delta)^\beta \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases} \tag{$\mathcal{P}$}$$

where $\alpha, \beta \in (0,1]$ such that $4\alpha + 2\beta \geq 3$. We suppose that $k(x)$ and $V(x)$ are nonnegative functions, where the potential $V(x)$ is locally integrable. In addition, we assume the following hypotheses:

(K) $k \in L^r(\mathbb{R}^3) \cup L^\infty(\mathbb{R}^3)$ such that

$$\begin{cases} r > r_* := \frac{6}{4\alpha+2\beta-3}, & \text{if } 4\alpha + 2\beta > 3, \\ r = r_* = \infty, & \text{if } 4\alpha + 2\beta = 3. \end{cases}$$

(V_1) There exists $\delta_0 > 0$ such that for the level set $\mathcal{G}_{\delta_0} := \{x \in \mathbb{R}^3 : V(x) < \delta_0\}$, we have $0 < |\mathcal{G}_{\delta_0}| < \infty$.

(V_2) For each $\delta > 0$ and level set $\mathcal{G}_\delta := \{x \in \mathbb{R}^3 : V(x) < \delta\}$, we have $0 \leq |\mathcal{G}_\delta| < \infty$.

We suppose that the Kirchhoff function $m \in C(\mathbb{R}_+, \mathbb{R}_+)$ satisfies the following assumption:

(M) $m(t) \geq m_0 > 0$, for all $t \in [0, +\infty)$ and there exist constants $a_1, a_2 > 0$ and $t_0 > 0$ such that for some $\sigma \geq 0$

$$M(t) := \int_0^t m(\tau)\, \mathrm{d}\tau \leq a_1 t + \frac{a_2}{2} t^{\sigma+2}, \quad \text{for all } t \leq t_0.$$

On the nonlinear term $f(x, s)$, we suppose the following local conditions:

(f_1) $f \in C(\mathbb{R}^3 \times [-\delta_1, \delta_1], \mathbb{R})$ for some $\delta_1 > 0$ and there exist $\nu \in (1, 2)$, $\mu \in (3/(2\alpha), 2/(2 - \nu))$ and a nonnegative function $\xi \in L^\mu(\mathbb{R}^3)$ such that

$$|f(x, s)| \leq \nu \xi(x) |s|^{\nu-1}, \quad \text{for all } (x, s) \in \mathbb{R}^3 \times [-\delta_1, \delta_1].$$

(f_2) There exist $x_0 \in \mathbb{R}^3$ and a constant $r_0 > 0$ such that

$$\liminf_{s\to 0} \left(\inf_{x \in B_{r_0}(x_0)} \frac{F(x, s)}{s^2} \right) > -\infty \quad \text{and} \quad \limsup_{s\to 0} \left(\inf_{x \in B_{r_0}(x_0)} \frac{F(x, s)}{s^2} \right) = +\infty.$$

(f_3) There exists $\delta_2 > 0$ such that $f(x, -s) = -f(x, s)$, for all $(x, s) \in \mathbb{R}^3 \times [-\delta_2, \delta_2]$.

2 Main Result

Theorem 2.1. *Suppose* (K), (V_1), (V_2), (M), (f_1)*–*(f_3) *hold. Then, System* (1) *has infinitely many non-trivial solutions* $(u_n, \phi_n)_{n\in\mathbb{N}}$ *such that* $u_n \to 0$ *as* $n \to +\infty$ *and*

$$\frac{1}{2} M([u_n]_\alpha^2) + \frac{1}{2} \int_{\mathbb{R}^3} V(x) u_n^2\, \mathrm{d}x + \frac{1}{4} \int_{\mathbb{R}^3} k(x) \phi_{u_n} u_n^2\, \mathrm{d}x - \int_{\mathbb{R}^3} F(x, u_n)\, \mathrm{d}x \leq 0.$$

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XIII ENAMA - Novembro 2019 *45–46*

TWO LINEAR NONCOERCIVE DIRICHLET PROBLEMS IN DUALITY

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Abstract

In this talk we give a self-contained and simple approach to prove the existence and uniqueness of a weak solution to a linear elliptic boundary value problem with drift in divergence form. Taking advantage of the method of continuity, we also deal with the relative dual problem. The complete results and proofs can be found in [3].

1 Introduction

In this talk we present existence and uniqueness result of solution to the following boundary value problem

$$\begin{cases} -\operatorname{div}(M(x)\nabla u) = -\operatorname{div}(u\,E(x)) + F, & \text{in} \quad \Omega, \\ u = 0, & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded, open subset of $\mathbb{R}^N$ with $N > 2$, $M(x)$ is a measurable matrix such that

$$\alpha|\xi|^2 \le M(x)\xi\cdot\xi, \quad |M(x)| \le \beta, \quad \text{a.e. } x \in \Omega, \quad \forall\, \xi \in \mathbb{R}^N, \tag{2}$$

$$F \in W^{-1,2}(\Omega) \tag{3}$$

and

$$E \in (L^N(\Omega))^N. \tag{4}$$

If the $\|E\|_{L^N(\Omega)}$ is not too large with respect to α or $\operatorname{div}(E) = 0$, the existence of a unique *weak* solution of (2.3), that is

$$u \in W_0^{1,2}(\Omega): \int_\Omega M(x)\nabla u \nabla\varphi = \int_\Omega u\,E(x)\nabla\varphi + \langle F, \varphi\rangle, \tag{5}$$

for any $\forall\, \varphi \in W_0^{1,2}(\Omega)$, is an easy consequence of the Lax-Milgram theorem.

If no assumptions on the size of $\|E\|_{L^N(\Omega)}$ are required, the problem is studied in [5], even for nonlinear principal part, by using the theory of rearrangements and in [1]. The key point in the approach used in [1] is the estimate

$$\left[\int_\Omega |\log(1+|u|)|^{2^*}\right]^{\frac{2}{2^*}} \le \frac{1}{\mathcal{S}^2\alpha^2}\int_\Omega |E|^2 + \frac{2}{\mathcal{S}^2\alpha}\int_\Omega |F(x)|, \tag{6}$$

where $\mathcal{S}$ is the Sobolev constant.

Here we propose an alternative approach to solve (5) based on the classical a priori estimate

$$\|u\|_{W_0^{1,2}(\Omega)} \le C_0\|F\|_{W^{-1,2}(\Omega)}, \tag{7}$$

where the constant C_0 does not depend on u neither on F.

The estimate (7) is obtained by contradiction, with a shorter proof than the ones given in [5] and in [1].

Moreover, due to the simple form of the inequality (7), it is natural to study the existence of a weak solution of the dual problem

$$\begin{cases} -\mathrm{div}(M^*(x)\nabla v) = \nabla v \cdot E(x) + G, & \text{in} \quad \Omega, \\ v = 0, & \text{on} \quad \partial\Omega, \end{cases} \tag{8}$$

that is

$$v \in W_0^{1,2}(\Omega) : \int_\Omega M^*(x)\nabla v \nabla\varphi = \int_\Omega E(x)\cdot\nabla v\,\varphi + \langle G, \varphi\rangle, \tag{9}$$

for any $\varphi \in W_0^{1,2}(\Omega)$. Here $M^*(x)$ denotes the transpose matrix of $M(x)$, $E(x)$ satisfies (4) and

$$G \in W^{-1,2}(\Omega). \tag{10}$$

We point out that due to the noncoercivity of the differential operators, the duality method is not so straightforward. Neverthelees, the estimate (7) and the duality between the two drifts terms

$$\int_\Omega u\,E(x)\nabla\varphi \quad \text{and} \quad \int_\Omega E(x)\cdot\nabla v\,\varphi \quad \text{with } \varphi \in W^{-1,2}(\Omega)$$

allow us to obtain an a priori estimate for any solution of (9) and, as a consequence, using also the method of continuity, we will prove the existence and uniqueness of solution of the dual problem (9).

For other existence and regularity results we refer to [4], [6] .

2 Main Results

Theorem 2.1. *Let the assumptions* (2), (3) *and* (4) *be satisfied. Then, there exists a unique solution* $u \in W_0^{1,2}(\Omega)$ *of the problem* (5).
Moreover, assuming (10), *there exists a unique solution* $v \in W_0^{1,2}(\Omega)$ *of the dual problem* (8).

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SECOND EIGENVALUE OF THE CR YAMABE OPERATOR

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1 Introdction

A question was proposed in the CR geometry context by Jerison and Lee in [4]. Precisely:

The CR Yamabe Problem. Given a closed CR manifold (M, θ) of dimension $2n + 1 \geq 3$, find a contact form conformal to θ with positive orientation and constant Webster scalar curvature.

The CR Yamabe problem was partially solved by Jerison and Lee, in the deep works [3], [4], [6], [5], for a closed CR manifold of dimension greater than 3 and non-locally CR-equivalent to the CR sphere. In short, given a compact CR manifold M of dimension $2n + 1 \geq 3$, they proved the existence of a geometric invariant $\lambda(M)$, analogous to the Yamabe invariant in the Riemannian geometry context, satisfying:

(a) $\lambda(M)$ depends only on the CR structure on M;

(b) $\lambda(M) \leq \lambda(\mathbf{S}^{2n+1})$, where $\mathbf{S}^{2n+1}$ denotes the CR sphere in $\mathbf{C}^{n+1}$;

(c) if M is not CR-equivalent to $\mathbf{S}^{2n+1}$ and $n > 1$, then $\lambda(M) < \lambda(\mathbf{S}^{2n+1})$;

(d) if $\lambda(M) < \lambda(\mathbf{S}^{2n+1})$, then M admits a conformal pseudo-hermitian structure with constant Webster scalar curvature.

The remaining cases $n = 1$ or CR manifolds locally CR-equivalent to the CR sphere in any dimension $2n+1 \geq 3$ were completed by Gamara and Yacoub in the works [1] and [2].

Let (M, θ) be a closed, connected, pseudo-hermitian CR manifold. We defined the **Second CR Yamabe Invariant** as

$$\mu_2(M, \theta) = \inf_{\tilde{\theta} \in [\theta]} \lambda_2(\tilde{\theta}) V_{\tilde{\theta}}^{\frac{1}{n+1}},$$

where $\lambda_2(\tilde{\theta})$ is the second eigenvalue of the CR Yamabe operator. We obtained the following result in the CR geometry context ([8]).

Teorema 1.1. *Let (M, θ) be a closed, connected, strictly pseudo-convex pseudo-hermitian CR manifold, with dimension $2n + 1 \geq 3$ and $\mu_1(M, \theta) = \lambda(M) > 0$. Then $\mu_2(M, \theta) \leq \mu_2(\mathbf{S}^{2n+1}))$. Furthermore, equality occurs if and only if (M, θ) is locally CR equivalent to the CR sphere $\mathbf{S}^{2n+1}$.*

W establish some properties of the eigenvalues of the CR Yamabe operator. We extend their definition to what we call generalized pseudo-Hermitian structure when possible and prove that sign is conformal invariant. Our main results are the following theorems

Teorema 1.2. *Suppose (M, θ) is a compact, strictly pseudoconvex, $2n+1$-dimensional CR manilfold. If $n \geq 3$ and $\mu(M.\theta) < 0$, then*

$$\mu_2(M, \theta) \leq \mu(\mathbf{S}^{2n+1}),$$

where $\mu(\mathbf{S}^{2n+1})$ is realized by "standard" pseudo-Hermitian form $\hat{\theta}$.

Teorema 1.3. *Let (M,θ) be a compact, strictly pseudoconvex, $2n+1$-dimensional CR manifold such that $\lambda_2(\theta) > 0$ and $\mu_1(M,\theta) < 0$ with $\mu_2(M,\theta) < \mu(\mathbf{S}^{2n+1})$. Suppose also that there is $B_0(M,\theta) > 0$ such that*

$$\mu(\mathbf{S}^{2n+1}) \leq \frac{\int_M (p\|\nabla_H u\|_\theta^2 + B_0(M,\theta)u^2)dV_\theta}{(\int_M u^p dV_\theta)^{\frac{2}{p}}} \tag{1}$$

with $u \in S_1^2(M)\backslash\{0\}$. Then there exists a non-negative function $u \in L^p(M)$ that we normalize by $\int_M u^p dV_\theta = 1$ and a function w which verifies in the sense of distribution the following equation

$$L_\theta w = \mu_2(M,\theta)|u|^{p-2}w \tag{2}$$

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EXISTÊNCIA DE SOLUÇÕES POSITIVAS PARA UMA CLASSE DE PROBLEMAS ELÍPTICOS QUASILINEARES COM CRESCIMENTO EXPONENCIAL EM DOMÍNIO LIMITADO.

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Abstract

Neste trabalho, estudamos resultados de existência de solução positiva para a seguinte classe de problemas elípticos :

$$-div(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) = f(u) \text{ em } \Omega, \quad u = 0 \text{ sobre } \partial\Omega,$$

onde Ω é um domínio limitado do $\mathbb{R}^N$ com $N \geq 3$ e $1 < p < N$. As hipóteses sobre a função a nos permitem estender o nosso resultado para uma grande classe de problemas e a função f possui crescimento crítico exponencial. As principais ferramentas utilizadas são Métodos Variacionais, Lema de Deformação e Desigualdade de Trundinger-Moser.

Palavras-chave: Crescimento crítico exponencial, Métodos Variacionais, Desigualdade de Trudinger-Moser.

1 Introdução

Neste trabalho estudamos existência de soluções positivas de energia mínima para o problema

$$(P_1) \qquad \begin{cases} -div(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) &= f(u) \text{ em } \Omega, \\ u &= 0, \text{ sobre } \partial\Omega, \end{cases}$$

onde $\Omega \subset \mathbb{R}^N$ é um domínio limitado e $1 < p < N$. As hipóteses sobre a função a são:

a_1) A função a é de classe C^1 e existem constantes k_1, k_3, $k_4 \geq 0$ e $k_2 > 0$ tais que

$$k_1 + k_2 t^{\frac{N-p}{p}} \leq a(t) \leq k_3 + k_4 t^{\frac{N-p}{p}}, \text{ para todo } t > 0.$$

a_2) As funções $t \mapsto a(t^p)t^p$, $\dfrac{1}{p}A(t^p) - \dfrac{1}{N}a(t^p)t^p$ são convexas em $(0, \infty)$, onde $A(t) = \displaystyle\int_0^t a(s)ds$.

a_3) A função $t \longmapsto \dfrac{a(t^p)}{t^{(N-p)}}$ é não crescente para todo $t > 0$.

E existe uma constante real $\gamma \geq \frac{N}{p}$ tal que

$$A(t) \geq \frac{1}{\gamma}a(t)t, \quad \text{para} \quad t \geq 0.$$

As hipóteses sobre a função $f : \mathbb{R} \longrightarrow \mathbb{R}$ contínua são:

f_1) Existe $\alpha_0 \geq 0$ tal que

$$\lim_{t\to+\infty} \frac{f(t)}{\exp(\alpha|t|^{\frac{N}{N-1}})} = 0 \text{ para } \alpha > \alpha_0 \text{ e } \lim_{t\to+\infty} \frac{f(t)}{\exp(\alpha|t|^{\frac{N}{N-1}})} = +\infty \text{ para } \alpha < \alpha_0;$$

f_2) A função f verifica o limite $\displaystyle\lim_{t\to 0^+} \frac{f(t)}{t^{p-1}} = 0$.

f_3) Existe $\theta > p\gamma$ tal que $0 < \theta F(t) \leq f(t)t, \quad \forall \quad t > 0$ em que γ é a mesma constante que aparece como consequência de a_3) e $F(s) = \displaystyle\int_0^s f(t)dt$.

f_4) A função $\dfrac{f(t)}{t^{(N-1)}}$ é crescente em $(0, \infty)$.

f_5) Existem $r > N$, $\tau > \tau^*$ e $\delta > 0$ tais que $f(t) \geq \tau t^{r-1}, \quad \forall \quad t \geq 0$, onde

$$\tau^* := \max\left\{1, \left[\frac{2^{N-1}\theta p\gamma c_r Nr(r-p)(\alpha_0+\delta)^{N-1}}{k_2(\theta - p\gamma)(r-N)pr\alpha_N^{N-1}}\right]^{\frac{r-p}{p}}\right\},$$

$$c_r = \inf_{\mathcal{N}_r} I_r,$$

$$I_r(u) = \frac{k_3}{p}\int_\Omega |\nabla u|^p dx + \frac{k_4}{N}\int_\Omega |\nabla u|^N dx - \frac{1}{r}\int_\Omega |u|^r dx$$

e

$$\mathcal{N}_r = \left\{u \in W_0^{1,N}(\Omega) \ \text{ e } \ u \neq 0 : I_r^{'}(u)u = 0\right\}.$$

2 Resultados Principais

Teorema 2.1. *(Subcrítico) Assumindo as condições* $(a_1) - (a_3)$, (f_1) *com* $\alpha_0 = 0$ *e* $(f_2) - (f_4)$, *o problema* (P_1) *tem solução positiva com energia mínima.*

Teorema 2.2. *(Crítico) Assumindo as condições* $(a_1) - (a_3)$, (f_1) *com* $\alpha_0 > 0$ *e* $(f_2) - (f_5)$, *o problema* (P_1) *tem solução positiva com energia mínima.*

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NONLOCAL SINGULAR ELLIPTIC SYSTEM ARISING FROM THE AMOEBA-BACTERIA POPULATION DYNAMICS

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Abstract

This talk is based on [2] where we prove the existence of coexistence states for a nonlocal singular elliptic system that arises from the interaction between amoeba and bacteria populations. For this, we use fixed point arguments and a version of the Bolzano's Theorem, for which we will first analyze a local system by bifurcation theory. Moreover, we study the behavior of the coexistence region obtained and we interpret our results according to the growth rate of both species.

1 Introduction

In this work, we deal with the existence of coexistence states of the following nonlocal singular elliptic system:

$$\begin{cases} -\Delta u = \lambda u - u^2 - buv & \text{in} \quad \Omega, \\ -\Delta v = \delta v \left(\dfrac{\int_\Omega u(x)v(x)\,dx}{\int_\Omega v(x)\,dx} \right) - \dfrac{\gamma uv}{1+v} & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\lambda, \delta, \gamma, b > 0$ and Ω is a bounded and regular domain of $\mathbb{R}^N$, $N \geq 1$. This system is the stationary counterpart of a reaction-diffusion-chemotaxis predator-prey mathematical model proposed in [3] to understand the interaction of two populations, one of amoebae and one of virulent bacteria. The main characteristic of the model is that predation of the amoeboid population on bacteria is governed by a nonlocal law through the integral term, this is due to fact that amoebae behave like a sole organism when food supply is low, in order to redistribute the food among all cells (see [3] and [5] for more details).

Observe that system (1) possesses a singular term, which makes our study even more complex. In fact, due to the presence of the singular term, we can not apply directly classical bifurcation results for systems, as in [4], for instance. Thus, to solve (1), we will follow the ideas contained in [1], which consist of transforming the nonlocal and singular system (1) into a local and nonsingular system. More precisely, note that to obtain a coexistence state (u, v) for (1) is equivalent to obtain the coexistence state (u, v) of the local system:

$$\begin{cases} -\Delta u = \lambda u - u^2 - buv & \text{in} \quad \Omega, \\ -\Delta v = \delta R v - \dfrac{\gamma uv}{1+v} & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

with

$$R = \frac{\int_\Omega u(x)v(x)\,dx}{\int_\Omega v(x)\,dx}.$$

Hence, by Bolzano's Theorem (see Section 3), it suffices to find a suitable continuum Σ_0 (*i.e.* a closed and connected subset) of coexistence states of (2) for which the function

$$h(R, u, v) = R - \frac{\int_\Omega u(x)v(x)\,dx}{\int_\Omega v(x)\,dx}.$$

is well defined, it is continuous and changes sign over Σ_0. We want to emphasize that the argument above requires the continuity of h just in Σ_0. This will be very important, because we can not define h, for example, over whole set $C_0(\overline{\Omega})$, once that h has a singularity. Thus, we will apply the classical results of bifurcation for systems (more precisely, the theory presented in [4]) to obtain a continuum of coexistence states of (2) for which the function h is well defined, it is continuous and changes sign over such continuum.

2 Main Results

For $m \in L^\infty(\Omega)$, we will denote by $\lambda_1(-\Delta + m(x))$ the principal eigenvalue of the problem:

$$\begin{cases} -\Delta u + m(x)u = \lambda u & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{1}$$

We obtain the following result for local system (2):

Theorem 2.1. *For each $\lambda > \lambda_1(-\Delta)$, there exists a point $(R_\lambda, u_\lambda, 0)$ such that from this point emanates a bounded continuum $\mathcal{C}^+$ of coexistence states of (2). Moreover, there exists at least one coexistence state of (2) if, and only if,*

$$\frac{\lambda_1(-\Delta)}{\delta} < R < \frac{\lambda_1(-\Delta + \gamma u_\lambda(x))}{\delta}.$$

With the help of this result, we can define h over whole continuum $\mathcal{C}^+$, prove that h is continuous and changes sign over $\mathcal{C}^+$. Consequently, using the Bolzano's Theorem, we show the following result for nonlocal system (1):

Theorem 2.2. *For each $\lambda > \lambda_1(-\Delta)$, there exists a point $F(\lambda) > 0$ such that, if*

$$\delta > F(\lambda),$$

then (1) has at least one coexistence state.

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POHOZAEV-TYPE IDENTITIES FOR A PSEUDO-RELATIVISTIC SCHRÖDINGER OPERATOR AND APPLICATIONS

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1 Introduction

We prove a Pohozaev-type identity for both the problem $(-\Delta+m^2)^s u = f(u)$ in $\mathbb{R}^N$ and its harmonic extension to $\mathbb{R}^{N+1}_+$ when $0<s<1$. So, our setting includes the pseudo-relativistic operator $\sqrt{-\Delta+m^2}$ and the results showed here are original, to the best of our knowledge. The identity is first obtained in the extension setting and then "translated" into the original problem. In order to do that, we develop a specific Fourier transform theory for the fractionary operator $(-\Delta+m^2)^s$, which lead us to define a weak solution u to the original problem if the identity

$$\int_{\mathbb{R}^N}(-\Delta+m^2)^{s/2}u(-\Delta+m^2)^{s/2}v\mathrm{d}x = \int_{\mathbb{R}^N} f(u)v\mathrm{d}x \tag{S}$$

is satisfied by all $v\in H^s(\mathbb{R}^N)$.

Comparison between the operators $(-\Delta)^s$ and $(-\Delta+m^2)^s$. At first sight, one supposes that the treatment of both operators might be similar. In fact, there are huge differences between them.

(a) $(-\Delta)^s$ satisfies $(-\Delta)^s u(\lambda x) = \lambda^{2s}(-\Delta)^s u(x)$, while such a property is not valid for $(-\Delta+m^2)^s$.

(b) As will see, $(-\Delta+m^2)^s$ generates a norm in $H^s(\mathbb{R}^N)$ and this is not the case for $(-\Delta)^s$. In consequence, the adequate spaces to handle both operators are quite different.

(c) Some results about fractionary Laplacian spaces are now standard, but not so easy to find for $(-\Delta+m^2)^s$.

Why to handle $(-\Delta+m^2)^s$ instead of $\sqrt{-\Delta+m^2}$.

In this paper we deal with a generalized version of the operator $\sqrt{-\Delta+m^2}$, namely the operator $T(u) = (-\Delta+m^2)^s u$, $0<s<1$. We study the problem

$$(-\Delta+m^2)^s u = f(u), \quad x\in\mathbb{R}^N. \tag{1}$$

2 Main Results

Theorem 2.1. *A solution $u\in H^s(\mathbb{R}^N)$ of problem (1) satisfies*

$$\frac{N-2s}{2}\int_{\mathbb{R}^N}\left|(m^2-\Delta)^{s/2}u(x)\right|^2\mathrm{d}x + sm^2\int_{\mathbb{R}^N}\frac{|\hat{u}(\xi)|^2\mathrm{d}\xi}{\left(m^2+4\pi^2|\xi|^2\right)^{1-s}} = N\int_{\mathbb{R}^N}F(u)\mathrm{d}x.$$

Theorem 2.2. *The problem*

$$(-\Delta+m^2)^s u = |u|^{p-2}u \quad \text{in} \quad \mathbb{R}^N$$

has no non-trivial solution if $p\geq 2^*_s$, where

$$2^*_s = \frac{2N}{N-2s}.$$

Theorem 2.3. *The problem*

$$(-\Delta + m^2)^s u = f(u) \quad in \quad \mathbb{R}^N, \tag{1}$$

when f satisfies

(f_1) *$f\colon \mathbb{R} \to \mathbb{R}$ is a C^1 function such that $f(t)/t$ is increasing if $t > 0$ and decreasing if $t < 0$;*

(f_2) $\displaystyle\lim_{t\to 0} \frac{f(t)}{t} = 0$ *and* $\displaystyle\lim_{t\to\infty} \frac{f(t)}{t} = k \in (m^{2s}, \infty]$*;*

(f_3) $\displaystyle\lim_{|t|\to\infty} tf(t) - 2F(t) = \infty$*, where* $F(t) = \int_0^t f(\tau)\mathrm{d}\tau$*,*

has a ground state solution $w \in H^s(\mathbb{R}^N)$.

Theorem 2.4. *Let $f\colon [0, \infty) \to \mathbb{R}$ be a continuous function that satisfies*

(s_1) *$f'(t) \geq 0$ and $f''(t) \geq 0$ for all $t \in [0, \infty)$.*

(s_2) *For any $\beta \in (1, 2_s^* - 1)$, there exists $q \in [2, 2_s^*]$ with $q > \max\{\beta, \frac{N(\beta-1)}{2s}\}$ such that $f'(w) \in L^{q/(\beta-1)}(\mathbb{R}^N)$, $\forall w \in H^s(\mathbb{R}^N)$.*

For any $0 < s < 1$, $N > 2s$ and $m \in \mathbb{R} \setminus \{0\}$, if $u(x)$ is a positive solution of

$$(-\Delta + m^2)^s u = f(u) \quad in \quad \mathbb{R}^N,$$

then u is radially symmetric and decreasing with respect to the origin.

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ON THE CRITICAL CASES OF LINEARLY COUPLED CHOQUARD SYSTEMS

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Abstract

We study the existence and nonexistence results for the linearly coupled Choquard system in critical cases.

$$-\Delta u + u = (I_\alpha * |u|^p)\,|u|^{p-2}u + \lambda v \;\; , \;\; -\Delta v + v = (I_\alpha * |v|^q)\,|v|^{q-2}v + \lambda u, \;\; x \in \mathbb{R}^N, \qquad (S_\alpha)$$

where $0 < \alpha < N$, $0 < \lambda < 1$,

1 Main Results

We consider existence and nonexistence of Ground State (GS) solutions for linearly Choquard coupled System (S_α) where $N \geq 3$, $\alpha \in (0, N)$, $\lambda \in (0,1)$ and $I_\alpha : \mathbb{R}^N \backslash \{0\} \to \mathbb{R}$ is the Riesz potential defined by

$$I_\alpha(x) := \mathcal{A}_\alpha / |x|^{N-\alpha}, \quad \text{where} \quad \mathcal{A}_\alpha := \Gamma\left((N-\alpha)/2\right) / \left[\Gamma\left(\alpha/2\right) \pi^{\frac{N}{2}} 2^\alpha\right]$$

where Γ is the Gamma function.

A *positive GS solution* is a solution such that $u > 0$ and $v > 0$ which has minimal energy among all nontrivial solutions. If $\lambda = 0$ and $p = q$, then System (S_α) reduces to the scalar Choquard equation

$$-\Delta u + u = (I_\alpha * |u|^p)\,|u|^{p-2}u, \quad x \in \mathbb{R}^N. \qquad (1)$$

Physical motivations arise from the case $N = 3$ and $\alpha = 2$. In 1954, Pekar[11] described a polaron at rest in the quantum theory. In 1976, to model an electron trapped in its own hole, Choquard[6] considered equation (1) as an approximation to Hartree-Fock theory of one-component plasma. In particular cases, Penrose [12] investigated the selfgravitational collapse of a quantum mechanical wave function. The system of weakly coupled equations has been widely considered in recent years and it has applications especially in nonlinear optics [9, 10]. Furthermore, nonlocal nonlinearities have attracted considerable interest as a means of eliminating collapse and stabilizing multidimensional solitary waves. It appears naturally in optical systems [8] and is known to influence the propagation of electromagnetic waves in plasmas [1]. In [1] is studied the semiclassical limit problem for the singularly perturbed Choquard equation in $\mathbb{R}^N$ and characterized the concentration behavior. In [4, 5], under a perturbation method and for a bounded domain Ω, it is established existence, multiplicity and nonexistence of solutions for following Brézis-Nirenberg type problem

$$-\Delta u = (I_\alpha * |u(y)|^{\frac{N+\alpha}{N}})|u|^{\alpha/N}u + \lambda u \;\text{ in } \Omega.$$

In order to use a variational approach, in the range $p, q \in \left[\frac{N+\alpha}{N}, \frac{N+\alpha}{N-2}\right]$, limited by the *lower and upper critical exponents*, $\frac{N+\alpha}{N}$ and $\frac{N+\alpha}{N-2}$, for the well defined even nonlocal terms

$$D_p^\alpha(u) := \int_{\mathbb{R}^N} (I_\alpha * |u|^p)\,|u|^p \,\mathrm{d}x \quad \text{and} \quad D_q^\alpha(v) := \int_{\mathbb{R}^N} (I_\alpha * |v|^q)\,|v|^q \,\mathrm{d}x,$$

we need to use the HLS inequality:

Theorem 1.1 (HLS inequality [7]). *Let $t, r > 1$ and $0 < \alpha < N$ with $\frac{1}{t} + \frac{1}{r} = 1 + \frac{\alpha}{N}$, $f \in L^t(\mathbb{R}^N)$ and $h \in L^r(\mathbb{R}^N)$. There exists a sharp constant $C(t, N, \alpha, r) > 0$, independent of f and h, such that*

$$\int_{\mathbb{R}^N} \int_{\mathbb{R}^N} [f(x)h(y)]/|x-y|^{N-\alpha} \, \mathrm{d}x\mathrm{d}y \leq C(t, N, \alpha, r)\|f\|_t \|h\|_r,$$

where $\|\cdot\|_s$ denotes the standard $L^s(\mathbb{R}^N)$-norm for $s \geq 1$.

In [3] the existence of GS solutions for (S_α) in the *subcritical case* is studied, precisely, when $p = q$ lies between the lower and upper critical exponents. Hence, a natural question arises: What occurs if $p \neq q$ lie in critical ranges? Motivated by this question, our goal is to establish existence and nonexistence of GS Solutions results for (S_α) in all critical cases. We establish our results under the cases

$$\text{“half-critical” case 1,} \quad (N+\alpha)/N < p < (N+\alpha)/(N-2) \quad \text{and} \quad q = (N+\alpha)/(N-2), \tag{HC1}$$

$$\text{“half-critical” case 2,} \quad p = (N+\alpha)/N \quad \text{and} \quad (N+\alpha)/N < q < (N+\alpha)/(N-2), \tag{HC2}$$

$$\text{“doubly critical” case,} \quad p = (N+\alpha)/N \quad \text{and} \quad q = (N+\alpha)/(N-2), \tag{DC}$$

$$\text{“inferior or superior supercritical” cases,} \quad p, q \leq (N+\alpha)/N \quad \text{or} \quad p, q \geq (N+\alpha)/(N-2). \tag{SC}$$

Theorem 1.2 (Existence). *If p, q satisfy (2.1), (5) or (6), then there exists $\alpha_0 > 0$ such that System (S_α) has at least one positive radial GS solution for $\alpha_0 < \alpha < N$.*

Theorem 1.3 (Nonexistence). *If p, q satisfy (7), then System (S_α) has no nontrivial solution.*

Remark 1.1. *It is usual introduce a parameter on critical nonlinearities in order to overcome the “lack of compactness”. However, we handle with this by using the behavior of the I_α when α is close to N.*

Remark 1.2. *For $\lambda > 0$, $u \neq 0$ and $v \neq 0$, the System (S_α) does not admit semitrivial solutions $(u, 0)$ and $(0, v)$.*

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EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR A SINGULAR $P\&Q$-LAPLACIAN PROBLEM VIA SUB-SUPERSOLUTION METHOD

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Abstract

In this work we show existence and multiplicity of positive solutions using the sub-supersolution method in a general singular elliptic problem which the operator is not homogeneous neither linear. More precisely, using the sub-supersolution method, we study this general class of problem

$$-\operatorname{div}(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) = h(x)u^{-\gamma} + f(x,u),\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega, \tag{1}$$

where $\gamma > 0$, Ω is a bounded domain in $\mathbb{R}^N$, $N \geq 3$, a, h and f are functions that the hypotheses we give later and $1 < p < N$.

1 Introduction

Consider the semilinear problem given by

$$-\Delta u = m(x,u),\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega. \tag{2}$$

The classical method of sub-supersolution asserts that if we can find sub-supersolution $v_1, v_2 \in H_0^1(\Omega)$ with $v_1(x) \leq v_2(x)$ a.e in Ω, then there exists a solution $v \in H_0^1(\Omega)$ such that $v_1(x) \leq v(x) \leq v_2(x)$ a.e in Ω. In general, a candidate to subsolution of problem (2) is given by $v_1 = \epsilon\phi_1$, where ϕ_1 is a eigenfunction associated with λ_1, the first eigenvalue of operator $(-\Delta, H_0^1(\Omega))$. A candidate to supersolution, in general, is the unique positive solution of the problem $-\Delta u = M$, $u > 0$ in Ω, $u = 0$ on $\partial\Omega$. The sizes of ϵ and the constant M together with Comparison Principle to operator $(-\Delta, H_0^1(\Omega))$ allow to show that the sub-supersolution are ordered. If the operator is not linear and nonhomogeneous, in general we do not have eigenvalues and eigenfunctions. However, we show in this work that the sub-supersolution method still can be applied.

The hypotheses on the C^1-function $a : \mathbb{R}^+ \to \mathbb{R}^+$, the nontrivial mensurable function $h \geq 0$ and the Caracthéodory function f are the following:

(h) There exists $0 < \phi_0 \in C_0^1(\overline{\Omega})$ such that $h\phi_0^{-\gamma} \in L^\infty(\Omega)$.

(f_1) There exists $0 < \delta < \frac{1}{2}$ such that $-h(x) \leq f(x,t) \leq 0$, for every $0 \leq t \leq \delta$, a.e in Ω.

(f_2) There exists $q < r < q^* = \frac{Nq}{(N-q)}$ ($q^* = \infty$ if $q \geq N$) such that

$$f(x,t) \leq h(x)(t^{r-1} + 1), \text{ for every } t \geq 0, \text{ a.e in } \Omega.$$

(f_3) There exists $t_0 > 0$ such that $0 < \theta F(x,t) \leq tf(x,t)$, for every $t \geq t_0$, a.e in Ω, where θ appeared in (a_4).

(a_1) There exist constants k_1, k_2, k_3, $k_4 > 0$ and $1 < p < q < N$ such that

$$k_1 t^p + k_2 t^q \leq a(t^p)t^p \leq k_3 t^p + k_4 t^q, \text{ for all } t \geq 0.$$

(a_2) The function $t \longmapsto A(t^p)$ is strictly convex and the function $t \longmapsto a(t^p)t^{p-2}$ is increasing.

(a_4) There exist constants μ and θ such that $\theta \in (q, q^*)$ and $\frac{1}{\mu}a(t)t \leq A(t) = \int_0^t a(s)\,ds$, for all $t \geq 0$, with $1 < \frac{q}{p} \leq \mu < \frac{\theta}{p}$.

2 Main Results

Theorem 2.1. *Assume that conditions* (h), (f_1) *and* $(a_1) - (a_2)$ *hold. If* $\|h\|_\infty$ *is small, then problem (1) has a weak solution.*

Proof Firstly, we use [2], [1, Lemma 2.1 and Lemma 2.2] to show that $0 < \underline{u}(x) \leq \overline{u}(x)$ a.e in Ω, where $\underline{u}$ is a subsolution and $\overline{u}$ is a supersolution for (1). Then, considering the function

$$g(x,t) = \begin{cases} h(x)\overline{u}(x)^{-\gamma} + f(x, \overline{u}(x)), & t > \overline{u}(x) \\ h(x)t^{-\gamma} + f(x,t), & \underline{u}(x) \leq t \leq \overline{u}(x) \\ h(x)\underline{u}(x)^{-\gamma} + f(x, \underline{u}(x)), & t < \underline{u}(x) \end{cases}$$

and the auxiliary problem $-\mathrm{div}(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) = g(x,u)$ $u > 0$ in Ω, $u = 0$ on $\partial\Omega$, we obtain that the functional $\Phi : W_0^{1,q}(\Omega) \to \mathbb{R}$ associated with auxiliary problem is bounded from below in $M = \{u \in W_0^{1,q}(\Omega); \underline{u} \leq u \leq \overline{u}$ a.e *in* $\Omega\}$ and attains its infimum at a point $u \in M$. So, u is a weak solution of auxiliary auxiliary problem and hence, since $g(x,t) = h(x)t^{-\gamma} + f(x,t)$, for every $t \in [\underline{u}, \overline{u}]$, problem (1) has a positive weak solution.

Theorem 2.2. *Assume that conditions* (h), $(f_1) - (f_3)$ *and* $(a_1) - (a_4)$ *hold. If* $\|h\|_\infty$ *is small, then problem (1) has two weak solutions.*

Proof Now, consider the auxiliary problem $-\mathrm{div}(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) = \widehat{g}(x,u)$ in Ω, $u = 0$ on $\partial\Omega$, where

$$\widehat{g}(x,t) = \begin{cases} h(x)t^{-\gamma} + f(x,t), & t \geq \underline{u}(x), \\ h(x)\underline{u}(x)^{-\gamma} + f(x, \underline{u}(x)), & t < \underline{u}(x). \end{cases} \tag{1}$$

Note that $g(x,t) = \widehat{g}(x,t)$, for all $t \in [0, \overline{u}]$, then $\Phi(u) = \widehat{\Phi}(u)$, for all $u \in [0, \overline{u}]$. Therefore, $\widehat{\Phi}(w) = \inf_M \Phi$, where M is given in the proof of Theorem 2.1 and w is a weak solution of (1). Thus, there exists a local minimizer $w \in B_R(0)$ such that $\widehat{\Phi}(w) \leq \inf_{u \in B_R(0)} \hat{\Phi}(u) \leq \widehat{\Phi}(\underline{u}) \leq \alpha$. Furthermore, by the Mountain Pass Theorem, there exists $v \in W_0^{1,q}(\Omega)$ such that $\beta \leq \widehat{\Phi}(v) = c$, where c is the minimax value of $\widehat{\Phi}$. So, the auxiliary problem has two positive weak solutions $w, v \in W_0^{1,q}(\Omega)$ such that $\widehat{\Phi}(w) \leq \widehat{\Phi}(\underline{u}) \leq \alpha < \beta \leq \widehat{\Phi}(v) = c$.

Finally, since $\underline{u} \leq v$ it follows from (1) that $\widehat{g}(x,v) = h(x)v^{-\gamma} + f(x,v)$ in Ω, which implies that $v, w \in W_0^{1,q}(\Omega)$ are two weak solutions for problem (1).

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XIII ENAMA - Novembro 2019 *59–60*

EXISTENCE OF SOLUTIONS FOR A NONLOCAL EQUATION IN $\mathbb{R}^2$ INVOLVING UNBOUNDED OR DECAYING RADIAL POTENTIALS

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Abstract

In this work, we study the following class of nonlinear equations:

$$-\Delta u+V(x)u=\left[|x|^{-\mu}*(Q(x)F(u))\right]Q(x)f(u),\quad x\in\mathbb{R}^2,\tag{1}$$

where V and Q are continuous, unbounded or decaying to zero radial potentials in $\mathbb{R}^2$, $f(s)$ is a continuous function, $F(s)$ is the primitive of $f(s)$, $*$ is the convolution operator and $0<\mu<2$. Assuming that the nonlinearity $f(s)$ has exponential critical growth in the sense of Trudinger-Moser, we establish the existence of solutions by using variational methods.

1 Introduction

The study of Eq. (1) is in part motivated by works concerning the equation

$$-\Delta u+V(x)u=\left(|x|^{-\mu}*|u|^p\right)|u|^{p-2}u,\quad x\in\mathbb{R}^N,\tag{2}$$

where $N\geq 3$, $0<\mu<N$, $V:\mathbb{R}^N\to\mathbb{R}$ is a continuous potential function, and $u:\mathbb{R}^N\to\mathbb{R}$, which is is generally named as *Choquard equation* or *Hartree type equation* and appear in various physical contexts. For example, in the case $N=3$, $V(x)=1$, $p=2$ and $\mu=2$, the Eq. (2) first appeared in the seminal work by S. I. Pekar [4] describing the quantum mechanics of a polaron at rest. As mentioned by Lieb [3], in 1976 and under same case, Ph. Choquard used Eq. (2) to model an electron trapped in its own hole, as a certain approximation to Hartree-Fock theory of one-component plasma. The nonlocal Choquard type equation with critical exponential growth in the planar case was first considered in [1, 2]. In these works, the authors considered the existence of nontrivial ground state solution for the following critical nonlocal equation with periodic potential $-\Delta u+W(x)u=(|x|^{-\mu}*F(u))f(u)$, $x\in\mathbb{R}^2$. Under a set of assumptions on potential W and nonlinear term f, they obtained the existence of nontrivial ground state solution in $H^1(\mathbb{R}^2)$. The goal of the present work is to continue the study of the critical *nonlocal* equation, that is, it is not a pointwise identity with the appearance of the term $|x|^{-\mu}*(Q(x)F(u))$, when the nonlinear term f behaves at infinity like $e^{\alpha s^2}$ for some $\alpha>0$.

In this work, we impose the following hypotheses on the potential V and the weight Q:

$(V0)$ $V\in C(0,\infty)$, $V(r)>0$ and there exists $a_0>-2$ and $a>-2$ such that

$$\limsup_{r\to 0^+}\frac{V(r)}{r^{a_0}}<\infty\quad\text{and}\quad\liminf_{r\to+\infty}\frac{V(r)}{r^{a}}>0;$$

$(Q0)$ $Q\in C(0,\infty)$, $Q(r)>0$ and there exist $b_0>-\frac{4-\mu}{2}$ and $b<\frac{a(4-\mu)}{4}$ such that

$$\limsup_{r\to 0^+}\frac{Q(r)}{r^{b_0}}<\infty\quad\text{and}\quad\limsup_{r\to+\infty}\frac{Q(r)}{r^{b}}<\infty.$$

Hereafter, we say that $(V,Q) \in \mathcal{K}$ if $(V0)$ and $(Q0)$ hold. The following hypotheses on $f(s)$ will be imposed:

(f_1) $f : \mathbb{R}_+ \to \mathbb{R}$ is continuous and $\lim_{s\to 0^+} f(s)/s^{\frac{2-\mu}{2}} = 0$;

(f_2) there exists $\theta > 1$ such that $\theta \int_0^s f(t)\,\mathrm{d}t = \theta F(s) \leq f(s)s, \ \forall s \geq 0$;

(f_3) there exist $q > 1$ and $\xi > 0$ such that $F(s) \geq \xi s^q$ for all $s \in [0,1]$.

In order to state our main results, we need to introduce some notations. We define the functional space $Y := \left\{u \in L^1_{\mathrm{loc}}(\mathbb{R}^2) : |\nabla u| \in L^2(\mathbb{R}^2) \text{ and } \int_{\mathbb{R}^2} V(|x|)u^2\,\mathrm{d}x < \infty\right\}$ endowed with the norm $\|u\| := \sqrt{\langle u,u\rangle}$ induced by the scalar product $\langle u,v\rangle := \int_{\mathbb{R}^2} (\nabla u \cdot \nabla v + V(|x|)uv)\,\mathrm{d}x$, which we prove that is a Hilbert space. Furthermore, the subspace $Y_{\mathrm{rad}} := \{u \in Y : u \text{ is radial}\}$ is closed in Y and thus it is a Hilbert space itself.

2 Main Results

Let $C_0^\infty(\mathbb{R}^2)$ be the set of smooth functions with compact support. We say that $u : \mathbb{R}^2 \to \mathbb{R}$ is a weak solution for (1) if $u \in Y$ and it holds the equality $\int_{\mathbb{R}^2} (\nabla u \cdot \nabla \phi + V(|x|)u\phi)\,\mathrm{d}x - \int_{\mathbb{R}^2} \left[|x|^{-\mu} * (Q(|x|)F(u))\right] Q(|x|)f(u)\phi\,\mathrm{d}x = 0$, for all $\phi \in C_0^\infty(\mathbb{R}^2)$. Our main results read as follow.

Theorem 2.1. *Assume that $0 < \mu < 2$ and $(V,Q) \in \mathcal{K}$. If $f(s)$ has exponential critical growth and satisfies $(f_1) - (f_3)$ with $\xi > 0$, given in (f_3), verifying* $\xi \geq \max\left\{\xi_1, \left[\dfrac{\frac{\|Q\|^2_{L^1(B_{1/2})}}{2}(q-1)\left(\frac{\xi_1^2}{q}\right)^{q/(q-1)}}{\frac{4-\mu}{\alpha_0}\left(1+\frac{2b_0}{4-\mu}\right)\frac{\pi(\theta-1)}{2\theta}}\right]^{(q-1)/2}\right\}$, *where* $\xi_1 := \dfrac{\left[\pi + \|V\|_{L^1(B_1)}\right]^{\frac{1}{2}}}{\|Q\|_{L^1(B_{1/2})}}$, *then Eq. (1) has a nontrivial weak solution in Y_{rad}.*

Theorem 2.2. *Under the conditions of Theorem 2.1 and supposing that $f(s)/s$ is increasing for $s > 0$, then the solution obtained in Theorem 2.1 is a ground state.*

For our second existence result, we replace condition (f_3) by the following conditions:

(f_4) there exist $s_0 > 0$, $M_0 > 0$ and $\vartheta \in (0,1]$ such that $0 < s^\vartheta F(s) \leq M_0 f(s), \ \forall s \geq s_0$;

(f_5) $\liminf\limits_{s\to+\infty} \dfrac{F(s)}{e^{\alpha_0 s^2}} =: \beta_0 > 0.$

Theorem 2.3. *Assume that $0 < \mu < 2$, $(V,Q) \in \mathcal{K}$, $f(s)$ has exponential critical growth and satisfies (f_1), (f_2), (f_4) and (f_5). If we also assume that $\liminf\limits_{r\to 0^+} Q(r)/r^{b_0} > 0$, then Eq. (1) has a nontrivial weak solution in Y_{rad}.*

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UM SISTEMA NÃO LINEAR EM ÁGUAS RASAS 1D

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Abstract

Soluções de um de sistema algébrico não linear resultante da simplificação do modelo de águas rasas com orografia são obtidas. Análise de propriedades das soluções em função de contantes que ocorrem nas equações também são realizadas. Este trabalho está associado a pesquisas de modelos simplificados de Águas Rasas com orografia que por sua vez têm utilidade na pequisa de métodos numéricos precisos e eficientes em previsão numérica de tempo.

1 Introdução

A partir das equações de águas rasas em 2 dimensões espaciais ([1]),

$$\begin{aligned}\frac{du}{dt} - f_0 v + \frac{\partial \phi}{\partial x} + \frac{\partial \phi^S}{\partial x} &= 0,\\ \frac{dv}{dt} + f_0 u + \frac{\partial \phi}{\partial y} + \frac{\partial \phi^S}{\partial y} &= 0,\\ \frac{d\phi}{dt} + \frac{(\partial \phi u)}{\partial x} + \frac{(\partial \phi v)}{\partial y} &= 0, \end{aligned} \tag{1}$$

em que (u, v) representa o vetor velocidade horizontal, $f_0 = 2\Omega \sin\theta_0$ a força de Coriolis na latitude θ_0, ϕ^S o geopotencial da superfície da terra, ϕ a diferença entre o geopotencial da superfície livre e o da superfície da terra, e $\frac{d.}{dt} = \frac{\partial .}{\partial t} + u\frac{\partial .}{\partial x} + v\frac{\partial .}{\partial y}$ a derivada material, com algumas condições podemos obter o seguinte modelo simplificado de águas rasas em uma dimensão espacial ([2],[3]):

$$u_t + uu_x - f_0 v + \phi_x + \phi^S_x = 0, \tag{2}$$

$$v_t + uv_x + f_0 u = f_0 U, \tag{3}$$

$$\phi_t + (\phi u)_x = 0 \quad \text{em } (t, x) \in (0, +\infty) \times [0, L]. \tag{4}$$

Se assumirmos que a segunda equação do sistema acima seja satisfeita, por exemplo pela introdução de um gradiente adequado de pressão, e procurando soluções estacionárias do mesmo, obtemos o seguinte sistema não-linear algébrico nas incógnitas u e ϕ para cada x fixado ([4],[2]):

$$\begin{aligned} u^2/2 + \phi + A &= \frac{B^2}{2} + C,\\ \phi u &= BC. \end{aligned} \tag{5}$$

sendo $A = \phi_S(x)$, $B = u(0)$, $C = \phi(0)$ e $\phi_S(0) = 0$, com A, B e C constantes e incógnitas que satisfazem as seguintes condições:

$$u \geq 0, \quad \phi > 0, \quad A \text{ e } B \geq 0, \quad C > 0. \tag{6}$$

Neste trabalho estabelecemos condições para a **existência e unicidade** do sistema (5) e condições (6) acima, o qual é importânte no estudo de modelos simplificados de águas rasas.

2 Resultados Principais

Teorema 2.1. *Sejam* $u, \phi \in R^+ \quad C > A \geq 0 \quad e \quad k = \frac{B^2}{2} + C - A$

1. *Se* $B = 0$ *então o sistema* (5)-(6) *possui uma única solução dada por* $u = 0, \quad \phi = C - A$.

2. *Se* $B > 0$ *e* $k \leq 0$ *então o sistema* (5)-(6) *não possui solução.*

3. *Se* $B > 0$ *e* $k > 0$ *então uma condição suficiente e necessária para o sistema* (5)-(6) *possuir solução é que*

$$k \geq \frac{3}{2}(BC)^{2/3}. \tag{1}$$

Teorema 2.2. *Sejam as hipóteses do caso 3) do teorema* (2.1) *acima.*
Se ocorrer a igualdade na condição (1) *então a solução do sistema* (5)-(6) *é única, dada por*

$$(u, \phi) = \left(\sqrt[3]{BC}, \sqrt[3]{(BC)^2}\right).$$

Caso contrário, ocorrer a desiguadade estrita em (1), *então existem duas soluções.*

Teorema 2.3. *Sejam as condições do teorema* (2.2) *no caso de haver duas soluções do sistema* (5)-(6).

1. *Se* $B < \sqrt{C}$ *então as soluções* u *satisfazem*

$$B < u < \frac{-B + \sqrt{B^2 + 8C}}{2}$$

2. *Se* $B > \sqrt{C}$ *então as soluções* u *satisfazem*

$$\frac{-B + \sqrt{B^2 + 8C}}{2} < u < B$$

3. *Se* $B = \sqrt{C}$ *então* $A = 0$ *e as soluções serão*

$$u = B \quad e \quad u = \frac{1}{2}(-B + \sqrt{B^2 + 8C})$$

Teorema 2.4. *O maior valor possivel da constante* A *para que o sistema* (5)-(6) *possua solução ocorre justamente quando a solução é única, ou seja, quando ocorre a igualdade na condição* (1) *acima.*

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HOMOGENIZATION OF THE LARGE DEVIATIONS REGIME FOR VISCOUS CONSERVATION LAWS

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Abstract

The Feynman-Kac formula is a classic subject that brings together Probability theory and Partial Differential Equations. In a nutshell a Feynman-Kac formula is a way of expressing the solution of the heat equation in terms of an average of a functional of the Brownian motion, a stochastic process that is closely linked to the Laplace operator. This type of formulas extends to a much wider class of probabilistic objects (in general Markovian) that are connected to certain but generic differential operators. It is our intention to show the natural link between a class of stochastic differential equations called forward-backward stochastic differential equations (FBSDEs for short) and the associated terminal value problems for certain quasilinear evolution PDEs. Within this connection we formulate a multi-scale system of stochastic differential equations related with a class of quasilinear viscous conservation laws and we study the homogenization problem in the small noise regime under this association. The work that we present generalizes to the qasilinear case the results obtained [2] for homogenization problem under the small noise regime that is not treated there.

1 Introduction

Lévy flights is a popular term in Physics for random walks in which the step lenghts U have a heavy-tailed distribution, i.e. $\mathbb{P}(U > u) = O(u^{-\alpha})$ for some $\alpha \in (1, 2)$. They are appropriate models that capture non Gaussian effects and where diffusive behavior is not adequate. Their use is well-known in climate modeling, animal hunting patterns and in the modeling of molecular gases in non-homogeneous media. Let us fix a terminal time $T > 0$. If we consider a system of particles whose motion is governed by *Lévy flights* and perform the hydrodynamic limit, in the presence of some additional assumptions, we end up with the so-called fractal Burgers Equations,

$$\begin{cases} \partial_t v^\nu(t,x) = -\nu(-\Delta)^{\frac{\alpha}{2}} v^\nu(t,x) - \langle v^\nu(t,x), \nabla_x v^\nu(t,x)\rangle + F^\nu(t,x) = 0, \\ v^\nu(0,x) = g(x), \quad t \in [0,T], \quad x \in \mathbb{R}^d. \end{cases}$$

The solution v^ν of the fractal Burgers equations models the velocity of a compressible fluid with nonlocal viscosity parameter $\nu > 0$ that shows a fractional (nonlocal) diffusive behavior captured by the presence of the fractional Laplacian $(-\Delta)^{\frac{\alpha}{2}}$, $\alpha \in (0, 2)$, and affected by a force F^ν that captures local and non-local sources of interaction depending eventually on the velocity of the fluid itself. We stress that this semilinear term F^ν is not stochastic. The initial condition g is the initial configuration of the velocity field in all space $\mathbb{R}^d$. The fractional Laplacian is an integral-differential operator defined by

$$(-\Delta)^{\frac{\alpha}{2}} f(x) = c_{d,\alpha} \lim_{\varepsilon \to 0} \int_{|y-x|>\varepsilon} \frac{|f(x) - f(y)|}{|x-y|^{d+\alpha}} dy,$$

for all the measurable functions f whenever the limit above exists and is well-defined. The constant $c_{d,\alpha}$ is defined by

$$c_{d,\alpha} := \frac{\alpha\Gamma\Big(\frac{d+\alpha}{2}\Big)}{2^{1-\alpha}\pi^{d-2}\Gamma\Big(1-\frac{\alpha}{2}\Big)},$$

where Γ is *Euler's Gamma function*.

The presence of $(-\Delta)^{\frac{\alpha}{2}}$ in the structure of the equations is not surprising since, via the *Kolmogorov functional limit theorem*, the distance from the origin of the *Lévy flights* converges, after a large number of steps, to an α-stable law and $(-\Delta)^{\frac{\alpha}{2}}$ is the infinitesimal generator of an α-stable process.

We do not enter in details for the functional study of this operator and refer the reader to [5]. The fractal Burgers equations form an example of a system of partial-integral differential equations (PIDEs for short). PIDEs are a preeminent topic of active research in mathematics with the growing demand of the use of differential equations that take into account nonlocal effects of interaction and non-isotropic propagation of energy. Fractal Burgers equations increased interest in models involving fractional dissipation, in particular in Navier-Stokes equations, combustion models and the surface geostrophic equation. These equations have been studied in [10]. In [12] the author studies probabilistically the fractal Navier Stokes equation which turns as an example in favor of probabilistic approaches to the study of nonlocal hydrodynamic models, as was made before to the Navier Stokes systems. We refer the reader to [5] and [6] as examples of probabilistic studies of Navier-Stokes equations. We will associate a certain class of partial-integral differential equations, including the fractal Burgers equation, with a certain system of stochastic differential equations and via this probabilistic object we will address the problem of the vanishing viscosity limit $\nu \to 0$ linked to a related mean field game.

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GLOBAL SOLUTIONS FOR A STRONGLY COUPLED FRACTIONAL REACTION-DIFFUSION SYSTEM

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Abstract

We study the well-posedness of the initial value problem for a strongly coupled fractional reaction-diffusion system in Marcinkiewicz spaces $L^{(p_1,\infty)}(\mathbb{R}^n) \times L^{(p_2,\infty)}(\mathbb{R}^n)$. The exponents p_1, p_2 of the initial value space are chosen to allow the existence of self-similar solutions. The result strongly depends on a fractional version of the Yamazaki's estimate [3].

1 Introduction

Here, we are interested in the following Cauchy problem

$$\begin{cases} u_t = \partial_t \int_0^t g_\alpha(t-s)\Delta u(s) + g_1(u,v), & x \in \mathbb{R}^n,\ t > 0, \\ v_t = \partial_t \int_0^t g_\alpha(t-s)\Delta v(s) + g_2(u,v), & x \in \mathbb{R}^n,\ t > 0, \\ u(0,x) = u_0, \quad v(0,x) = v_0, & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $g_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$, $0 < \alpha \leq 1$, and

$$g_1(u,v) = |u|^{(\rho_1-1)}u|v|^{(\rho_2-1)}v \text{ and } g_2(u,v) = |u|^{(r_1-1)}u|v|^{(r_2-1)}v, \tag{2}$$

for $1 < \rho_i, r_i < \infty$, $i = 1, 2$. We study the well-posedness of (1) in Marcinkiewicz spaces $L^{(p_1,\infty)}(\mathbb{R}^n) \times L^{(p_2,\infty)}(\mathbb{R}^n)$. The exponents p_1, p_2 of the initial value space are chosen to allow the existence of self-similar solutions.

The system (1) has the following scalling: $(u,v) \to (u_\lambda, v_\lambda)$ where

$$u_\lambda(t,x) = \lambda^{k_1} u(\lambda^2 t, \lambda x) \text{ and } v_\lambda(t,x) = \lambda^{k_2} v(\lambda^2 t, \lambda x)$$

and

$$k_1 = \frac{1}{\alpha}\frac{2(\rho_2 - r_2 + 1)}{r_1\rho_2 - (\rho_1 - 1)(r_2 - 1)} \quad \text{and} \quad k_2 = \frac{1}{\alpha}\frac{2(r_1 - \rho_1 + 1)}{r_1\rho_2 - (\rho_1 - 1)(r_2 - 1)}, \tag{3}$$

provided that

$$r_1\rho_2 - (\rho_1 - 1)(r_2 - 1) \neq 0. \tag{4}$$

Definition 1.1. *Let k_i be given by (3), and $p_i = \frac{n}{k_i} > 1$. We define the following Banach space*

$$E \equiv BC((0,\infty), L^{(p_1,\infty)} \times L^{(p_2,\infty)}\}$$

with respective norm given by

$$\|(u,v)\|_E = \max\left\{\sup_{t>0} \|u\|_{(p_1,\infty)}, \sup_{t>0} \|v\|_{(p_2,\infty)}\right\}. \tag{5}$$

Next, according to Duhamel's principle, we introduce the notion of a solution we use here for the initial value problem (1).

Definition 1.2. *A global mild solution of the initial value problem (1) in E is a pair $(u(t), v(t))$ satisfying*

$$(u(t), v(t)) = (G_\alpha(t)u_0, G_\alpha(t)v_0) + B(u, v)(t). \tag{6}$$

In (6),

$$B(u, v)(t) = \left(\int_0^t G_\alpha(t-s)|u|^{\rho_1 - 1}u|v|^{\rho_2 - 1}v ds \ , \ \int_0^t G_\alpha(t-s)|u|^{r_1 - 1}u|v|^{r_2 - 1}v ds \right)$$

and

$$G_\alpha(x, t) = \int_0^\infty M_\alpha(\eta) G(x, \eta t^\alpha) d\eta, \tag{7}$$

where $G(x, t)$ is given by $G(x, t) = (4\pi t)^{-\frac{N}{2}} e^{-\frac{|x|^2}{4t}}$, and M_α is a Wright-type function.

The case $\alpha = 1$ was studied by Ferreira and Mateus [2], whereas fractional reaction–diffusion equations with power-type nonlinearities have been studied recently in [1, 4], where local and global well-posedness is addressed, as well as the nonexistence of global bounded positive solutions and existence of self-similar solutions. Therefore, the results we present here generalizes the global existence results in both [2] and [1].

2 Main Results

Next, we state the most important results fo the work.

Lemma 2.1 (Fractional Yamazaki's estimate)**.** *Let $1 < p < q < \infty$ be such that $\frac{n}{2}\left(\frac{1}{p} - \frac{1}{q}\right) < 1$. There exists a constant $C > 0$ such that,*

$$\int_0^\infty t^{\frac{n\alpha}{2}(\frac{1}{p} - \frac{1}{q}) - 1} \|G_\alpha(t)\phi\|_{(q,1)} \, ds \leq C\|\phi\|_{(p,1)} \tag{1}$$

for all $\phi \in L^{(p,1)}(\mathbb{R}^n)$.

Theorem 2.1. *Let $n > \frac{2}{\alpha}$, $1 < r_i, \rho_i < p_i < \infty$ and $p_i \geq \frac{n\alpha}{n\alpha - 2}$, $i = 1, 2$. Assume that $(u_0, v_0) \in L^{(p_1,\infty)} \times L^{(p_2,\infty)}$.There exist $\varepsilon > 0$ and $\delta = \delta(\varepsilon) > 0$ such that if $\|u_0\|_{(p_1,\infty)} < \delta, \|v_0\|_{(p_2,\infty)} < \delta$, then the initial value problem (1) has a global mild solution $(u(t, x), v(t, x)) \in E$, with initial data (u_0, v_0), which is the unique one satisfying $\|(u, v)\|_E \leq 2\varepsilon$*

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GRADIENT FLOW APPROACH TO THE FRACTIONAL POROUS MEDIUM EQUATION IN A PERIODIC SETTING

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Abstract

We consider a fractional porous medium equation that extends the classical porous medium and fractional heat equations. The flow is studied in the space of periodic probability measures endowed with a non-local transportation distance constructed in the spirit of the Benamou-Brenier formula. For initial periodic probability measures, we show the existence of absolutely continuous curves that are generalized minimizing movements associated to Rényi entropy.

1 Introduction

We consider a fractional porous medium equation

$$\begin{cases} \partial_t \rho + (-\Delta)^\sigma \rho^m = 0, & (x,t) \in \mathbb{T}^d \times (0,\infty) \\ \rho(0,x) = \rho_0(x), & x \in \mathbb{T}^d \end{cases} . \tag{1}$$

where $d \geq 1$, $0 < \sigma < 1$, $0 < m \leqslant 2$ and $\mathbb{T}^d$ is the d-dimensional torus.

Due to the conservation of mass and positiveness for solutions, we can formally consider the solution $\rho(t,x)$ of (1) as a curve $t \mapsto \rho(t,.) \in \mathcal{P}(\mathbb{T}^d)$ in the set of probability measures on the d-dimensional torus. This curve satisfies a gradient flow problem of the type

$$\dot{\rho} = -\nabla_{\mathcal{W}} \mathcal{U}_m(\rho) \tag{2}$$

where $\nabla_{\mathcal{W}}$ is a gradient induced by a metric $\mathcal{W}$ defined on $\mathcal{P}(\mathbb{T}^d)$ and $\mathcal{U}_m$ Rényi entropy

$$\mathcal{U}_m(\rho) = \frac{1}{m-1} \int_{\mathbb{T}^d} \rho^m(x)\, dx \ , \quad \text{for} \quad m \neq 1 \quad \text{and} \quad \mathcal{U}_1(\rho) = \int_{\mathbb{T}^d} \rho(x) \log \rho(x)\, dx.$$

In this work, we study the problem (1) by moving in the opposite direction of the above arguments. We defined on $\mathcal{P}(\mathbb{T}^d)$ a pseudo-metric $\mathcal{W}$ that incorporates the fractional nonlocal character of the operator $(-\Delta)^\sigma$ and use it to construct a solution to the gradient flow equation. This is done using a steepest descent minimizing movement described in the next section.

2 Main Results

For a fixed initial periodic probability measure $\rho_0 \in \mathcal{P}(\mathbb{T}^d)$ and a $\tau > 0$ we define the functional

$$\mu \in \mathcal{P}(\mathbb{T}^d) \mapsto \Phi(\tau, \rho_0; \mu) := \frac{1}{2\tau} \mathcal{W}^2(\rho_0, \mu) + \mathcal{U}_m(\mu) \tag{3}$$

The next result show a coerciveness property for the entropy functional $\mathcal{U}_m$:

Theorem 2.1. *For any $\tau > 0$ and $\rho_0 \in \mathcal{P}(\mathbb{T}^d)$, the function $\Phi(\tau, \mu_*; \,.\,)$ is bounded from below in $\mathcal{P}(\mathbb{T}^d)$. Moreover, there exists a unique $\rho^* \in \mathcal{P}(\mathbb{T}^d)$ (depending on τ and ρ_0) such that*

$$\Phi(\tau, \rho_0; \rho^*) \leqslant \Phi(\tau, \rho_0; \mu) \ , \ \ \forall\, \mu \in \mathcal{P}(\mathbb{T}^d).$$

We may inductively apply the previous result to define the following sequence: given a initial periodic probability measure ρ_0 and a $\tau > 0$, let $(\rho_\tau^n)_n$ the sequence given by

$$\begin{cases} \rho_\tau^0 := \rho_0 \\ \rho_\tau^n := \operatorname{argmin}\left\{ \Phi(\tau, \rho_0^{n-1}; \mu) \middle|\ \mu \in \mathcal{P}(\mathbb{T}^d) \right\} \ , \quad \forall n \in \mathbb{N}. \end{cases}$$

Now let $\rho_\tau : [0, \infty) \to \mathcal{P}(\mathbb{T}^d)$ the piecewise constant curve given by

$$\rho_\tau(t) := \rho_\tau^n \quad \text{for} \quad t \in [n\tau, (n+1)\tau) \ , \quad \text{and} \ \ n \in \mathbb{N} \cup \{0\}. \tag{4}$$

The main result of this work is the following:

Theorem 2.2. *Given $\rho_0 \in \mathcal{P}(\mathbb{T}^d)$ such that $\mathcal{U}_m(\rho_0) < \infty$ we can define the net of piecewise constant curves $(\rho_\tau)_{\tau>0} \subseteq \mathcal{P}(\mathbb{T}^d)$. Then, there exists a curve $\rho \in AC_{\mathrm{loc}}([0, \infty), \mathcal{P}(\mathbb{T}^d))$ such that (up to a subsequence)*

$$\rho_\tau(t) \rightharpoonup \rho(t), \quad \text{as} \ \ \tau \to 0 \ \ \forall\, t \geqslant 0.$$

Furthermore, the curve ρ satisfies the gradient flow equation (2).

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A ULTRA-SLOW REACTION-DIFFUSION EQUATION

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Abstract

We present results concerning the existence and uniqueness of solutions for a reaction-diffusion ultra-slow equation. We also show that they can be extended up a maximal time and are stable as long as they exist, and we give conditions to obtain symmetric and positive solutions. These results are published in the paper [9].

1 Introduction

Define the distributed-order fractional derivative $\mathbb{D}^{(\mu)}$ by

$$\mathbb{D}^{(\mu)}\varphi(t) = \int_0^t k(t-\tau)\varphi'(\tau)d\tau,$$

where

$$k(s) = \int_0^1 \frac{s^{-\alpha}}{\Gamma(1-\alpha)}\mu(\alpha)d\alpha. \tag{1}$$

Several diffusion equations may appear in the form of the distributed-order fractional diffusion equation

$$\mathbb{D}_t^{(\mu)}u = \Delta u, \quad (t,x) \in (0,T) \times \mathbb{R}^N. \tag{2}$$

See e.g [1, 2, 3, 4, 5, 7]. Kochubei [5] called (2) the ultra-slow diffusion equation and gave an adequate physical interpretation and has done a detailed mathematical analysis of the fundamental solution of this equation under the initial condition

$$u(0,x) = u_0(x),$$

for $x \in \mathbb{R}^N$, provided that it is Hölder continuous. Nevertheless, even though semilinear problems are of great interest in evolution equations, we can cite only a few papers where (3) is perturbed by f depending on u, see e.g. [6, 8].

Therefore, we are motivated to study the local well-posedness theory for the nonlinear distributed-order fractional diffusion equation

$$\begin{cases} \mathbb{D}_t^{(\mu)}u = \Delta u + f(u), & (t,x) \in (0,T) \times \mathbb{R}^N \\ u(0,x) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{3}$$

where, $\rho > 1$, Δ denotes the Laplace operator and the initial data are in $L^\infty(\mathbb{R}^N)$. Also, we shall consider $\mu \in C^3[0,1]$, $\mu(1) \neq 0$, with either $\mu(\alpha) = a\alpha^\nu$, for some $\nu > 0$, or $\mu(0) \neq 0$, in a such way that it complies some key lemmas from [5]. The nonlinearity f we consider behaves like $f(u) = |u|^{\rho-1}u$. Besides the local well-posedness, we prove the existence of the maximal solution and its blow-up alternative, which can be also useful to prove global existence. A stability result is also established. Some additional qualitative aspects of the solutions are also studied, namely, we show the existence of symmetric and positive solutions.

2 Main Results

Theorem 2.1. *If $v_0 \in L^\infty(\mathbb{R}^N)$, there exists $0 < T < \delta$ and $r > 0$ such that, for each $u_0 \in B_{L^\infty}(v_0, r)$ there exists a unique local mild solution $u : [0,T] \to L^\infty(\mathbb{R}^N)$ for the Cauchy problem (3) and*

$$\|u(t,\cdot) - u_0\|_{L^\infty} \to 0$$

as $t \to 0^+$. Furthermore, for any $u_0, w_0 \in B_{L^\infty}(v_0, r)$, there exists $M > 0$ such that

$$\|u - w\|_X \leq M\|u_0 - w_0\|_{L^\infty}, \tag{1}$$

where u and w are the solutions starting at u_0 and w_0, respectively. The solution found can be uniquely continued up a maximal time $T_{max} > 0$ and, if $T_{max} < \infty$, it satisfies

$$\limsup_{t \to T_{max}^-} \|u(t,\cdot)\|_{L^\infty} = \infty. \tag{2}$$

Moreover, if u and w are the maximal solutions of (3) starting at u_0 and w_0, respectively. Then, for each $\bar{T} \in (T, \min\{T_{max}(u_0), T_{max}(w_0)\})$, there exists $K(\bar{T}) = K$ such that

$$\|u(t,\cdot) - w(t,\cdot)\|_{L^\infty} \leq K\|u_0 - w_0\|_{L^\infty}, \tag{3}$$

for every $t \in [0, \bar{T}]$.

Theorem 2.2. *Let the hypotheses of Theorem 2.1 be satisfied . The solution $u(t,\cdot)$ is symmetric for all $t > 0$, whenever u_0 is symmetric under the action of $\mathcal{A}$. In particular, if u_0 is a radial function, then the solution $u(t,\cdot)$ is also a radial function, for all $t \in [0, T_{max})$. If in addition u_0 is a non-negative function that is not identically null, then the solution $u(t,\cdot)$ is positive for all $t \in [0, T_{max})$.*

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LIMITES POLINOMIAIS PARA O CRESCIMENTO DAS NORMAS DA SOLUÇÃO DA EQUAÇÃO DE KLEIN-GORDON SEMILINEAR EM ESPAÇOS DE SOBOLEV

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Abstract

Neste trabalho consideramos a equação de Klein Gordon semilinear em uma variedade Riemanniana M de dimensão três com ou sem bordo, e analisamos o comportamento das normas $H^{m+1}(M) \times H^m(M)$, $m \in \mathbb{N}$, da solução desta equação. A partir de um argumento de indução, combinado com as estimativas de Strichartz provamos que estas normas podem ser limitadas por funções polinomiais.

1 Introdução

Consideramos, neste trabalho, o seguinte modelo

$$\partial_t^2 u - \Delta_g u + \beta u + \gamma(x)\partial_t u + f(u) = 0 \quad \text{em } \mathbb{R}_+ \times M, \tag{1}$$

$$u = 0 \quad \text{sobre } \mathbb{R}_+ \times \partial M, \quad \text{se } \partial M \neq \emptyset, \tag{2}$$

$$u(0) = u_0, \quad \partial_t u(0) = u_1, \tag{3}$$

onde (M, g) é uma variedade Riemanniana compacta de dimensão 3 com fronteira ∂M e Δ_g é o operador de Laplace-Beltrami com a condição de Dirichlet no bordo, se $\partial M \neq \emptyset$. Estamos interessados em estimar a taxa de crescimento da norma da solução $(u, \partial_t u)$ de $(1) - (3)$ em espaços de Sobolev $H^{m+1}(M) \times H^m(M)$, $m \in \mathbb{N}$.

Assumimos $\beta > 0$ tal que, para alguma constante $C > 0$ seja válida a desigualdade de Poincaré $\int_M |\nabla_g u|^2 dx + \beta \int_M |u|^2 dx \geq C \int_M |u|^2 dx$, onde $dx = dvol_g$ é o elemento de volume induzido por g. Em particular $\beta > 0$ se $\partial M = \emptyset$. Assumimos a não linearidade f sendo suficientemente regular e tal que existe uma constante $C > 0$ para a qual é válido, para todo $s \in \mathbb{R}$,

$$f(0) = 0, \quad sf(s) \geq 0, \quad |f(s)| \leq C(1 + |s|)^p, \quad |f'(s)| \leq C(1 + |s|)^{p-1} \tag{4}$$

com $1 \leq p < 5$. Consideramos $\gamma \in C^\infty(M)$ uma função a valores reais não negativa.

Nas condições acima, dado $(u_0, u_1) \in H_0^1(M) \times L^2(M)$, existe uma única solução $u \in C(\mathbb{R}_+; H_0^1(M)) \cap C^1(\mathbb{R}_+; L^2(M))$ para o problema $(1) - (3)$. Além disso, o funcional de energia definido por

$$E(t) = \frac{1}{2}\left(\|\partial_t u(t)\|^2_{L^2(M)} + \|\nabla_g u(t)\|_{L^2(M)} + \beta\|u(t)\|^2_{L^2(M)}\right) + \int_M V(u)(x,t)dx, \quad t \in \mathbb{R}, \tag{5}$$

com $V(u) = \int_0^u f(s)ds$, é bem definido, tendo em vista a imersão de Sobolev $H_0^1(M) \hookrightarrow L^6(M)$. Veja que, o funcional (5) é decrescente, desta forma, devido a hipótese (4) temos que, para todo $t \in \mathbb{R}_+$, $\|(u, \partial_t u)(t)\|^2_{H_0^1(M)\times L^2(M)} \leq CE(t) \leq CE(0)$, com $C > 0$ uma constante. Isto é, a norma $H_0^1(M) \times L^2(M)$ de $(u, \partial_t u)$ é uniformemente limitada, com respeito a $t \in \mathbb{R}_+$. Nos propomos a responder a questão: Quais estimativas podemos obter acerca das normas de $(u, \partial_t u)$ nos espaços $H^{m+1}(M) \times H^m(M)$?

O problema de descrever o crescimento das normas da solução de uma EDP em espaço de Sobolev de ordem alta possui grande interesse físico, uma vez que descreve a velocidade em que o sistema considerado transfere energia de baixas frequências para altas frequências.

2 Resultado Principal

Teorema 2.1. *Seja $m \in \mathbb{N}$, $\gamma \in C^\infty(M)$ uma função não negativa, $f \in C^\infty(M)$ satisfazendo (4) com $1 \leq p < 5$ e $(u, \partial_t u) \in C(\mathbb{R}_+; H^{m+1}(M) \times H^m(M))$ a solução de (1) − (3). Então existe uma constante $C > 0$ tal que,*

$$\sup_{t\in[0,T]} \|(u, \partial_t u)(t)\|_{H^2(M)\times H^1(M)} \leq C(1+T)^{\frac{4}{5-p}} \tag{1}$$

e, se $m > 1$,

$$\sup_{t\in[0,T]} \|(u, \partial_t u)(t)\|_{H^{m+1}(M)\times H^m(M)} \leq C(1+T) \tag{2}$$

com $C = C(\|u_0\|_{H^1(M)}, \|u_1\|_{L^2(M)}, f, \|\gamma\|_{W^{m,\infty}(M)}, M) > 0$.

Proof. (Ideia) Inspirados no trabalho pioneiro [2] de Bourgain almejamos provar

$$\|(u, \partial_t u)(t)\|^2_{H^{m+1}(M)\times H^m(M)} \leq C\|(u, \partial_t u)(0)\|^2_{H^{m+1}(M)\times H^m(M)} + \|(u, \partial_t u)(0)\|^{2-\delta}_{H^{m+1}(M)\times H^m(M)} \tag{3}$$

para todo $t \in (0, T)$, onde $T \in (0, 1)$ é convenientemente escolhido, $m = 0, 1, 2, ...$ e $\delta > 0$ dependendo de m. A desigualdade (1) nos leva então a (2.2) e (2.3). A prova de (1) é baseada em um argumento de indução sobre $m \in \mathbb{N}$ combinado com as estimativas de Strichartz provadas em [1] e com uma adaptação do método desenvolvido em [3]. □

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FLUIDOS MICROPOLARES COM CONVECÇÃO TÉRMICA: ESTIMATIVAS DE ERRO PARA O MÉTODO DE GALERKIN

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Abstract

Consideramos o problema que descreve o movimento de um fluido micropolar, viscoso e incompressível com convecção térmica em um domínio limitado $\Omega \subsetneq \mathbb{R}^3$. Estamos interessados nas estimativas de erro no tempo das aproximações de Galerkin.

1 Introdução

No presente trabalho, aplicamos o método de *Galerkin espectral* ao sistema (1)-(2) que descreve o movimento de um fluido micropolar com convecção térmica, a fim de estimar o erro por potências do inverso dos autovalores $\lambda_{k+1}, \gamma_{k+1}$ e $\widetilde{\gamma}_{k+1}$ dos operadores de Stokes, Laplace e Lamé, respectivamente, considerando-se aproximações nos subespaços V_k, H_k e $\widetilde{H}_k$. Estas estimativas de erro para o método de Galerkin são importantes pela ampla aplicação de tais métodos em experimentos numéricos. Em 1980, Rautmann [2] sistematizou as estimativas de erro para o método de Galerkin espectral aplicado às equações de Navier-Stokes clássicas. O caso para fluidos magneto-micropolares foi tratado por Ortega-Torres, Rojas-Medar e Cabrales em [1]. Inspirados nestas ideias, obtemos, no Teorema 2.1, estimativas na norma $L^2(\Omega)$ para o erro que se comete ao aproximar (u, w, θ) por (u^k, w^k, θ^k), suas respectivas aproximações de Galerkin. No Teorema 2.2, fizemos o mesmo para u e w na norma $H^1(\Omega)$. Por fim, tratamos de outras normas no Teorema 2.3. O sistema que estudamos na região $Q_T \equiv \Omega \times (0, T)$ é o seguinte:

$$\left\{\begin{aligned} u_t - (\mu+\mu_r)\Delta u + (u\cdot\nabla)u + \nabla p &= 2\mu_r \text{rot } w + f(\theta), \\ w_t - (c_a+c_d)\Delta w + (u\cdot\nabla)w + 4\mu_r w &= -(c_0+c_d-c_a)\nabla \text{div } w + 2\mu_r \text{rot } u + g(\theta), \\ \theta_t + u\cdot\nabla\theta - \kappa\Delta\theta &= \Phi(u,w) + h, \\ \text{div } u &= 0, \end{aligned}\right. \tag{1}$$

junto com as seguintes condições de fronteira e iniciais

$$\left\{\begin{array}{l} u = 0, \quad w = 0, \quad \theta = 0 \text{ em } S_T, \\ u(\cdot,0) = u_0, \ w(\cdot,0) = w_0, \ \theta(\cdot,0) = \theta_0, \ \text{em } \Omega, \end{array}\right. \tag{2}$$

onde $S_T \equiv \partial\Omega \times (0, T)$. As funções vetoriais $u = (u_1, u_2, u_3)$ e $w = (w_1, w_2, w_3)$ e as funções escalares p e θ são as incógnitas, e denotam, respectivamente, a velocidade linear, a velocidade angular de rotação das partículas, a pressão e a temperatura do fluido. Por outro lado, as funções vetoriais f e g, e a função escalar h são conhecidas e denotam, respectivamente, as fontes externas de momento linear, angular e a entrada de calor. As constantes positivas μ, μ_r, c_a, c_d e c_0 são coeficientes dos tipo viscosidade satisfazendo a seguinte desigualdade $c_0 + c_d > c_a$ e a constante positiva κ é a condutividade de calor. A função real Φ é dada por $\Phi := \sum_{i=1}^{5} \Phi_i$, onde

$$\Phi_1(u) := \frac{\mu}{2}\sum_{i,j=1}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2, \quad \Phi_2(u,w) := 4\mu_r\left(\frac{1}{2}\text{rot } u - w\right)^2,$$

$$\Phi_3(w) := c_0(\text{div } w)^2, \quad \Phi_4(w) := (c_a + c_d)\sum_{i,j=1}^{3}\left(\frac{\partial w_i}{\partial x_j}\right)^2, \quad \Phi_5(w) := (c_d - c_a)\sum_{i,j=1}^{3}\frac{\partial w_i}{\partial x_j}\frac{\partial w_j}{\partial x_i}.$$

Além disso, supomos que as funções f, g e h satisfazem

$$|f(s) - f(t)| \le M_f|t - s|, \qquad |g(s) - g(t)| \le M_g|t - s|, \tag{3}$$

para $s, t \in \mathbb{R}$ e constantes $M_f, M_g > 0,\ f(0) = g(0) = 0$ e $h \in L^2\left(0, T; L^2(\Omega)\right)$.

2 Resultados Principais

Proposição 2.1. *Suponha que f, g, f_t e g_t verificam a condição (3), para constantes positivas M_f, M_g, M_{f_t} e M_{g_t} respectivamente. Existe $T_2 > 0$ e uma única solução do problema* (1)-(2) *no intervalo* $[0, T_2]$. *Ademais,*

$$u \in L^\infty(0, T_2; D(A)),\ w \in L^\infty(0, T_2; D(B)),\ \theta \in L^\infty(0, T_2; H_0^1(\Omega)).$$

O mesmo resultado vale para a solução (u^k, w^k, θ^k) do sistema com as aproximações de Galerkin.

Em nosso primeiro resultado estabelecemos a estimativa na norma $L^2(\Omega)$ do erro da aproximação de Galerkin.

Teorema 2.1. *Sob as hipóteses da Proposição 2.1, as aproximações (u^k, w^k, θ^k) satisfazem*

$$\|u(t) - u^k(t)\|^2 + \|w(t) - w^k(t)\|^2 + \|\theta(t) - \theta^k(t)\|^2 \le \frac{C}{\lambda_{k+1}^2} + \frac{C}{\gamma_{k+1}^2} + \frac{C}{\tilde{\gamma}_{k+1}^2} + \frac{C}{\lambda_{k+1}} + \frac{C}{\gamma_{k+1}}, \tag{1}$$

para todo $t \ge 0$ e $C > 0$ uma constante genérica que não depende de $k \in \mathbb{N}$.

De maneira análoga estabelecemos para u e w na norma $H^1(\Omega)$ o seguinte

Teorema 2.2. *Sob as hipóteses da Proposição 2.1, as aproximações (u^k, w^k) satisfazem*

$$\|\nabla\big(u - u^k\big)(t)\|^2 + \|L^{1/2}\big(w - w^k\big)(t)\|^2 \le \frac{C}{\gamma_{k+1}} + \frac{C}{\lambda_{k+1}} + \frac{C}{\tilde{\gamma}_{k+1}}, \quad \forall\, t \ge 0,$$

onde $C > 0$ é uma constante independente de k.

Obtemos também

Teorema 2.3. *Sob as hipóteses da proposição 2.1, as aproximações (u^k, w^k, θ^k) satisfazem*

$$\left\|u_t(t) - u_t^k(t)\right\|_{V^*}^2 + \left\|w_t(t) - w_t^k(t)\right\|_{H^{-1}}^2 + \int_0^t \left\|\theta_t(\tau) - \theta_t^k(\tau)\right\|_{H^{-1}}^2 \mathrm{d}\tau \le \frac{C}{\gamma_{k+1}} + \frac{C}{\lambda_{k+1}} + \frac{C}{\tilde{\gamma}_{k+1}}, \quad \forall\, t \ge 0,$$

onde $C > 0$ é uma constante que não depende de k.

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ON THE SOLUTIONS FOR THE EXTENSIBLE BEAM EQUATION WITH INTERNAL DAMPING AND SOURCE TERMS

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Abstract

In this manuscript, we consider the nonlinear beam equation with internal damping and source term

$$u_{tt} + \Delta^2 u + M(|\nabla u|^2)(-\Delta u) + u_t = |u|^{r-1}u$$

where $r > 1$ is a constant, $M(s)$ is a continuous function on $[0, +\infty)$. The global solutions are constructed by using the Faedo-Galerkin approximations, taking into account that the initial data is in appropriate set of stability created from the Nehari manifold. The asymptotic behavior is obtained by the Nakao method.

1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$. In this paper, we study the existence and the energy decay estimate of global solutions for the initial boundary value problem of the following equation with internal damping and source terms

$$u_{tt} + \Delta^2 u + M(|\nabla u|^2)(-\Delta u) + u_t = |u|^{r-1}u \quad \text{in} \quad \Omega \times (0, T), \tag{1}$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega, \tag{2}$$

$$u(x, t) = \frac{\partial u}{\partial \eta}(x, t) = 0, \quad x \in \partial\Omega, t \geq 0, \tag{3}$$

where $r > 1$ is a constant, $M(s)$ is a continuous function on $[0, +\infty)$. In (3), $u = 0$ is the homogeneous Dirichlet boundary condition and the normal derivative $\partial u/\partial\eta = 0$ is the homogeneous Neumann boundary condition where η is the unit outward normal on $\partial\Omega$. The equation (1) without source terms was studied by several authors in different contexts. In this work we use the potential well theory.

2 The Potencial Well

It is well-known that the energy of a PDE system, in some sense, splits into the kinetic and the potential energy. By following the idea of Y. Ye [2], we are able to construct a set of stability. We will prove that there is a valley or a well of the depth d created in the potential energy. If d is strictly positive, then we find that, for solutions with the initial data in the good part of the potential well, the potential energy of the solution can never escape the potential well. In general, it is possible that the energy from the source term to cause the blow-up in a finite time. However, in the good part of the potential well, it remains bounded. As a result, the total energy of the solution remains finite on any time interval [0; T), providing the global existence of the solution.

3 Existence of Global Solutions

We consider the following hypothesis

$$(H) \quad M \in C([0,\infty]) \text{ with } M(\lambda) \geq -\beta, \forall\ \lambda \geq 0, 0 < \beta < \lambda_1,$$
$$\lambda_1 \text{ is the first eigenvalue of the problem } \Delta^2 u - \lambda(-\Delta u) = 0.$$

Remark 3.1. *Let λ_1 the first eigenvalue of $\Delta^2 u - \lambda(-\Delta u) = 0$ then (see Miklin [1])*

$$\lambda_1 = \inf_{u \in H_0^2(\Omega)} \frac{|\Delta u|^2}{|\nabla u|^2} > 0 \quad \textit{and} \quad |\nabla u|^2 \leq \frac{1}{\lambda_1}|\Delta u|^2.$$

Theorem 3.1. *Let us take $u_0 \in W_1$, $E(0) < d$, $u_1 \in L^2(\Omega)$, $1 < r \leq 5$ and let suppose the hyphotesis (H) holds then there exists a function $u : [0,T] \to L^2(\Omega)$ in the class*

$$u \in L^\infty(0,T; H_0^2(\Omega)) \cap L^\infty(0,T; L^{r+1}(\Omega)) \tag{4}$$
$$u_t \in [L^\infty(0,T; L^2(\Omega)) \tag{5}$$
$$\tag{6}$$

such that, for all $w \in H_0^2(\Omega)$

$$\frac{\mathrm{d}}{\mathrm{d}t}(u_t(t), w) + \langle \Delta u(t), \Delta w \rangle + M(|\nabla u|^2)(-\Delta u, w) + (u_t(t), w) - (|u(t)|^{r-1}u(t), w) = 0,$$
$$u(0) = u_0,\ u_t(0) = u_1,$$

in $\mathcal{D}'(0,T)$

Proof. We use the Faedo-Galerkin's method and potencial well to prove the global existence of solutions. □

4 Asymptotic Behavior

Theorem 4.1. *Under the hypotheses of Theorem 3.1, the solution of problem (1)-(3) satisfies:*

$$\frac{1}{2}|u_t(t)|^2 + \frac{1}{2}\left(1 - \frac{\beta}{\lambda_1}\right)|\Delta u(t)|^2 - \frac{1}{r+1}|u(t)|_{r+1}^{r+1} + \int_t^{t+1} |u_t(s)|^2 \,\mathrm{d}\,s \leq Ce^{-\alpha t},$$

$\forall\ t \geq 0$, *where C and α are positive constants.*

Proof. See [3]. □

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STRONG SOLUTIONS FOR THE NONHOMOGENEOUS MHD EQUATIONS IN THIN DOMAINS

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Abstract

We prove the global existence of strong solutions to the nonhomogeneous incompressible Magnetohydrodynamic equations in a thin domain $\Omega \subsetneq \mathbb{R}^3$.

1 Introduction

The governing equations of nonhomogeneous incompressible MHD are (see [2])

$$\begin{cases} \rho \boldsymbol{u}_t + \rho(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \mu\Delta\boldsymbol{u} + \nabla\left(P + \frac{1}{2}|\boldsymbol{b}|^2\right) = (\boldsymbol{b}\cdot\nabla)\boldsymbol{b}, \\ \boldsymbol{b}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{b} - \eta\Delta\boldsymbol{b} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{u}, \\ \rho_t + \boldsymbol{u}\cdot\nabla\rho = 0, \\ \operatorname{div}\boldsymbol{u} = \operatorname{div}\boldsymbol{b} = 0. \end{cases} \tag{1}$$

These equations are considered in the set $\Omega \times (0,T)$, where $\Omega \stackrel{\text{def}}{=} \mathbb{R}^2 \times (0,\epsilon)$. Here, $\epsilon \in (0,1]$ is a parameter and $T > 0$. In system (1), the unknowns are $\rho(\boldsymbol{x},t) \in \mathbb{R}^+$, $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^3$, $P(\boldsymbol{x},t) \in \mathbb{R}$ and $\boldsymbol{b}(\boldsymbol{x},t) \in \mathbb{R}^3$. They represent, respectively, the density, the incompressible velocity field, the hydrostatic pressure and the magnetic field of the fluid as functions of the position $\boldsymbol{x} \in \Omega$ and of the time $t \geq 0$. The function $|\boldsymbol{b}|^2/2$ is the magnetic pressure. So, we denote by $p \stackrel{\text{def}}{=} P + \frac{1}{2}|\boldsymbol{b}|^2$ the total pressure of the fluid. The positive constants μ and η represent, respectively, the viscosity and the resistivity coefficient which is inversely proportional to the electrical conductivity constant and acts as the magnetic diffusivity of magnetic field. We supplement the system (1) with given initial conditions

$$\rho(\boldsymbol{x},0) = \rho_0(\boldsymbol{x}), \quad \boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x}) \quad \text{and} \quad \boldsymbol{b}(\boldsymbol{x},0) = \boldsymbol{b}_0(\boldsymbol{x}) \quad \text{in } \Omega, \tag{2}$$

and homogeneous Dirichlet boundary conditions

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{0}, \quad \boldsymbol{b}(\boldsymbol{x},t) = \boldsymbol{0} \quad \text{on } \partial\Omega \times (0,\infty), \tag{3}$$

where $\partial\Omega = \left\{(x_1,x_2,x_3) \,/\, (x_1,x_2) \in \mathbb{R}^2, x_3 = 0 \text{ or } x_3 = \epsilon\right\}$.

2 Main Result

From now on, we denote by $\boldsymbol{V}$ the closure of $\boldsymbol{\mathcal{V}}(\Omega) \stackrel{\text{def}}{=} \{\boldsymbol{v} \in \boldsymbol{C}_0^\infty(\Omega)\,;\, \operatorname{div}\boldsymbol{v} = 0 \text{ in } \Omega\}$ in $\boldsymbol{H}_0^1(\Omega)$. Our main result is the following [1]

Theorem 2.1. *Assume that the initial data ρ_0, $\boldsymbol{u}_0$ and $\boldsymbol{b}_0$ satisfy*

$$0 < \alpha \leq \rho_0(x) \leq \beta < \infty \ \text{in } \Omega, \ \text{with } \alpha,\, \beta \in \mathbb{R}^+,$$

$$\boldsymbol{u}_0,\, \boldsymbol{b}_0 \in \boldsymbol{V},$$

$$\epsilon^{\frac{1}{2}}\left(\|\nabla\boldsymbol{u}_0\|_{\boldsymbol{L}^2(\Omega)} + \|\nabla\boldsymbol{b}_0\|_{\boldsymbol{L}^2(\Omega)}\right) \leq c_0,$$

for some positive constant c_0 *small enough depending solely on* α *and* β. *Then the problem* (1)–(3) *has a unique global in time strong solution* $(\rho, \boldsymbol{u}, p, \boldsymbol{b})$ *such that, for any* $T > 0$,

$$
\begin{aligned}
&\rho(\boldsymbol{x}, t) \in [\alpha, \beta] \text{ a.e. } t \in [0, T],\ \boldsymbol{x} \in \Omega,\\
&\|(\boldsymbol{u}, \boldsymbol{b})\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} + \|(\sqrt{\mu}\,\nabla \boldsymbol{u}, \sqrt{\eta}\,\nabla \boldsymbol{b})\|^2_{L^2(0,T;\boldsymbol{L}^2(\Omega))} \leq C\left(\|\boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right),\\
&\|(\nabla \boldsymbol{u}, \nabla \boldsymbol{b})\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} + \|(\boldsymbol{u}_t, \boldsymbol{b}_t, \Delta \boldsymbol{u}, \Delta \boldsymbol{b})\|^2_{L^2(0,T;\boldsymbol{L}^2(\Omega))} \leq M\left(\|\nabla \boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\nabla \boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right),\\
&\|(\boldsymbol{u}, \boldsymbol{b})\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} \leq C\left(\|\boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right) e^{-\gamma\, T/\epsilon^2},\\
&\|(\nabla \boldsymbol{u}, \nabla \boldsymbol{b})\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} \leq M\left(\|\nabla \boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\nabla \boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right) e^{-\overline{\gamma}\, T/\epsilon^2},
\end{aligned}
$$

where $C = C(\alpha, \beta) > 0$, $M = M(\alpha, \beta, \mu, \eta, c_0) > 0$, $\gamma \overset{def}{=} \min\{\mu/\beta, \eta\}$ *and* $\overline{\gamma} \overset{def}{=} \min\left\{\frac{\widetilde{\gamma}}{\mu}, \frac{\widetilde{\gamma}}{\eta}\right\}$, *with* $\widetilde{\gamma} \overset{def}{=} \min\left\{\frac{\mu^2}{16\beta}, \frac{\eta^2}{16}\right\}$. *Furthermore, if* $\boldsymbol{u}_0, \boldsymbol{b}_0 \in \boldsymbol{V} \cap \boldsymbol{H}^2(\Omega)$, *then*

$$
\begin{aligned}
&\|(\boldsymbol{u}_t, \boldsymbol{b}_t, \nabla^2 \boldsymbol{u}, \nabla^2 \boldsymbol{b}, \nabla p)\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} + \|(\nabla \boldsymbol{u}_t, \nabla \boldsymbol{b}_t)\|^2_{L^2(0,T;\boldsymbol{L}^2(\Omega))} \leq M\left(\|\nabla^2 \boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\nabla^2 \boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right),\\
&\|(\nabla^2 \boldsymbol{u}, \nabla^2 \boldsymbol{b})\|^2_{L^\infty(0,T;\boldsymbol{L}^2(\Omega))} \leq M\left(\|\nabla^2 \boldsymbol{u}_0\|^2_{\boldsymbol{L}^2(\Omega)} + \|\nabla^2 \boldsymbol{b}_0\|^2_{\boldsymbol{L}^2(\Omega)}\right) e^{-\sigma\, T/\epsilon^2},
\end{aligned}
$$

where $\sigma \overset{def}{=} \min\{\gamma, \overline{\gamma}\}$. *In particular, for any* $t_* \in (0, \infty)$, *one concludes that*

$$
\lim_{\epsilon \to 0^+} (\boldsymbol{u}, \boldsymbol{b}) = (\boldsymbol{0}, \boldsymbol{0}) \text{ uniformly in } C\left([t_*, \infty); \boldsymbol{H}^2(\Omega)\right).
$$

Remark 2.1. *The global existence for strong solutions of the nonhomogeneous Navier-Stokes equations in a thin 3D domain was studied by Xian Liao in the paper [3].*

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ON A VARIATIONAL INEQUALITY FOR A PLATE EQUATION WITH P-LAPLACIAN END MEMORY TERMS

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Abstract

In this paper we investigate the unilateral problem for a plate equation with memory terms and lower order perturbation of p-Laplacian type $u'' + \Delta^2 u - \Delta_p u + \int_0^t g(t-s)\Delta u(s)ds + \Delta u' + f(u) = 0$ in $\Omega \times \mathbb{R}^+$, where Ω is a bounded domain of $\mathbb{R}$, $g > 0$ is a memory kernel and $f(u)$ is a nonlinear perturbation. Making use of the penalty method an Faedo-Galerkin's approximation, we establish our result on existence and uniqueness of strong solutions.

1 Introduction

In [1] the authors establish existence of global solution to the problem

$$u''+\Delta^2 u - \Delta_p u + \int_0^t g(t-s)\Delta u(s)ds - \Delta u' + f(u) = 0, u = \Delta u = 0 \text{ on } \Sigma \times \mathbb{R}^+, u(.,0) = u_0, u'(.,0) = u_1 \text{ in } \Omega, \quad (1)$$

where Ω is a bounded domain of $\mathbb{R}^n$ and $\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian operator.

Problem (1), with its memory term $\int_0^t g(t-s)\Delta u(s)ds$, can be regarded as a fourth order viscoelastic plate equation with a lower order perturbation of the p-Laplacian type. It can be also regarded as an elastoplastic flow equation with some kind of memory effect.

We observe that for viscoelastic plate equation, it is usual consider a memory of the form $\int_0^t g(t-s)\Delta^2 u(s)ds$ (e. g. [2, 3]). However, because the main dissipation of the system (1) is given by strong damping $-\Delta' u$, here we consider a weaker memory, acting only on Δu. There is a large literature about stability in viscoelasticity. We refer the reader to, for example [4, 5].

A nonlinear perturbation of problem (1) is given by $u'' + \Delta^2 u - \Delta_p u + \int_0^t g(t-s)\Delta u(s)ds - \Delta u' + f(u) \geq 0$. In the present work we investigated the unilateral problem associated with this perturbation, (see [10]). Making use of the penalty method and Galerkin's approximations, we establish existence and uniqueness of strong solutions.

Unilateral problem is very interesting too, because in general, dynamic contact problems are characterized by nonlinear hyperbolic variational inequalities. For contact problem on elasticity and finite element method see Kikuchi-Oden [6] and reference there in. For contact problems on viscoelastic materials see [3]. For contact problems on Klein-Gordon operator see [7]. For contact problems on Oldroyd Model of Viscoelastic fluids see [9]. For contact problems on Navier-stokes Operator with variable viscosity see [8].

2 Main Results

Theorem 2.1. *Consider space* $H^3_\Gamma(\Omega) = \{u \in H^3(\Omega) | u = \Delta u = 0 \text{ on } \Gamma\}$. *If* $(u_0, u_1) \in H^2(\Omega) \cap H^1_0(\Omega) \cap H^3_\Gamma(\Omega) \times H^1_0(\Omega)$ *holds, then there exists a function* u *such that*

$$u \in L^\infty(\mathbb{R}^+; H^1_0(\Omega) \cap H^2(\Omega)) \cap L^\infty(0,T; H^3_\Gamma(\Omega)), u'(t) \in K \ a.e. \ in \ [0,T] \tag{1}$$

$$u' \in L^\infty(\mathbb{R}^+; L^2(\Omega)) \cap L^2(\mathbb{R}^+; H^1_0(\Omega)) \cap L^2(0,T; H^1_0(\Omega) \cap H^2(\Omega)), u'' \in L^2(0,T; H^{-1}(\Omega)), \tag{2}$$

satisfying

$$\begin{aligned} &\int_0^T \left[\langle u'', v-u' \rangle + (\Delta^2 u, v-u') - (\Delta_p u, v-u') \quad - \left(\int_0^t g(t-s)\Delta u(s) ds, v-u' \right) \right. \\ &-(\Delta u', v-u') + (f(u), v-u')] \, dt \geq 0, \forall v \in L^2(0,T; H^1_0(\Omega)), \ v(t) \in K \ a.e. \ in \ t, u(0) = u_0, \ \ u'(0) = u_1 \end{aligned} \tag{3}$$

Proof: *Existence* - The proof of Theorem 2.1 is made by the penalty method. It consists in considering a perturbation of the problem (1) adding a singular term called penalty, depending on a parameter $\epsilon > 0$. We solve the mixed problem in Q for the penalty operator and the estimates obtained for the local solution of the penalized equation, allow to pass to limits, when ϵ goes to zero, in order to obtain a function u which is the solution of our problem. uniqueness follows in a standard way through the energy method.

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BLOWING UP SOLUTION FOR A NONLINEAR FRACTIONAL DIFFUSION EQUATION

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Abstract

In this work we study results of the existence of solutions for the semilinear fractional diffusion equation and we still give sufficient conditions to obtain the blowing up behavior of the solution.

1 Introduction

In the recent years anomalous diffusion has attracted much interest of the scientific community since this subject involves a large variety of natural science. Among the mathematical models of such theory, the so-called fractional diffusion equations

$$u_t(t,x) = \partial_t(g_\alpha * \Delta u)(t,x) + r(t,x) \quad t > 0,\ x \in \Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ and $g_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$, $0 < \alpha \leq 1$, have attracted a great attention mostly due to their success in the modeling of a large variety of subdiffusive phenomena.

From the mathematical point of view, the study of (1) was initiated by Schneider and Wyss [4] where Fox H functions are used to obtain the corresponding Green functions in a closed form for arbitrary space dimensions. In [1], de Andrade and Viana consider the nonlinear fractional diffusion equation and prove a global well-posedness result for initial data $u_0 \in L^q(\mathbb{R}^N)$ in the critical case $q = \frac{\alpha N}{2}(\rho - 1)$. They also provide sufficient conditions to obtain self-similar solutions to the problem. Viana [5] consider a more general version of the previous nonlinear problem where concentrated and non concentrated nonlinear sources are taken into account.

We had obtained results about a local well-posedness theory for the semilinear fractional diffusion equation

$$u_t(t,x) = \partial_t \int_0^t g_\alpha(s)\Delta u(t-s,x)ds + |u(t,x)|^{\rho-1}u(t,x),\ \text{in } (0,T)\times\Omega, \tag{2}$$

$$u(t,x) = 0,\ \text{on } (0,T)\times\partial\Omega, \tag{3}$$

$$u(x,0) = u_0(x),\ \text{in } \Omega, \tag{4}$$

where $g_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$, for $\alpha \in (0,1)$, Δ is the Laplace operator, and Ω is a sufficiently smooth domain in $\mathbb{R}^N$.

We will talk about results of the local well-posedness result and we give sufficient conditions to produce the blowing up behavior of solution. Such results are part of the work [2] that was submitted.

2 Main Results

The local well-posedness result (Theorem 2.1 below) is motivated by [3, Th. 1].

Theorem 2.1. *Let $v_0 \in L^q(\Omega)$, $q \geq \rho$ and $q > \frac{N}{2}(\rho-1)$. Then, there exist $T > 0$ and $R > 0$ such that (2)-(4) has a L^q-mild solution $u : [0,T] \to L^q(\Omega)$ which is unique in $C([0,T];L^q(\Omega))$, for any $u_0 \in B_{L^q(\Omega)}(v_0, R/4)$. This solution depends continuously on the initial data, that is, if u and v are solutions of (2)-(4) starting in u_0 and v_0, then*

$$\sup_{t\in(0,T]} \|u(t,\cdot) - v(t,\cdot)\|_{L^q(\Omega)} \leq \tilde{C}\|u_0 - v_0\|_{L^q(\Omega)}.$$

We give sufficient conditions to obtain a blowing-up behavior for the solution of (2)-(4). To do this, recall that there exists a L^1-normalized eigenfunction φ_1 of the Dirichlet Laplacian associated to its first eigenvalue λ_1.

Theorem 2.2. *Let $\rho > 2-\alpha$, $u_0 \in L^\infty(\Omega)$ a nonnegative nonzero function and suppose that the solution u given by Theorem 2.1 is a classical solution of (2)-(4) starting at u_0. If*

$$\int_0^1 \int_\Omega u(s,x)\varphi_1(x)dxds > \left[\frac{\rho-1}{\rho-2+\alpha} \cdot \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1)+\lambda_1}\right]^{\frac{1}{1-\rho}} := c_\alpha \tag{1}$$

then $T_{max} < \infty$ and u blows-up in the L^∞-norm.

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STABILITY RESULTS FOR NEMATIC LIQUID CRYSTALS

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Abstract

In 1994, Ponce et al [4] analyzed the stability of mildly decaying global strong solutions for the Navier-Stokes equations. In this work, we try to apply the same approach for a nematic liquid crystal model, that is a coupled model including a Navier-Stokes type-system for the velocity of the liquid crystal ("liquid part") and a parabolic system for the orientation vector field for the molecules of the liquid crystal ("solid part"). We will focus on the similarities and differences with respect to Ponce et al [4], depending on the boundary data chosen for the solid part.

1 Introduction

Suppose Ω a bounded, simply-connected and open set in $\mathbb{R}^3$ having a smooth boundary and lying at one side of $\partial\Omega$. Let $Q = \Omega \times (0, \infty)$ and $\Sigma = \partial\Omega \times (0, \infty)$. If we denote by $\boldsymbol{v} = \boldsymbol{v}(t, \boldsymbol{x})$ the velocity vector, $\pi(t, \boldsymbol{x})$ the pressure of the fluid, $\boldsymbol{e} = \boldsymbol{e}(t, \boldsymbol{x})$ the orientation of the liquid crystal molecules, and $\boldsymbol{x} = (x_1, x_2, x_3) \in \Omega$ the space point, then the model for the phenomenon in 3D of liquid crystals of nematic type can be described, for example, by the coupled system:

$$\begin{array}{lll}
\partial_t \boldsymbol{v} - \nu\Delta\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \nabla\pi = -\lambda(\nabla\boldsymbol{e})^t\Delta\boldsymbol{e} + \boldsymbol{g} & \text{in} & Q,\\
\nabla\cdot\boldsymbol{v} = 0 & \text{in} & Q,\\
\partial_t\boldsymbol{e} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{e} - \gamma\left(\Delta\boldsymbol{e} - \boldsymbol{f}_\delta(\boldsymbol{e})\right) = 0 & \text{in} & Q,\\
\boldsymbol{v} = \boldsymbol{0}, \ \text{ and either } \ \boldsymbol{e} = \boldsymbol{a}, \ \text{ or } \ \partial_{\mathbf{n}}\boldsymbol{e} = \boldsymbol{0} & \text{on} & \Sigma,\\
\boldsymbol{v}(\boldsymbol{x},0) = \boldsymbol{v}_0(\boldsymbol{x}), \quad \boldsymbol{e}(\boldsymbol{x},0) = \boldsymbol{e}_0(\boldsymbol{x}) & \text{in} & \Omega,
\end{array} \tag{1}$$

where $\nu > 0$ is the fluid viscosity, $\lambda > 0$ is the elasticity constant, $\gamma > 0$ is a relaxation in time constant, the function $\boldsymbol{f}_\delta$ is defined by

$$\boldsymbol{f}_\delta(\boldsymbol{e}) = \frac{1}{\delta^2}\left(|\boldsymbol{e}|^2 - 1\right)\boldsymbol{e} \quad \text{with} \quad |\boldsymbol{e}| \le 1, \tag{2}$$

where $|\cdot|$ is the euclidian norm in $\mathbb{R}^3$, $\delta > 0$ is a penalization parameter, $\boldsymbol{g}$ is a known function defined in Q.

Let $\boldsymbol{V} = \{\boldsymbol{y} \in \boldsymbol{H}^1(\Omega);\ \nabla\cdot\boldsymbol{y} = 0,\ \boldsymbol{y}_{|\partial\Omega} = 0\}$ and $\boldsymbol{H} = \{\boldsymbol{y} \in \boldsymbol{L}^2(\Omega);\ \nabla\cdot\boldsymbol{y} = 0, \boldsymbol{y}\cdot\mathbf{n}_{||\partial\Omega} = 0\}$. Assuming the compatibility hypothesis

$$|\boldsymbol{e}_0| \le 1 \ \text{ a.e. in } \ \Omega \quad \text{and} \quad |\boldsymbol{a}| \le 1 \ \text{ a.e. on } \ \Sigma, \tag{3}$$

and that

$$\boldsymbol{v}_0 \in \boldsymbol{V}, \quad \boldsymbol{e}_0 \in \boldsymbol{H}^2(\Omega), \quad \boldsymbol{a} \in \mathbf{H}^{5/2}(\partial\Omega) \quad \text{and} \quad \boldsymbol{g} \in L^2(0, \infty, \boldsymbol{H}), \tag{4}$$

Lin & Liu [3] showed, for fixed $\delta > 0$, that system (1)-(4) has global strong solutions $(\boldsymbol{v}, \pi, \boldsymbol{e})$ with the following regularity:

$$\begin{array}{l}
\boldsymbol{v} \in L^\infty\big(0, +\infty; \boldsymbol{H}\cap\boldsymbol{H}^1(\Omega)\big) \cap L^2\big(0, +\infty; \boldsymbol{H}^2(\Omega)\cap\boldsymbol{V}\big), \quad \partial_t\boldsymbol{v} \in L^2\big(0, +\infty; \boldsymbol{H}\big),\\
\boldsymbol{e} \in L^\infty\big(0, +\infty; \boldsymbol{H}^2(\Omega)\big) \cap L^2\big(0, +\infty; \boldsymbol{H}^3(\Omega)\big), \quad \partial_t\boldsymbol{e} \in L^\infty\big(0, +\infty; \boldsymbol{L}^2(\Omega)\big) \cap L^2\big(0, +\infty; \boldsymbol{H}^1(\Omega)\cap\boldsymbol{V}\big).
\end{array}$$

Our mean contribution here is to prove the stability of the strong global solutions of system (1) considering only $\boldsymbol{v}=\boldsymbol{0}$ and $\boldsymbol{e}=\boldsymbol{a}$ on Σ. For this purpose, we consider the open neighborhood containing $(\boldsymbol{v}_0,\boldsymbol{e}_0,\boldsymbol{g},\boldsymbol{a})$,

$$\begin{aligned}&\mathcal{O}_\epsilon\big((\boldsymbol{v}_0,\boldsymbol{e}_0,\boldsymbol{g},\boldsymbol{a})\big)=\Big\{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t})\in\boldsymbol{V}\times\boldsymbol{H}^2(\Omega)\times L^2\big(0,\infty;\boldsymbol{H}\big)\times\boldsymbol{H}^{5/2}(\partial\Omega);\\&\|\nabla(\boldsymbol{v}_0-\boldsymbol{x})\|^2+\|\boldsymbol{e}_0-\boldsymbol{y}\|^2_{\boldsymbol{H}^2}+\int_0^\infty\|(\boldsymbol{g}-\boldsymbol{z})(t)\|^2dt+\|\boldsymbol{a}-\boldsymbol{t}\|^2_{\boldsymbol{H}^{5/2}}<\epsilon\Big\},\end{aligned}\tag{5}$$

such that, for all $(\boldsymbol{u}_0,\boldsymbol{d}_0,\boldsymbol{h},\boldsymbol{b})\in\mathcal{O}_\epsilon\big((\boldsymbol{v}_0,\boldsymbol{e}_0,\boldsymbol{g},\boldsymbol{a})\big)$ there exists a unique strong global solution $(\boldsymbol{u},\theta,\boldsymbol{d})$ of the perturbed system

$$\begin{array}{lll}\partial_t\boldsymbol{u}-\nu\Delta\boldsymbol{u}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}+\nabla\theta=-\lambda(\nabla\boldsymbol{d})^t\Delta\boldsymbol{d}+\boldsymbol{h} & \text{in} & Q,\\ \nabla\cdot\boldsymbol{u}=0 & \text{in} & Q,\\ \partial_t\boldsymbol{d}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{d}-\gamma(\Delta\boldsymbol{d}-\boldsymbol{f}_\delta(\boldsymbol{d}))=0 & \text{in} & Q,\\ \boldsymbol{u}=\boldsymbol{0},\quad \boldsymbol{e}=\boldsymbol{b} & \text{on} & \Sigma,\\ \boldsymbol{u}(\boldsymbol{x},0)=\boldsymbol{u}_0(\boldsymbol{x}),\quad \boldsymbol{d}(\boldsymbol{x},0)=\boldsymbol{d}_0(\boldsymbol{x}) & \text{in} & \Omega,\end{array}\tag{6}$$

where $\boldsymbol{b}$ is a time-independent datum and $\|\cdot\|$ is the norm in $\boldsymbol{L}^2(\Omega)$.

2 Main result

To establish the main result of this work, we need to assume that there exist a strong solution of (1) satisfying the Leray [2] global criterion of regularity

$$\|\nabla\boldsymbol{v}(t)\|^4\quad\text{and}\quad\|\nabla(t)\|^4_{H^1(\Omega)}\quad\text{belong to}\quad L^1(0,\infty),\tag{1}$$

or, equivalently, see Beirão da Veiga [1],

$$\|\nabla\boldsymbol{v}(t)\|^{\frac{2p}{2p-3}}_{\boldsymbol{L}^p(\Omega)}\quad\text{and}\quad\|\nabla\boldsymbol{e}(t)\|^{\frac{2q}{2q-3}}_{W^{1,q}(\Omega)}\quad 2\le p,\,q\le 3\quad\text{belong to}\quad L^1(0,\infty),\tag{2}$$

Our stability results for the system (1) can be state as follows:

Theorem 2.1. *Suppose that there exists a global strong solution* $(\boldsymbol{v},\pi,\boldsymbol{e})$ *of system* (1) *and that satisfies the Leray global criterion of regularity* (1). *If*

$$(\boldsymbol{u}_0,\boldsymbol{d}_0,\boldsymbol{h},\boldsymbol{b})\in\mathcal{O}_\epsilon\big((\boldsymbol{v}_0,\boldsymbol{e}_0,\boldsymbol{g},\boldsymbol{a})\big)\tag{3}$$

then

$$\lim_{\epsilon\to0}\big(\|\nabla(\boldsymbol{u}-\boldsymbol{v})(t)\|+\|(\boldsymbol{d}-\boldsymbol{e})(t)\|_{H^2(\Omega)}\big)=0.\tag{4}$$

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CONTROLE EXATO-APROXIMADA INTERNA PARA O SISTEMA DE BRESSE TERMOELÁSTICO

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Abstract

Neste trabalho será apresentado o controle exato-aproximado interno para o sistema de Bresse termoelástico, cujo controle age em um subintervalo do domínio. O controle é obtido minimizando-se o funcional associado ao sistema de Bresse termoelástico, como feito em [2], este trabalho faz parte da tese de doutorado em [1].

1 Introdução

Nosso objetivo é obter o controle exato-aproximada em (l_1, l_2), com $(l_1, l_2) \subset (0, L)$, para o sistema de Bresse termoelástico

$$\left\{\begin{array}{l} \rho_1\varphi_{tt} - k(\varphi_x + \psi + lw)_x - k_0 l(w_x - l\varphi) = f_1\chi_{(l_1,l_2)}, \quad em \quad (0,L)\times(0,T) \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + lw) + \gamma\theta_x = f_2\chi_{(l_1,l_2)}, \quad em \quad (0,L)\times(0,T) \\ \rho_1 w_{tt} - k_0(w_x - l\varphi)_x + kl(\varphi_x + \psi + lw) = f_3\chi_{(l_1,l_2)}, \quad em \quad (0,L)\times(0,T) \\ \theta_t - k_1\theta_{xx} + m\psi_{xt} = 0, \quad em \quad (0,L)\times(0,T) \\ \varphi(0,t) = \varphi(L,t) = \psi(0,t) = \psi(L,t) \\ = w(0,t) = w(L,t) = \theta(0,t) = \theta(L,t) = 0, \quad t \in (0,T) \\ \varphi(.,0) = \varphi_0, \quad \varphi_t(.,0) = \varphi_1, \quad em \quad (0,L) \\ \psi(.,0) = \psi_0, \quad \psi_t(.,0) = \psi_1, \quad em \quad (0,L) \\ w(.,0) = w_0, \quad w_t(.,0) = w_1, \quad em \quad (0,L) \\ \theta(.,0) = \theta_0, \quad em \quad (0,L). \end{array}\right. \tag{1}$$

Para o controle exato-aproximada interna encontramos um espaço de Hilbert $\mathcal{H} = H_0^1(0,L)\times L^2(0,L)\times H_0^1(0,L)\times L^2(0,L)\times H_0^1(0,L)\times L^2(0,L)\times L^2(0,L)$, tal que para cada dados inicial e final $(\varphi_0, \varphi_1, \psi_0, \psi_1, w_0, w_1, \theta_0), (\Phi_0, \Phi_1, \Psi_0, \Psi_1, W_0, W_1, \eta_0) \in \mathcal{H}$ e $\varepsilon > 0$, é possível encontrar controles f_1, f_2, f_3 tais que a solução de (1) satisfaça

$$\begin{array}{l} \varphi(T) = \Phi_0, \quad \varphi_t(T) = \Phi_1 \\ \psi(T) = \Psi_0, \quad \psi_t(T) = \Psi_1 \\ w(T) = W_0, \quad w_t(T) = W_1 \\ |\theta(T) - \eta_0|_{L^2(0,L)} \leq \varepsilon. \end{array} \tag{2}$$

Para obter tal controle fizemos como em [1],[2] e [3].

2 Resultados Principais

O processo usado para se obter-se o controle exato-aproximada interna consiste em encontrar uma estimativa de observabilidade para o sistema homogêneo (1) (isto é $f_1 = f_2 = f_3 = 0$). Para obter tal estimativa de observabilidade usaremos uma desigualdade de observabilidade para o sistema desacoplado associado

$$\left\{\begin{array}{l}\rho_1\widetilde{\varphi}_{tt}-k(\widetilde{\varphi}_x+\widetilde{\psi}+l\widetilde{w})_x-k_0l(\widetilde{w}_x-l\widetilde{\varphi})=0, \quad em \quad (0,L)\times(0,T)\\ \rho_2\widetilde{\psi}_{tt}-b\widetilde{\psi}_{xx}+k(\widetilde{\varphi}_x+\widetilde{\psi}+l\widetilde{w})+\frac{m\gamma}{k_1}P\widetilde{\psi}_t=0, \quad em \quad (0,L)\times(0,T)\\ \rho_1\widetilde{w}_{tt}-k_0(\widetilde{w}_x-l\widetilde{\varphi})_x+kl(\widetilde{\varphi}_x+\widetilde{\psi}+l\widetilde{w})=0, \quad em \quad (0,L)\times(0,T)\\ \widetilde{\theta}_t-k_1\widetilde{\theta}_{xx}+m\widetilde{\psi}_{xt}=0, \quad em \quad (0,L)\times(0,T)\\ \widetilde{\varphi}(0,t)=\widetilde{\varphi}(L,t)=\widetilde{\psi}(0,t)=\widetilde{\psi}(L,t)\\ =\widetilde{w}(0,t)=\widetilde{w}(L,t)=\widetilde{\theta}(0,t)=\widetilde{\theta}(L,t)=0, \quad t\in(0,T)\\ \widetilde{\varphi}(.,0)=\varphi_0, \quad \widetilde{\varphi}_t(.,0)=\varphi_1, \quad em \quad (0,L)\\ \widetilde{\psi}(.,0)=\psi_0, \quad \widetilde{\psi}_t(.,0)=\psi_1, \quad em \quad (0,L)\\ \widetilde{w}(.,0)=w_0, \quad \widetilde{w}_t(.,0)=w_1, \quad em \quad (0,L)\\ \widetilde{\theta}(.,0)=\theta_0, \quad em \quad (0,L),\end{array}\right. \tag{1}$$

onde

$$P\widetilde{\psi}_t=P\widetilde{\psi}_t-\frac{1}{L}\int_0^L P\widetilde{\psi}_t\,dx$$

e um teorema que diz, para $S(t)$ e $S^0(t)$ os semigrupos fortemente contínuos em $\mathcal{H}$ associados aos sistemas homogêneo (1) e (1) respectivamente tem-se que
$S(t)-S^0(t):\mathcal{H}\to C([0,T];\mathcal{H})$ é contínuo e compacto.

Por fim para obter-se o controle exata-aproximada interna minimizaremos o funcional $J:\widetilde{H}\to\mathbb{R}$ definido da seguinte forma:

$$\begin{array}{l}J(u_0,u_1,v_0,v_1,z_0,z_1,p_0)=\frac{1}{2}\int_0^T\int_{l_1}^{l_2}(|u|^2+|v|^2+|z|^2)\,dx\,dt\\ -\rho_1\int_0^L\Phi_1u_0dx-\rho_2\int_0^L\Psi_1v_0dx-\rho_1\int_0^L W_1z_0dx+\rho_1\langle\Phi_0,u_1\rangle+\rho_2\langle\Psi_0,v_1\rangle\\ +\rho_1\langle W_0,z_1\rangle-\int_0^L(\eta_0+m\Psi_x)p_0\,dx+\varepsilon\|p_0\|_{L^2(0,L)},\end{array} \tag{2}$$

onde
$\widetilde{H}=L^2(0,L)\times H^{-1}(0,L)\times L^2(0,L)\times H^{-1}(0,L)\times L^2(0,L)\times H^{-1}(0,L)\times L^2(0,L).$

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XIII ENAMA - Novembro 2019 *87–88*

ON THE L^2 DECAY OF WEAK SOLUTIONS FOR THE 3D ASYMMETRIC FLUIDS EQUATIONS

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Abstract

We study the long time behavior of weak solutions for the asymmetric fluids equations in the whole space $\mathbb{R}^3$. We prove that $\|(\boldsymbol{u},\boldsymbol{w})(\cdot,t)\|^2_{\boldsymbol{L}^2(\mathbb{R}^3)} \leq C\,(t+1)^{-3/2}$ for all $t \geq 0$ through Fourier splitting method.

1 Introduction

In the work, we use boldface letters to denote vector fields in $\mathbb{R}^n$, as well as to indicate spaces whose elements are of this nature. We consider, in $\mathbb{R}^3 \times \mathbb{R}^+$, the Cauchy problem

$$\begin{cases} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\mu+\mu_r)\Delta\boldsymbol{u} + \nabla p - 2\mu_r \operatorname{curl}\boldsymbol{w} = \boldsymbol{f}, \\ \operatorname{div}\boldsymbol{u} = 0, \\ \boldsymbol{w}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{w} - (c_a+c_d)\Delta\boldsymbol{w} - (c_0+c_d-c_a)\nabla(\operatorname{div}\boldsymbol{w}) + 4\mu_r\boldsymbol{w} - 2\mu_r\operatorname{curl}\boldsymbol{u} = \boldsymbol{g}, \\ \boldsymbol{u}\big|_{t=0} = \boldsymbol{u}_0, \quad \boldsymbol{w}\big|_{t=0} = \boldsymbol{w}_0, \end{cases} \tag{1}$$

complemented with Dirichlet conditions at infinity. This system, proposed by Eringen [1], describes the motion of viscous incompressible asymmetric (also known as micropolar) fluids with constant density $\rho = 1$ and generalized the classical Navier Stokes model. In system (1), the unknowns are the linear velocity $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^3$, the pressure distribution $p(\boldsymbol{x},t) \in \mathbb{R}$ and the angular (or micro-rotational) velocity of the fluid particles as functions of the position $\boldsymbol{x}$ and time t, $\boldsymbol{w}(\boldsymbol{x},t) \in \mathbb{R}^3$. The functions $\boldsymbol{u}_0 = \boldsymbol{u}_0(\boldsymbol{x})$, $\boldsymbol{w}_0 = \boldsymbol{w}_0(\boldsymbol{x})$, $\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{x},t)$ and $\boldsymbol{g} = \boldsymbol{g}(\boldsymbol{x},t)$ denote, respectively, a given initial linear velocity, initial angular velocity and external forces. The positive constants μ, μ_r, c_0, c_a and c_d represent viscosity coefficients and satisfy the inequality $c_0 + c_d > c_a$.Without loss of generality to our goals, we fix $\mu = 1/2 = \mu_r$ and $c_a + c_d = 1 = c_0 + c_d - c_a$. Besides that, We denote the Fourier transform either by F or $\widehat{\ }$, i. e.

$$F\{\varphi\}(\boldsymbol{\xi}) = \widehat{\varphi}(\boldsymbol{\xi}) = \int_{\mathbb{R}^3} e^{-i\,\boldsymbol{\xi}\cdot\boldsymbol{x}}\varphi(\boldsymbol{x})\,d\boldsymbol{x}. \tag{2}$$

2 Main Results

The main results are similar to the problems solved to Navier-Stokes equations in [3] by M. Schonbek through a method now known as the “Fourier splitting method” developed by her and first applied in the context of parabolic conservation laws (see [4]). These results as well as their proofs can be seen in [6]

Theorem 2.1. *Let $(\boldsymbol{u},p,\boldsymbol{w})$ be a smooth solution of the Cauchy problem (1) with $\boldsymbol{f} = \boldsymbol{g} = \boldsymbol{0}$. If $\boldsymbol{u}_0, \boldsymbol{w}_0 \in \boldsymbol{L}^1(\mathbb{R}^3) \cap \boldsymbol{L}^2(\mathbb{R}^3)$, with $\operatorname{div}\boldsymbol{u}_0 = 0$, then there exists a constant $C > 0$ such that*

$$\|\boldsymbol{u}(\cdot,t)\|_2^2 + \|\boldsymbol{w}(\cdot,t)\|_2^2 \leq C\,(t+1)^{-3/2}, \qquad \forall\, t \geq 0. \tag{1}$$

The constant C depends only on the $\boldsymbol{L}^1$ and $\boldsymbol{L}^2$ norms of $\boldsymbol{u}_0$ and $\boldsymbol{w}_0$.

Proof To prove Theorem 2.1, we use the following results which proofs can be seen in [6].

Lemma 2.1. *Let* $(\boldsymbol{u}, p, \boldsymbol{w})$ *be a smooth solution of the Cauchy problem* (1) *with* $\boldsymbol{f} = \boldsymbol{g} = \mathbf{0}$*. If* $\boldsymbol{u}_0, \boldsymbol{w}_0 \in \boldsymbol{L}^1(\mathbb{R}^3) \cap \boldsymbol{L}^2(\mathbb{R}^3)$*, with* $\operatorname{div} \boldsymbol{u_0} = 0$*, then one has, for all* $t \geq 0$ *and* $\boldsymbol{\xi} \in \mathbb{R}^3$*,*

$$|F\{(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\}(\boldsymbol{\xi},t)| + |F\{(\boldsymbol{u}\cdot\nabla)\boldsymbol{w}\}(\boldsymbol{\xi},t)| + |F\{\nabla p\}(\boldsymbol{\xi},t)| \leq \|\boldsymbol{u}(\cdot,t)\|_2 (2\,\|\boldsymbol{u}(\cdot,t)\|_2 + \|\boldsymbol{w}(\cdot,t)\|_2)\,|\boldsymbol{\xi}|. \tag{2}$$

In particular, $|F\{(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\}(\boldsymbol{\xi},t)| + |F\{(\boldsymbol{u}\cdot\nabla)\boldsymbol{w}\}(\boldsymbol{\xi},t)| + |F\{\nabla p\}(\boldsymbol{\xi},t)| \leq C|\boldsymbol{\xi}|$*, where* $C \in \mathbb{R}^+$ *depends only on* $\|\boldsymbol{u}_0\|_2$ *and* $\|\boldsymbol{w}_0\|_2$*.*

Proposition 2.1. *Let* $\mathcal{K} \subset \mathbb{R}^3$ *be a compact set. Under the assumptions of Lemma 2.1, one has*

$$|\,\widehat{\boldsymbol{u}}(\boldsymbol{\xi},t)| + |\,\widehat{\boldsymbol{w}}(\boldsymbol{\xi},t)|\, \leq\, C\,|\boldsymbol{\xi}|^{-1}, \tag{3}$$

for all $t \geq 0$ *and* $\boldsymbol{\xi} \in \mathcal{K}$*, with* $\boldsymbol{\xi} \neq \mathbf{0}$*, where the constant* $C > 0$ *depends only on the set* $\mathcal{K}$ *and on the* $\boldsymbol{L}^1$ *and* $\boldsymbol{L}^2$ *norms of the initial data.*

By the construction of approximate solutions of (1), we prove

Theorem 2.2. *Let* $\boldsymbol{u_0} \in \boldsymbol{H} \cap \boldsymbol{L}^1(\mathbb{R}^3)$ *and* $\boldsymbol{w}_0 \in \boldsymbol{L}^2(\mathbb{R}^3) \cap \boldsymbol{L}^1(\mathbb{R}^3)$*. There exists a weak solution* $(\boldsymbol{u}, p, \boldsymbol{w})$ *of problem* (1) *with* $\boldsymbol{f} = \boldsymbol{g} = \mathbf{0}$ *such that*

$$\big\|\,\boldsymbol{u}(\cdot,t)\big\|_2^2 + (t+1)\,\big\|\,\boldsymbol{w}(\cdot,t)\big\|_2^2 \,\leq\, C\,(t+1)^{-3/2} \tag{4}$$

for all $t \geq 0$*, where the constant* C *depends only on the* $\boldsymbol{L}^1$ *and* $\boldsymbol{L}^2$ *norms of* $\boldsymbol{u_0}$ *and* $\boldsymbol{w}_0$ *and*

$$\boldsymbol{H} := \text{the closure of } \big\{\boldsymbol{v} \in \{\boldsymbol{C}_0^\infty(\mathbb{R}^3)\,/\,\mathit{div}\,\boldsymbol{v} = 0 \text{ in } \mathbb{R}^3\}\big\} \text{ on } \boldsymbol{L}^2(\mathbb{R}^3).$$

Remark 2.1. *The results can be generalized to the case that the external forces satisfy some decay estimates.*

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XIII ENAMA - Novembro 2019 *89–90*

LOCAL EXISTENCE FOR A HEAT EQUATION WITH NONLOCAL TERM IN TIME AND SINGULAR INITIAL DATA

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Abstract

We prove sharp results for the local existence of non-negative solutions for a semilinear parabolic equation with memory. The initial data is singular in the sense that it belongs to the Lebesgue space.

1 Introduction

Let Ω be either a smooth bounded domain or the whole space $\mathbb{R}^N$. We consider the nonlocal in time parabolic problem

$$u_t - \Delta u = \int_0^t m(t,s) f(u(s)) ds \text{ in } \Omega \times (0,T), \tag{1}$$

with boundary and initial conditions

$$u = 0 \text{ in } \partial\Omega \times (0,T), \quad u(0) = u_0 \geq 0 \text{ in } \Omega, \tag{2}$$

where $f \in C([0,\infty))$, $m \in C(\mathcal{K}, [0,\infty))$, $\mathcal{K} = \{(t,s) \in \mathbb{R}^2; 0 < s < t\}$ and $u_0 \in L^r(\Omega)$, $r \in [1,\infty)$.

Problem (1) models diffusion phenomena with memory effects and can be widely encountered in models of population dynamics, as for example the Volterra diffusion equation. This problem has been considered by many authors, see for instance [2, 6] and the references therein. In particular, when $m(t,s) = (t-s)^{-\gamma}$, $\gamma \in (0,1)$, and $u_0 \in C_0(\mathbb{R}^N)$, problem (1) was studied in [2].

We are interested in the local existence of solutions of (1) considering initial data in $L^r(\Omega)$. The first works in this direction are dued to F. Weissler, who treated the nonlinear parabolic problem

$$u_t - \Delta u = f(u) \text{ in } \Omega \times (0,T) \tag{3}$$

with conditions (2), $u_0 \in L^r(\mathbb{R}^N)$ and $f(u) = u^p$, $p > 1$. From the results of [1], [3] and [7] it is well known that there exists a critical value $p^* = 1 + 2r/N$ such that problem (3) has a solution in $L^r(\Omega)$ if either $p < p^*$ and $r \geq 1$ or $p = p^*$ and $r > 1$. Moreover, if either $p > p^*$ and $r \geq 1$ or $p = p^*$ and $r = 1$, one can find a nonnegative initial data in $L^r(\Omega)$ for which there is no local nonnegative solution.

Recently, these results were extended for the general case $f \in C([0,\infty))$ assuming that f is a non-decreasing function, see [5]. It was shown, in the case Ω a bounded domain, that problem (3) has a solution in $L^r(\Omega)$ with $r > 1$ if and only if $\limsup_{t\to\infty} t^{-p^*} f(t) < \infty$. If $r = 1$ problem (3) has a solution in $L^1(\Omega)$ if and only if $\int_1^\infty t^{-(1+2/N)} F(t) dt < \infty$, where $F(t) = \sup_{1\leq\sigma\leq t} f(\sigma)/\sigma$. Similar results were obtained when $\Omega = \mathbb{R}^N$, but in this case is needed the additional condition $\limsup_{t\to 0} f(t)/t < \infty$.

2 Main results

We assume that the function m verifies the following conditions:
H1) The function m is a nonnegative continuous function defined in the set $\mathcal{K} = \{(t, s) \in \mathbb{R}^2; 0 < s < t\}$.
H2) The function m verifies: there exists a constant $\gamma \in \mathbb{R}$ such that $m(\lambda t, \lambda s) = \lambda^{-\gamma} m(t, s),$ for all $(t, s) \in \mathcal{K}$.
H3) $m(1, \cdot) \in L^1(0, 1)$, and
H4) $\limsup_{\eta \to 0^+} \eta^l |m(1, \eta)| < \infty$ for some $l \in \mathbb{R}$.

Theorem 2.1 (Existence)**.** *Assume that $f \in C([0, \infty))$, m verifies conditions H1)-H4) with $\gamma < 2, l < 1$. Let $a = \min\{1 - l, 2 - \gamma\}$, $r \geq 1$ and $p^* = \frac{2r}{N}(2 - \gamma) + 1$. Suppose that $p^*[N + 2a - 2(2 - \gamma)] > N + 2a$, $p^*(a + \gamma - 1) > a$, $\gamma > l$ and $p^* + \gamma > 2$, and some the following conditions hold:*

(i) $\limsup_{t \to \infty} t^{-p^*} f(t) < \infty$, *if Ω is a bounded domain.*

(ii) $\limsup_{t \to \infty} t^{-p^*} f(t) < \infty$ *and* $\limsup_{t \to 0^+} f(t)/t < \infty$ *if $\Omega = \mathbb{R}^N$.*

Then for every $u_0 \in L^r(\Omega), u_0 \geq 0$ problem (1) has a local solution.

Theorem 2.2 (Non-existence)**.** *Assume $f \in C([0, \infty))$ is a non-decreasing function.*

(i) If $\limsup_{t \to \infty} t^{-p^*} f(t) = \infty$ *and m verifies conditions H1)-H3) with $\gamma < 2$, then there exists $u_0 \in L^r(\Omega)$, $u_0 \geq 0$ so that problem (1) does not have a local solution.*

(ii) There exist $\gamma < 2$ and $l < 1$ in every situation: $p^*[N + 2a - 2(2 - \gamma)] \leq N + 2a$ or $p^*(a + \gamma - 1) \leq a$ or $p^* + \gamma \leq 2$ or $\gamma \leq l$. Moreover, for these values of γ and l there exist a function m satisfying H1)-H4) such that if $\limsup_{t \to \infty} t^{p^*} f(t) = \infty$*, then it is possible to find $u_0 \in L^r(\Omega), u_0 \geq 0$ such that problem (1) does not have any local solution.*

(iii) Suppose that $\Omega = \mathbb{R}^N$, $\limsup_{t \to)^+} f(t)/t = \infty$ *and m verifies conditions H1)-H3) with $\gamma < 2$, then there exists $u_0 \in L^r(\Omega)$, $u_0 \geq 0$ so that problem (1) does not have a local solution.*

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XIII ENAMA - Novembro 2019 *91–92*

ATRATOR PULLBACK PARA SISTEMAS DE BRESSE NÃO-AUTÔNOMOS

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Abstract

Neste trabalho investigamos a dinâmica a longo prazo de um sistema de Bresse não-autônomo. Garantimos a existência e unicidade de solução e o resultado principal estabelece a existência de atrator pullback. A semicontinuidade superior de atratores, quando se considera um parâmetro no sistema, é também estudada.

1 Introdução

Neste trabalho nós estudamos a dinâmica a longo prazo das soluções do seguinte sistema de Bresse

$$\begin{aligned} \rho_1\varphi_{tt} - k(\varphi_x+\psi+lw)_x - k_0 l(w_x - l\varphi) + g_1(\varphi_t) + f_1(\varphi,\psi,w) &= \epsilon h_1(x,t), \\ \rho_2\varphi_{tt} - b\psi_{xx} + k(\varphi_x+\psi+lw) + g_2(\psi_t) + f_2(\varphi,\psi,w) &= \epsilon h_2(x,t), \\ \rho_1 w_{tt} - k_0(w_x - l\varphi)_x + kl(\varphi_x+\psi+lw) + g_3(w_t) + f_3(\varphi,\psi,w) &= \epsilon h_3(x,t), \end{aligned} \tag{1}$$

definida em $(0,L)\times[\tau,+\infty[$, sujeita às condições de froteira de Dirichlet,

$$\varphi(0,t)=\varphi(L,t)=\psi(0,t)=\psi(L,t)=w(0,t)=w(L,t)=0, \quad t\geqslant\tau, \tag{2}$$

e condição inicial (para $t=\tau$),

$$\varphi(\cdot,\tau)=\varphi_0^\tau,\ \varphi_t(\cdot,\tau)=\varphi_1^\tau,\ \psi(\cdot,\tau)=\psi_0^\tau,\ \psi_t(\cdot,\tau)=\psi_1^\tau,\ w(\cdot,\tau)=w_0^\tau,\ w(\cdot,\tau)=w_1^\tau, \tag{3}$$

onde $g_1(\varphi_t)$, $g_2(\varphi_t)$ e $g_3(\varphi_t)$ são termos de damping não linear, $f_i(\varphi,\psi,w)$, $i=1,2,3$, são forças externas e $h_i=h_i(t)$, $i=1,2$, são perturbações dependentes do tempo, o que torna o sistema não-autônomo. Sob condições bastante gerais nós garantimos que o problema (1)-(3) é bem-posto no espaço de energia

$$V = H_0^1(0,L)\times H_0^1(0,L)\times H_0^1(0,L)\times L^2(0,L)\times L^2(0,L)\times L^2(0,L),$$

equipado com a norma

$$\|(\varphi,\psi,w,\tilde\varphi,\tilde\psi,\tilde w)\|_V = \rho_1\|\tilde\varphi\|^2+\rho_2\|\tilde\psi\|^2+\rho_1\|\tilde w\|^2+b\|\psi_x\|^2+k\|\varphi_x+\psi+lw\|^2+k_0\|w_x-l\varphi\|^2.$$

Consideramos que f_1, f_2 e f_3 são localmente Lipschitz e do tipo gradiente. Assumimos que existe uma função de classe C^2, $F:\mathbb{R}^3\to\mathbb{R}$ tal que $\nabla F=(f_1,f_2,f_3)$, e satisfaz as seguintes condições: existem β, $m_F\geqslant 0$ tais que

$$F(u,v,w)\geqslant -\beta(|u|^2+|v|^2+|w|^2)-m_F \quad \forall\, u,v,w\in\mathbb{R}, \tag{4}$$

e existem $p\geqslant 1$ e $C_f>0$ tais que, para $i=1,2,3$,

$$|\nabla f_i(u,v,w)|\leqslant C_f(1+|u|^{p-1}+|v|^{p-1}+|w|^{p-1}), \quad \forall\, u,v,w\in\mathbb{R}. \tag{5}$$

Em particular isso implica que existe $C_F>0$ tal que

$$F(u,v,w)\leqslant C_F(1+|u|^{p+1}+|v|^{p+1}+|w|^{p+1}), \quad \forall\, u,v,w\in\mathbb{R}. \tag{6}$$

Além disso, assumimos que, para todo $u, v, w \in \mathbb{R}$,

$$\nabla F(u,v,w) \cdot (u,v,w) - F(u,v,w) \geqslant -\beta(|u|^2 + |v|^2 + |w|^2) - m_F. \tag{7}$$

Em relação às funções damping $g_i \in C^1(\mathbb{R})$, $i = 1,2,3$, assumimos que g_i é crescente e $g_i(0) = 0$, e existem constantes $m_i, M_i > 0$ tais que

$$m_i \leqslant g_i'(s) \leqslant M_i, \quad \forall\, s \in \mathbb{R}. \tag{8}$$

Finalmente, assumimos $h_1, h_2 \in L^2_{\text{loc}}(\mathbb{R}; L^2(0,L))$ e mais algumas condições sobre estas funções.

2 Resultados Principais

Teorema 2.1. *Se as hipóteses (2.1)-(8) são válidas, então o processo de evolução gerado pelo problema (1)-(3) admite um atrator pullback $\mathcal{A}_\epsilon = \{\mathcal{A}_\epsilon(t)\}$ no espaço de fase V.*

Teorema 2.2. *Sob as condições do Teorema 2.1, o atrator pullback $\{\mathcal{A}_\epsilon\}$ é semicontínuo superiormente quando $\epsilon \to 0$, isto é,*

$$\lim_{\epsilon \to 0} dist(\mathcal{A}_\epsilon(t), \mathcal{A}_0) = 0, \quad \forall\, t \in \mathbb{R}. \tag{1}$$

A existência de atrator global para o problema (1)-(3) quando $\epsilon = 0$ foi demonstrada em [1].

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EXACT CONTROLLABILITY FOR AN EQUATION WITH NON-LINEAR TERM

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Abstract

Let Ω be a bounded domain of $\mathbb{R}^n$ with regular boundary of type C^2, so that Ω contains the origin of $\mathbb{R}^n$. Consider the non homogeneous problem

$$\begin{cases} u''(t) - \Delta\, u(t) \; + |u(t)|^{\rho}\, u(t) = 0 & \text{in} \quad Q = \Omega \times (0,T) \\ u(t) = \; v(t), \; \Sigma = \partial\Omega \times (0,T) & \\ u(0) = u_0 \quad u'(0) = u_1 & \text{on} \quad \Omega. \end{cases}$$

Our main objective is to study the exact controllability of problem. Where u is the displacement, Δ denotes the Laplace operator.

1 Main Results

Theorem 1.1. *For $T > T_0$, and for each $\{u_0, u_1\} \in L^2(\Omega) \times \left(H^{-1}(\Omega) + L^{p'}(\Omega)\right), p = \rho + 2$, exist a control function at the boundary $v \in L^2(\Sigma)$, such that the ultraweak solution u satisfies the final condition*

$$u(T) \; = \; u'(T) \; = \; 0, \qquad in \quad \Omega$$

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NEW DECAY RATES FOR THE LOCAL ENERGY OF WAVE EQUATIONS WITH LIPSCHITZ WAVESPEEDS

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Abstract

We consider the Cauchy problem for wave equations with variable coefficients in the whole space $\mathbf{R}^n$. We improve the rate of decay of the local energy, which has been recently studied by J. Shapiro [6], where he derives the log-order decay rates of the local energy under stronger assumptions on the regularity of the initial data.

1 Introduction

We consider in this work the Cauchy problem associated to the wave equation with variable coefficient in $\mathbf{R}^n$ $(n \geq 1)$ as follow

$$u_{tt}(t,x) - c(x)^2 \Delta u(t,x) = 0, \quad (t,x) \in (0,\infty) \times \mathbf{R}^n, \tag{1}$$

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbf{R}^n, \tag{2}$$

where (u_0, u_1) are initial data chosen as

$$u_0 \in H^1(\mathbf{R}^n), \quad u_1 \in L^2(\mathbf{R}^n),$$

and the function $c : \mathbf{R}^n \to \mathbf{R}$ satisfies the two assumptions below:
(A-1) $c(x) > 0$ $(x \in \mathbf{R}^n)$, $c, c^{-1} \in L^\infty(\mathbf{R}^n)$, $\nabla c \in (L^\infty(\mathbf{R}^n))^n$,
(A-2) there exists a constant $L > 0$ such that $c(x) = 1$ for $|x| > L$.
In particular, the condition (A-1) implies $c \in C^{0,1}(\mathbf{R}^n)$.

The local energy $E_R(t)$ on the zone $\{|x| \leq R\}$ $(R > 0)$ corresponding to the solution $u(t,x)$ of (1)-(2) is defined by

$$E_R(t) := \frac{1}{2}\int_{|x|\leq R} \big(\frac{1}{c(x)^2}|u_t(t,x)|^2 + |\nabla u(t,x)|^2\big)dx.$$

Shapiro [6] imposes rather stronger hypothesis on the regularity of the initial data such as
(I) the supports of initial data are compact, and as a result $[u_0, u_1] \in H^2(\mathbf{R}^n) \times H^1(\mathbf{R}^n)$.
Furthermore, in a sense,
(II) the decay order $(\log t)^{-2}$ obtained in [6] of the local energy seems to be rather slow.

Under weaker regularity assumptions on the initial data to modify **(I)**, one obtains faster algebraic decay rate which improves **(II)** in the case when the coefficient $c(x)$ and the parameter L have a special relation.

2 Main Results

Theorem 2.1. *Let* $n \geq 3$*, and assume* (A-1) *and* (A-2)*. If the initial data* $[u_0, u_1] \in H^1(\mathbf{R}^n) \times (L^2(\mathbf{R}^n) \cap L^1(\mathbf{R}^n))$ *further satisfies*

$$\int_{\mathbf{R}^n} (1 + |x|) \left(\frac{1}{c(x)^2}|u_1(x)|^2 + |\nabla u_0(x)|^2\right) dx < +\infty,$$

then the unique solution $u \in C_1^n$ *to problem* (1.1)-(1.2) *satisfies*

$$E_R(t) = O(t^{-(1-\eta)}) \quad (t \to \infty),$$

for each $R > L$ *provided that* $\eta := 2L\|\frac{1}{c(\cdot)}\|_\infty \|\nabla c\|_\infty \in [0, 1)$.

Theorem 2.2. *Let* $n = 2$*, and assume* (A-1) *and* (A-2). *Let* $\gamma \in (0, 1]$. *If* $[u_0, u_1] \in H^1(\mathbf{R}^n) \times (L^2(\mathbf{R}^n) \cap L^{1,\gamma}(\mathbf{R}^n))$ *further satisfies*

$$\int_{\mathbf{R}^2} (1 + |x|) \left(\frac{1}{c(x)^2} |u_1(x)|^2 + |\nabla u_0(x)|^2 \right) dx < +\infty,$$

and

$$\int_{\mathbf{R}^2} \frac{u_1(x)}{c(x)^2} dx = 0,$$

then the unique solution $u \in C_1^2$ *to problem* (1.1)-(1.2) *satisfies*

$$E_R(t) = O(t^{-(1-\eta)}) \quad (t \to \infty),$$

for each $R > L$ *provided that* $\eta := 2L\|\frac{1}{c(\cdot)}\|_\infty \|\nabla c\|_\infty \in [0, 1)$.

Theorem 2.3. *Let* $n = 1$*, and assume* (A-1) *and* (A-2). *If* $[u_0, u_1] \in H^1(\mathbf{R}^n) \times L^2(\mathbf{R}^n)$ *further satisfies*

$$\int_{\mathbf{R}} (1 + |x|) \left(\frac{1}{c(x)^2} |u_1(x)|^2 + |\nabla u_0(x)|^2 \right) dx < +\infty,$$

then the unique solution $u \in C_1^1$ *to problem* (1.1)-(1.2) *satisfies*

$$E_R(t) = O(t^{-(1-\eta)}) \quad (t \to \infty),$$

for each $R > L$ *provided that* $\eta := 2L\|\frac{1}{c(\cdot)}\|_\infty \|\nabla c\|_\infty \in [0, 1)$.

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XIII ENAMA - Novembro 2019 *97–98*

A SUMMABILITY PRINCIPLE AND APPLICATIONS

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Abstract

A new Inclusion Theorem for summing operators that encompasses several recent similar results as particular cases is presented. As applications, we improve estimates of classical inequalities for multilinear forms. This is a joint work with Gustavo Araújo and Joedson Santos.

1 Introduction

Summing operators date back to Grothendieck's Résumé and the seminal paper of Lindenstrauss and Pelczynski. In the 80's, the investigation of these was directed to the multilinear framework and several different lines of investigation emerged. In an attempt to unify most of the different approaches, the following notion of Λ-summability arose naturally (see [3] and the references therein):

Definition 1.1. *Let $E_1, \ldots, E_m, F$ be Banach spaces, m be a positive integer, $(\mathbf{r};\mathbf{p}) := (r_1, \ldots, r_m; p_1, \ldots, p_m) \in [1,\infty)^{2m}$ and $\Lambda \subset \mathbb{N}^m$ be a set of indexes. A multilinear operator $T : E_1 \times \cdots \times E_m \to F$ is Λ-$(\mathbf{r};\mathbf{p})$-summing if there exists a constant $C > 0$ such that for all $x^j \in \ell_{p_j}^w(E_j)$, $j = 1, \ldots, m$,*

$$\left(\sum_{i_1=1}^{\infty}\left(\cdots\left(\sum_{i_m=1}^{\infty}\left\|T\left(x_{i_1}^1, \ldots, x_{i_m}^m\right)\mathbf{1}_\Lambda(i_1,...,i_m)\right\|^{r_m}\right)^{\frac{r_{m-1}}{r_m}}\cdots\right)^{\frac{r_1}{r_2}}\right)^{\frac{1}{r_1}} \leq C \cdot \prod_{j=1}^{m}\left\|x^j\right\|_{w,p_j},$$

where $\mathbf{1}_\Lambda$ is the characteristic function of Λ. We represent the class of all Λ-$(\mathbf{r};\mathbf{p})$-summing multilinear operators from $E_1 \times \cdots \times E_m$ to F by $\Pi_{(\mathbf{r};\mathbf{p})}^\Lambda(E_1, \ldots, E_m; F)$. When $r_1 = \cdots = r_m = r$ and $p_1 = \cdots = p_m = p$, we will represent $\Pi_{(\mathbf{r};\mathbf{p})}^\Lambda$ by $\Pi_{(r;p)}^\Lambda$.

When $\Lambda = \Lambda_{\text{as}} := \{(i, \ldots, i) : i \in \mathbb{N}\}$, Definition 1.1 recovers the notion of $(r;\mathbf{p})$-absolutely summing operators, denoted by $\Pi_{(r;\mathbf{p})}^{\text{as}}$. When $\Lambda = \mathbb{N}^m$, we get the notion of $(\mathbf{r};\mathbf{p})$-multiple summing operators, denoted by $\Pi_{(\mathbf{r};\mathbf{p})}^{\text{mult}}$. Results of the type $\Pi_{(r;p)}^\Lambda \subset \Pi_{(s;q)}^\Lambda$ are called Inclusion Theorems, which role is very important in the literature. The main contribution we present is an Inclusion Theorem for the case in which the set Λ is formed by "blocks". The set Λ is called *block*, if

$$\Lambda = \left\{\mathbf{i} = \left(i_1, \overset{n_1 \text{ times}}{\ldots}, i_1, \ldots, i_d, \overset{n_d \text{ times}}{\ldots}, i_d\right) : i_1, \ldots, i_d \in \mathbb{N}\right\},$$

where $1 \leq n_1, \ldots, n_d \leq m$ are fixed positive integers such that $n_1 + \cdots + n_d = m$. The general block situation, on which Λ is called block of $\mathcal{I}$-type, corresponds to a partition $\mathcal{I} = \{I_1, \ldots, I_d\}$ of non-void disjoint set of $\{1, \ldots, m\}$, such that $\pi_j(\mathbf{i}) = i_k$, with $j \in I_k, k = 1, \ldots, d$, where π_j the projection on the j-th coordinate.

Provided that Λ is a block, we shall prove the inclusion

$$\Pi_{(r;\mathbf{p})}^\Lambda \subset \Pi_{(\mathbf{s};\mathbf{q})}^\Lambda,$$

for suitable values of $s_1, \ldots, s_m$. In the final section we apply our main result to the investigation of Hardy–Littlewood inequalities for multilinear forms.

2 Main Results

For multiple summing operators, Inclusion Theorems are more subtle. Recently, this subject was investigated by several authors and using different techniques (see [2, Theorem 1.2] and [4, Proposition 3.3]). Our main result recovers the aforementioned results. A useful notation is used: given $A \subset \{1, \ldots, m\}$, we set $\left|\frac{1}{\mathbf{p}}\right|_{j\in A} := \sum_{j\in A} \frac{1}{p_j}$. Also, for $1 \leq k \leq m$, we define $|1/\mathbf{p}|_{j\geq k} := |1/\mathbf{p}|_{j\in\{k,\ldots,m\}}$; we simply write $|1/\mathbf{p}|$ instead of $|1/\mathbf{p}|_{j\geq 1}$.

Theorem 2.1. *Let $1 \leq d \leq m$ be positive integers and $r \geq 1$, $\mathbf{p}, \mathbf{q} \in [1,\infty)^m$. Let also $\mathcal{I} = \{I_1, \ldots, I_d\}$ be a partition of $\{1, \ldots, m\}$ and suppose that Λ is a block-set of $\mathcal{I}$-type. Then*

$$\Pi^{\mathcal{I}}_{(r;\mathbf{p})}\left(E_1, \ldots, E_m; F\right) \subset \Pi^{\mathcal{I}}_{(\mathbf{s};\mathbf{q})}\left(E_1, \ldots, E_m; F\right),$$

for any Banach spaces $E_1, \ldots, E_m, F$, with

$$\frac{1}{s_k} - \left|\frac{1}{\mathbf{q}}\right|_{j\in\bigcup_{i=k}^{d} I_i} = \frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right|_{j\in\bigcup_{i=k}^{d} I_i}, \; k = 1, \ldots, d$$

whenever $q_j \geq p_j$, $j = 1, \ldots, m$, and

$$\frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right| + \left|\frac{1}{\mathbf{q}}\right| > 0$$

or $q_1 > p_1$, $q_j \geq p_j$, $j = 2, \ldots, m$, and

$$\frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right| + \left|\frac{1}{\mathbf{q}}\right| = 0.$$

Moreover, the inclusion operator has norm 1.

As application, we improve the exponent on a Hardy–Littlewood/Dimant–Sevilla's inequality. The following standard notation is used: e_i^n denotes the n-tuple $(e_i, \ldots, e_i)$, with e_i the canonical vector of the sequence space c_0; here $\mathbf{j} := (j_1, \ldots, j_m)$ stands for a multi-index; we shall denote $X_p = \ell_p$ for $1 \leq p < \infty$ and $X_\infty = c_0$.

Proposition 2.1. *Let $1 \leq d \leq m$ and let $n_1, \ldots, n_d$ be positive integers such that $n_1 + \cdots + n_d = m$. If $m < p \leq 2m$, then*

$$\left(\sum_{j_1=1}^{\infty}\left(\cdots\left(\sum_{j_d=1}^{\infty}\left|A\left(e_{i_1}^{n_1}, \ldots, e_{i_d}^{n_d}\right)\right|^{s_d}\right)^{\frac{s_{d-1}}{s_d}}\cdots\right)^{\frac{s_1}{s_2}}\right)^{\frac{1}{s_1}} \leq D^{\mathbb{K}}_{m,\mathbf{p},\mathbf{s}}\|A\|,$$

for all m-linear forms $A : \ell_p^n \times \cdots \times \ell_p^n \to \mathbb{K}$, with $s_k = \left[\frac{1}{2} - (n_k + \cdots + n_d)\cdot\left(\frac{1}{p} - \frac{1}{2m}\right)\right]^{-1}$, for $k = 1, \ldots, d$.

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XIII ENAMA - Novembro 2019 *99–100*

HOLOMORPHIC FUNCTIONS WITH LARGE CLUSTER SETS

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Abstract

We study linear and algebraic structures in sets of bounded holomorphic functions on the ball which have large cluster sets at every possible point (i.e., every point on the sphere in several complex variables and every point of the closed unit ball of the bidual in the infinite dimensional case). We show that this set is strongly $\mathfrak{c}$-algebrable for all separable Banach spaces. For specific spaces including ℓ_p or duals of Lorentz sequence spaces, we have strongly $\mathfrak{c}$-algebrability and spaceability even for the subalgebra of uniformly continous holomorphic functions on the ball.

1 Introduction and main results

There is an increasing interest in the search linear or algebraic structures in sets of functions with special (usually *bad*) non-linear properties. A seminal example of this kind of results is the construction in [4] of infinite dimensional subspaces of $C([0,1])$ containing only of nowhere differentiable functions (except the zero function). Since then, many efforts were devoted in this direction, especially in the last years. We refer the reader to [1] for a complete monograph in the subject.

In this work, we study linear and algebraic structures in the set of holomorphic functions with *large cluster sets at every point*, both for finite and infinitely many variables. Cluster values and cluster sets of holomorphic functions in the complex disk $\mathbb{D}$ were first considered by I. J. Schark (a fictitious name chosen by eight brilliant mathematicians of the time) in [6]. Their motivation was to relate the set of cluster values of a bounded function f at a point in the unit circle S with the set of evaluations $\varphi(f)$ of elements φ in the spectrum of the algebra H^∞ over that point. Different authors have studied the analogous problem in the infinite dimensional setting [2, 3, 5]. Let us remark that for a bounded holomorphic function f on $\mathbb{D}$, a large cluster set of f at some $z_0 \in S$ means that f has a wild behaviour as $z \to z_0$ (the cluster set consists of all limit values of $f(z)$ as $z \to z_0$). So, in the one dimensional case, we are interested in those functions that have this wild behaviour at every point of the unit circle. This is our non-linear property, which can be rated as bad, in opposition to continuity at the boundary, which plays the rôle of the good property. Since a cluster set is a compact connected subset of $\mathbb{C}$, it is considered large whenever it contains a disk.

Let us begin in the context of several complex variables. We consider a norm $\|\cdot\|$ in $\mathbb{C}^n$ and the corresponding finite dimensional Banach space $E = (\mathbb{C}^n, \|\,\|)$. We write B and S for the open unit ball and the unit sphere of E, respectively. Let $\mathcal{H}^\infty(B)$ denote the algebra of all bounded holomorphic functions on B. The cluster set of a function $f \in \mathcal{H}^\infty(B)$ at a point $z \in \overline{B}$ is the set $Cl(f,z)$ of all limits of values of f along sequences converging to z. For z in the open unit ball this cluster set contains just one point: $f(z)$; but for $z \in S$ the situation can be very different.

Theorem 1.1. *For $E = (\mathbb{C}^n, \|\,\|)$, the set of functions $f \in \mathcal{H}^\infty(B)$ such that there exists a (fixed) disk centered at the origin which is contained in $Cl(f,z)$ for every $z \in S$ is strongly $\mathfrak{c}$-algebrable.*

Now we consider an infinite dimensional complex Banach space E. The symbol B_E (or B if there is no ambiguity) represents the open unit ball of E, while S_E (or S) represents the unit sphere. Also, we write $B^{**} := B_{E^{**}}$ and $\overline{B}^{**} := \overline{B_{E^{**}}}$, where E^{**} denotes the topological bidual of E.

In this case, for $f \in \mathcal{H}^\infty(B)$ and $z \in \overline{B}^{**}$, the cluster set of f at z is the set $Cl(f,z)$ of all limits of values of f along nets in B weak-star converging to z. More precisely,

$$Cl(f,z) = \{\lambda \in \mathbb{C} : \text{there exists a net } (x_\alpha) \subset B \text{ such that } x_\alpha \overset{w(E^{**},E^*)}{\longrightarrow} z, \text{ and } f(x_\alpha) \to \lambda\}.$$

In the infinite dimensional case, the cluster set can be large even at points in the interior of the ball.

Theorem 1.2. *If E is a separable infinite-dimensional Banach space, then the set of functions $f \in \mathcal{H}^\infty(B)$ such that there exists a (fixed) disk centered at the origin which is contained in $Cl(f,z)$ for every $z \in \overline{B}^{**}$ is strongly $\mathfrak{c}$-algebrable.*

Recall that $A_u(B)$ is the Banach algebra of all uniformly continuous holomorphic functions on the unit ball B. As a consequence of [2, Corollary 2.5], functions in $A_u(B_{\ell_p})$ have trivial cluster sets at points of S_{ℓ_p} for $1 \le p < \infty$. Moreover, a function $f \in \mathcal{H}^\infty(B_{\ell_p})$ for which there exists a fixed disk contained in $Cl(f,z)$ for every $z \in B_{\ell_p}$ cannot belong to $A_u(B_{\ell_p})$ $(1 \le p < \infty)$. So we do not expect a result like Theorem 1.2 to hold for $A_u(B_{\ell_p})$. The same happens for some duals/preduals Lorentz sequence spaces. However, if we only ask cluster sets at $z \in B$ to contain disks (whose radii depend on the point), we have both strongly $\mathfrak{c}$-algebrability and spaceability.

Theorem 1.3. *Let E be either ℓ_p $(1 \le p < \infty)$ or $d(w,p)^*$ $(1 < p < \infty)$ or $d_*(w,1)$ with $w \in \ell_s$ for some $1 < s < \infty$. Then, the set of functions $f \in A_u(B_E)$ whose cluster set at every $x \in B$ contains a disk is strongly $\mathfrak{c}$-algebrable and contains (up to the zero function) an isometric copy of ℓ_∞. In particular, it is spaceable.*

We remark that the copy of ℓ_∞ obtained in the previous theorem is actually contained in the subspace of m-homogeneous polynomials, where $m \ge p$ for ℓ_p $(1 < p < \infty)$, $m \ge p'$ for $d(w,p)^*$ $(1 < p < \infty)$, $m \ge s'$ for $d_*(w,1)$ and m can be any even number for ℓ_1.

Finally, we state the following spaceability result for the case $E = c_0$.

Theorem 1.4. *The set of functions $f \in \mathcal{H}^\infty(B_{c_0})$ whose cluster set at every $x \in B_{c_0}$ contains a disk is strongly $\mathfrak{c}$-algebrable and contains (up to the zero function) an almost isometric copy of ℓ_1. In particular, it is spaceable.*

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RELATIONS BETWEEN FOURIER-JACOBI COEFFICIENTS

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Abstract

Positive definite functions on two-point homogeneous spaces were characterized by R. Gangolli some forty years ago and are very useful for solving scattered data interpolation problems on the spaces. Such characterization is related to the so called Fourier-Jacobi coefficients and can be found in [5]. This work provides relations between these coefficients.

1 Introduction

Let $\mathbb{M}^d$ denote a d dimensional compact two-point homogeneous space. It is well known that spaces of this type belong to one of the following categories ([8]): the unit spheres S^d, $d = 1, 2, \ldots$, the real projective spaces $\mathbb{P}^d(\mathbb{R})$, $d = 2, 3, \ldots$, the complex projective spaces $\mathbb{P}^d(\mathbb{C})$, $d = 4, 6, \ldots$, the quaternionic projective spaces $\mathbb{P}^d(\mathbb{H})$, $d = 8, 12, \ldots$, and the Cayley projective plane $\mathbb{P}^d(Cay)$, $d = 16$. In general this classification is decisive in analysis of problemas involving the compact two-point homogeneous spaces, as can be seen in [1, 2, 4] and others mentioned there.

A zonal kernel K on $\mathbb{M}^d$ can be written in the form $K(x, y) = K_r^d(\cos |xy|/2)$, $x, y \in \mathbb{M}^d$, for some function $K_r^d : [-1, 1] \to \mathbb{R}$, the *radial* or *isotropic part* of K. A result due to Gangolli ([5]) established that a continuous zonal kernel K on $\mathbb{M}^d$ is positive definite if and only if

$$K_r^d(t) = \sum_{k=0}^{\infty} a_k^{\alpha,\beta} P_k^{\alpha,\beta}(t), \quad t \in [-1, 1], \tag{1}$$

in which $\sum_{k=0}^{\infty} a_k^{\alpha,\beta} P_k^{\alpha,\beta}(1) < \infty$ and $a_k^{\alpha,\beta} \in [0, \infty)$, $k \in \mathbb{Z}_+$. Here, $\alpha = (d-2)/2$ and $\beta = (d-2)/2, -1/2, 0, 1, 3$, depending on the respective category $\mathbb{M}^d$ belongs to, among the five we have mentioned in the beginning of this section. The symbol $P_k^{(d-2)/2,\beta}$ stands for the Jacobi polynomial of degree k associated with the pair (α, β). The coefficients $a_k^{\alpha,\beta}$ are given by

$$a_k^{\alpha,\beta} := \frac{\left[P_k^{\alpha,\beta}(1)\right]^2 (2k+\alpha+\beta+1)\Gamma(k+1)\Gamma(k+\alpha+\beta+1)}{2^{\alpha+\beta+1}\,\Gamma(k+\alpha+1)\Gamma(k+\beta+1)} \int_{-1}^{1} f(t) R_k^{(\alpha,\beta)}(t)(1-t)^{\alpha}(1+t)^{\beta} dt,$$

and they are called *Fourier-Jacobi coefficients*.

2 Main Results

The main results to be proved in this work are based on those presented in [3] and are described below.

Theorem 2.1. *Let K be a continuous, isotropic and positive definite kernel on $\mathbb{M}^d$, and $a_k^{\alpha,\beta}$ the Fourier-Jacobi coefficients presented in* (1). *Then*

$$a_k^{\alpha,\beta} = \sum_{j=0}^{\infty} \left(\prod_{l=1}^{j} \omega_{k+l-1}^{\alpha,\beta} \right) \gamma_{k+j}^{\alpha,\beta} \; a_{k+j}^{\alpha+1,\beta} = \sum_{j=0}^{\infty} \left(\prod_{l=1}^{j} \varphi_{k+l-1}^{\alpha,\beta} \right) \xi_{k+j}^{\alpha,\beta} \; a_{k+j}^{\alpha,\beta+1}$$

in which,

$$\begin{aligned}
\omega_k^{\alpha,\beta} &= \frac{(k+1)(n+\beta+1)(2k+\alpha+\beta+1)}{(k+\alpha+1)(k+\alpha+\beta+1)(2k+\alpha+\beta+3)}\\
\gamma_k^{\alpha,\beta} &= \frac{(\alpha+1)(2k+\alpha+\beta+1)}{(k+\alpha+1)(k+\alpha+\beta+1)}\\
\varphi_k^{\alpha,\beta} &= \frac{2k+\alpha+\beta+1}{k+\alpha+\beta+1}\\
\xi_k^{\alpha,\beta} &= \frac{(k+1)(2k+\alpha+\beta+1)}{(k+\alpha+\beta+1)(2k+\alpha+\beta+3)}.
\end{aligned}$$

We can obtain an application of previous result involving the positive definiteness and strictly positive definiteness of a kernel on a two-point homogeneos space $\mathbb{M}^d$.

Theorem 2.2. *Let $d, d' \geq 2$ be integers. If K is a positive definite kernel on a two-point homogeneous space $\mathbb{M}^{2d}$ and a strictly postive definite kernel on $\mathbb{M}^{2d'}$, such that $\mathbb{M}^{2d}$ and $\mathbb{M}^{2d'}$ belong to same category we have mentioned in the beginning of previous section, then K is a strictly postive definite kernel on $\mathbb{M}^{2d}$.*

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XIII ENAMA - Novembro 2019 *103–104*

SEQUENTIAL CHARACTERIZATIONS OF LATTICE SUMMING OPERATORS

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Abstract

We prove that a linear operator from a Banach space to a Banach lattice is lattice summing if and only if it sends weakly summable sequences to sequences whose partial sums of the modulus are norm bounded if and only if it sends unconditionally summable sequences to modulus summable sequences. Applications are provided.

1 Introduction

The following class of operators, closely related to the class of absolutely summing operators, was introduced by Yanovskii [5] and Nielsen and Szulga [3] (see also [1, 4]): Given a Banach space E and a Banach lattice F, a linear operator $u\colon E \longrightarrow F$ is *lattice summing* if there exists a constant $C \geq 0$ such that, for any $n \in \mathbb{N}$ and all $x_1, \ldots, x_n \in E$,

$$\left\| \sum_{j=1}^{n} |u(x_j)| \right\| \leq C \cdot \sup_{x^* \in B_{E^*}} \sum_{j=1}^{n} |x^*(x_j)|.$$

The infimum of the constants C working in the inequality is denoted by $\lambda_1(u)$.

It is a natural question if lattice summing operators can be characterized by means of the transformation of weakly summable sequences in E to sequences in some Banach lattice formed by F-valued sequences. In this work we use the spaces $|\ell_1|(F)$ and $|\ell_1(F)|$ of F-valued sequences introduced in [2] to prove that an operator is lattice summing if and only if it sends weakly summable sequences to sequences in $|\ell_1|(F)$ if and only if it sends unconditionally summable sequences to sequences in $|\ell_1(F)|$.

E will always be a Banach space and F will be a Banach lattice. By $\ell_1^w(E)$ we denote the space of E-valued weakly summable sequences and by $\ell_1^u(E)$ the space of E-valued unconditionally summable sequences. Now we recall the spaces of Banach lattices-valued sequences introduced in [2]:

$$|\ell_1|(F) = \left\{ (x_n)_{n=1}^{\infty} \subseteq E : \|(x_n)_{n=1}^{\infty}\|_{|\ell_1|(F)} := \sup_n \left\| \sum_{j=1}^{n} |x_j| \right\|_F < +\infty \right\} \text{ and}$$

$$|\ell_1(F)| = \left\{ (x_n)_{n=1}^{\infty} \subseteq E : \sum_{n=1}^{\infty} |x_n| \text{ converges in } F \right\}.$$

2 Main Results

Proposition 2.1. (a) *$|\ell_1|(F)$ is a Banach lattice which contains $|\ell_1(F)|$ as a closed ideal.*
(b) *The containing relations $\ell_1(F) \subseteq |\ell_1(F)| \subseteq |\ell_1|(F)$ hold and are strict in general.*
(c) $\|(x_n)_{n=1}^{\infty}\|_{|\ell_1|(F)} = \left\| \sum_{n=1}^{\infty} |x_n| \right\|_F$ *for every* $(x_n)_{n=1}^{\infty} \in |\ell_1(F)|$.
(d) *$|\ell_1|(F) = |\ell_1(F)|$ if and only if F is weakly sequentially complete.*
(e) *If $u\colon F \longrightarrow G$ is a regular linear operator between Banach lattices, then $(u(x_j))_j \in |\ell_1|(G)$ whenever $(x_j)_j \in |\ell_1|(F)$.*

Our main result reads as follows:

Theorem 2.1. *The following are equivalent for a linear operator* $u \colon E \longrightarrow F$*:*
(a) *u is lattice summing.*
(b) $(u(x_j))_{j=1}^{\infty} \in |\ell_1|(F)$ *whenever* $(x_j)_{j=1}^{\infty} \in \ell_1^w(E)$.
(c) $(u(x_j))_{j=1}^{\infty} \in |\ell_1(F)|$ *whenever* $(x_j)_{j=1}^{\infty} \in \ell_1^u(E)$.
(d) *There exists a constant* $C \geq 0$ *such that*

$$\sup_n \left\| \sum_{j=1}^{n} |u(x_j)| \right\| \leq C \cdot \sup_{x^* \in B_{E^*}} \sum_{j=1}^{\infty} |x^*(x_j)|$$

for every $(x_j)_{j=1}^{\infty} \in \ell_1^w(E)$.
(e) *There exists a constant* $C \geq 0$ *such that*

$$\left\| \sum_{j=1}^{\infty} |u(x_j)| \right\| \leq C \cdot \sup_{x^* \in B_{E^*}} \sum_{j=1}^{\infty} |x^*(x_j)|$$

for every $(x_j)_{j=1}^{\infty} \in \ell_1^u(E)$.

In this case, the induced maps $\hat{u} \colon \ell_1^w(E) \longrightarrow |\ell_1|(F)$ *and* $\tilde{u} \colon \ell_1^u(E) \longrightarrow |\ell_1(F)|$*, given by*

$$\hat{u}\left((x_n)_{n=1}^{\infty}\right) = \tilde{u}\left((x_n)_{n=1}^{\infty}\right) = (u(x_j))_{n=1}^{\infty},$$

are well defined bounded linear operators and

$$\lambda_1(u) = \|\hat{u}\| = \|\tilde{u}\| = \inf\{C : \text{(d) holds}\} = \inf\{C : \text{(e) holds}\}.$$

If E is also a Banach lattice, then the operator u is positive (regular, a lattice homomorphism, respectively) if and only if the induced operators $\hat{u}$ and $\tilde{u}$ are positive (regular, lattice homomorphisms, respectively).

Corollary 2.1. (Ideal property) *If* $v \colon H \longrightarrow E$ *is a bounded linear operator,* $u \colon E \longrightarrow F$ *is a lattice summing operator and* $t \colon F \longrightarrow G$ *is a regular linear operator, then* $t \circ u \circ v \colon H \longrightarrow G$ *is a lattice summing operator.*

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(X, Y)-NORMS ON TENSOR PRODUCTS AND DUALITY

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Abstract

We introduce a class of abstract norms on the tensor product of Banach spaces E and F from sequence classes X and Y. These abstract norms recover known norms on the tensor product, such as the Chevet-Saphar norms, and generate new ones. A natural issue in this subject is how to characterize the dual of the tensor product endowed with a given norm. Instead of offering a characterization of the dual of our (X, Y)-normed tensor product as a class of linear operators, which is more common in the literature, we build one as a class of bilinear applications.

1 Introduction

In the work [1] of 2017, G. Botelho and J. R. Campos synthesize the study of Banach operator ideals and multi-ideals characterized by transformation of vector-valued sequences by introducing an abstract framework based in the new concept of sequence classes. This environment also accommodates the already studied ideals as particular instances. We refer to the books [2] and [3] for examples of classes of operators that fit in this subject and for the theory of operator ideals.

In the current paper we use the environment of sequence classes to introduce an abstract (X, Y)-norm on the tensor product and characterize its dual as a class of bilinear applications.

The letters E, F shall denote Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ and the symbol $E \overset{1}{=} F$ means that E and F are isometrically isomorphic. We refer to the book [4] for the theory and all symbology concerning tensor products used in this work. The theory, symbology, definitions and results concerning sequence classes and operator ideals will be used indistinctly and can be found in paper [1] and in the book [3], respectively.

2 Main Results

Let E and F be Banach spaces and X and Y sequence classes. Consider the function $\alpha_{X,Y}(\cdot) : E \otimes F \longrightarrow \mathbb{R}$, given by

$$\alpha_{X,Y}(u) = \inf \left\{ \left\| (x_j)_{j=1}^n \right\|_{X(E)} \left\| (y_j)_{j=1}^n \right\|_{Y(F)} ; u = \sum_{j=1}^n x_j \otimes y_j \right\},$$

taking the infimum over all representations of $u \in E \otimes F$.

Under certain conditions, the function $\alpha_{X,Y}(\cdot)$ is a reasonable crossnorm on $E \otimes F$:

Proposition 2.1. *Let E and F be Banach spaces and X, Y sequence classes. If $\alpha_{X,Y}(\cdot) : E \otimes F \longrightarrow \mathbb{R}$ is a function such that $\varepsilon(u) \leq \alpha_{X,Y}(u)$, for any tensor $u \in E \otimes F$, and $\alpha_{X,Y}(\cdot)$ satisfies the triangular inequality, then $\alpha_{X,Y}(\cdot)$ is a reasonable crossnorm on $E \otimes F$.*

We denote by $E \otimes_{\alpha_{X,Y}} F$ the tensor product $E \otimes F$ endowed with the (X, Y)-norm $\alpha_{X,Y}(\cdot)$ and its completion by $E \widehat{\otimes}_{\alpha_{X,Y}} F$. The Banach space $E \widehat{\otimes}_{\alpha_{X,Y}} F$ will be called (X, Y)-*normed tensor product* of the Banach spaces E and F. The Chevet-Saphar norms are recovered as (X, Y)-norms: taking $X = \ell_{p^*}^w(\cdot)$ and $Y = \ell_p(\cdot)$ or $X = \ell_p(\cdot)$ and

$Y = \ell_{p^*}^w(\cdot)$, we obtain $\alpha_{\ell_{p^*}^w,\ell_p}(\cdot) = d_p(\cdot)$ or $\alpha_{\ell_p,\ell_{p^*}^w}(\cdot) = g_p(\cdot)$, respectively. Of course, many other new reasonable crossnorms can be generated, for instance taking $X = \ell_{p^*}^w(\cdot)$ and $Y = \ell_p\langle\cdot\rangle$.

If α is a reasonable crossnorm we have $\left(E\widehat{\otimes}_\alpha F\right)' \subseteq \left(E\widehat{\otimes}_\pi F\right)' \stackrel{1}{=} \mathcal{B}(E \times F) \stackrel{1}{=} \mathcal{L}(E, F')$ and so we can interpret $\left(E\widehat{\otimes}_\alpha F\right)'$ as a class of bilinear forms or as a class of linear operators. This last interpretation is the most common in the literature and, as far as we known, the ε norm is one of the few where this dual is originally interpreted in the first form.

We now characterize de dual $\left(E\widehat{\otimes}_{\alpha_{X,Y}} F\right)'$ as a class of bilinear applications. Before that, we need a definition.

Definition 2.1. A sequence class X is *Hölder-limited* if for any Banach space E and all $(x_j)_{j=1}^\infty \in X(E)$ and $(\lambda_j)_{j=1}^\infty \in \ell_\infty$, it follows that $(\lambda_j x_j)_{j=1}^\infty \in X(E)$ and

$$\left\|(\lambda_j x_j)_{j=1}^\infty\right\|_{X(E)} \leq \left\|(\lambda_j)_{j=1}^\infty\right\|_\infty \cdot \left\|(x_j)_{j=1}^\infty\right\|_{X(E)}.$$

Theorem 2.1. *Let E and F be Banach spaces, X and Y finitely determined sequence classes, where X or Y is Hölder-limited, and $\alpha_{X,Y}(\cdot)$ a reasonable crossnorm. Then,*

$$\left(E\widehat{\otimes}_{\alpha_{X,Y}} F\right)' \stackrel{1}{=} \mathcal{L}_{X,Y;\ell_1}(E, F; \mathbb{K}).$$

Example 2.1. For the new abovementioned case, taking $X = \ell_{p^*}^w(\cdot)$ and $Y = \ell_p\langle\cdot\rangle$, our result asserts that

$$(E\widehat{\otimes}_{\alpha_{\ell_{p^*}^w,\ell_p\langle\cdot\rangle}} F)' \stackrel{1}{=} \mathcal{L}_{\ell_{p^*}^w,\ell_p\langle\cdot\rangle;\ell_1}(E, F; \mathbb{K}).$$

For the norm $d_p(\cdot)$, is well known that $\left(E\widehat{\otimes}_{d_p} F\right)' \stackrel{1}{=} \Pi_{p^*}(E, F')$, where $1 = 1/p + 1/p^*$. In this case, our result states that

$$\left(E\widehat{\otimes}_{d_p} F\right)' = (E\widehat{\otimes}_{\alpha_{\ell_{p^*}^w,\ell_p}} F)' \stackrel{1}{=} \mathcal{L}_{\alpha_{\ell_{p^*}^w,\ell_p};\ell_1}(E, F; \mathbb{K})$$

and so $\Pi_{p^*}(E, F') = \mathcal{L}_{\alpha_{\ell_{p^*}^w,\ell_p};\ell_1}(E, F; \mathbb{K})$ holds isometrically.

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XIII ENAMA - Novembro 2019 *107–108*

A PROPRIEDADE DA C_0-EXTENSÃO

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Esse trabalho é financiado parcialmente pela Fapesp, processo 2018/09797-2

Abstract

No presente trabalho, investigamos a classe dos espaços de Banach que possuem a propriedade da c_0-extensão. Mostramos que essa classe contém propriamente os espaços de Banach weakly Lindelöf determined. Também estabelecemos propriedades de fechamento e mostramos que diversos espaços não pertencem a essa classe.

1 Introdução

O problema da extensão de operadores limitados remonta aos primórdios da Geometria dos Espaços de Banach e é um assunto central nessa área de pesquisa. O mais famoso teorema sobre extensão de operadores limitados é o Teorema de Hahn–Banach. Esse teorema garante que todo funcional linear limitado definido num subespaço de um espaço normado admite uma extensão linear e limitada ao espaço todo. Um corolário simples do Teorema de Hahn–Banach é que se X é um espaço normado e Y é um subespaço de X, então todo operador limitado definido em Y e tomando valores em l_∞ admite uma extensão linear e limitada a X. No entanto, se trocarmos ℓ_∞ pelo seu subespaço c_0, então a situação muda drasticamente. Por exemplo, é um resultado clássico atribuído a Phillips [3] que a identidade do espaço c_0 não admite uma extensão linear e limitada a l_∞. Nesse contexto, o celebrado Teorema de Sobczyk [4] desempenha um papel central. Esse teorema garante que se um espaço de Banach X é separável, então todo operador limitado definido num subespaço fechado de X e tomando valores em c_0 admite uma extensão linear e limitada a X. Para estudar generalizações do Teorema de Sobczyk no contexto de espaços de Banach não separáveis, a seguinte definição foi introduzida em [2].

Definição 1.1. *Dizemos que um espaço de Banach X possui a propriedade da c_0-extensão (c_0-EP) se para todo subespaço fechado Y de X e todo operador limitado $T : Y \to c_0$ existe uma extensão $\tilde{T} : X \to c_0$ linear e limitada de T.*

Usando essa terminologia, o Teorema de Sobczyk diz que todo espaço de Banach separável possui a c_0-EP. Uma adaptação da prova do Teorema de Sobczyk nos mostra que os espaços de Banach weakly compactly generated também possuem a c_0-EP. Recorde que um espaço de Banach é dito *weakly compactly generated* (WCG) se ele possui um subconjunto fracamente compacto e linearmente denso e que a classe dos espaços WCG contém propriamente os espaços de Banach separáveis e reflexivos. No presente trabalho, investigamos a classe dos espaços de Banach que possuem a c_0-EP.

2 Resultados Principais

Esse é um trabalho em andamento. O principal resultado obtido até agora é o estabelecimento da c_0-EP para a classe dos espaços de Banach weakly Lindelöf determined (Teorema 2.1). Dizemos que um espaço de Banach é *weakly Lindelöf determined* (WLD) se a bola unitária fechada de seu espaço dual, munida da topologia fraca-estrela, é um compacto de Corson. Um espaço compacto Haudorff é dito um *compacto de Corson* se ele é homeomorfo a um subconjunto de $\Sigma(I)$, onde I é um conjunto, $\Sigma(I)$ está munido da topologia produto e:

$$\Sigma(I) = \big\{(x_i)_{i\in I} \in \mathbb{R}^I : \{i \in I : x_i \neq 0\} \text{ é enumerável}\big\}.$$

Note que se um espaço de Banach é WCG, então a bola unitária fechada de seu espaço dual, munida da topologia fraca-estrela, é um compacto de Eberlein e portanto, a classe dos espaços WLD generaliza a classe dos WCG, já que todo compacto de Eberlein é um compacto de Corson. Recorde que um espaço compacto Hausdorff é dito um *compacto de Eberlein* se ele é homeomorfo a um subconjunto fracamente compacto de um espaço de Banach, munido da topologia fraca.

Teorema 2.1. *Se X é um espaço de Banach WLD, então X possui a c_0-EP.*

É importante destacar que existem espaços de Banach que possuem a c_0-EP e não são WLD. Em [1] foi mostrado que se K é uma reta compacta monolítica, então $C(K)$ possui a c_0-EP e existem retas compactas monolíticas cujos espaços de funções contínuas não são WLD. Como usual, dado um espaço compacto Hausdorff K, denotamos por $C(K)$ o espaço de Banach das funções contínuas definidas em K e tomando valores na reta real, munido da norma do supremo.

Outro resultado estabelecido no presente trabalho é o seguinte.

Teorema 2.2. *Se κ é um cardinal não enumerável, então $C(2^\kappa)$ não possui a c_0-EP.*

O resultado acima é muito interessante, já que os espaços $C(2^\kappa)$ possuem uma propriedade mais fraca que a c_0-EP. A saber, se Y é um subespaço fechado e **separável** de $C(2^\kappa)$, então todo operador limitado $T : Y \to c_0$ admite uma extensão limitada $\tilde{T} : C(2^\kappa) \to c_0$.

Um outro resultado de destaque é o fato de que a c_0-EP é estável para quocientes. Mais precisamente, estabelecemos o seguinte resultado.

Teorema 2.3. *Sejam X e Z espaços de Banach e $Q : X \to Z$ uma transformação linear, limitada e sobrejetora. Se X possui a c_0-EP, então Z possui a c_0-EP*

Um corolário importante do Teorema 2.3 é que se K é um espaço compacto Hausdorff tal que $C(K)$ possui a c_0-EP e F é um subconjunto fechado de K, então $C(F)$ também possui a c_0-EP. Portanto, segue do Teorema 2.2 que se um espaço compacto Hausdorff K contém uma cópia de 2^κ, para um cardinal não enumerável κ, então $C(K)$ não possui a c_0-EP.

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ABOUT SINGULARITY OF TWISTED SUMS

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Abstract

In this talk we study some aspects of the structure of twisted sums. Although a twisted sums of Köthe spaces is not necessarily a Köthe space, those which are obtained by the complex interpolation method are equipped in a natural way with an L_∞ - module structure. In this case we study disjoint versions of basic notions of the theory of twisted sums. We also consider some properties in the direction of local theory.

1 Introduction

Recall that a twisted sum of two Banach spaces Y, Z is a quasi-Banach space X which has a closed subspace isomorphic to Y such that the quotient X/Y is isomorphic to Z. Equivalently, X is a twisted sum of Y, Z if there exists a short exact sequence

$$0 \longrightarrow Y \longrightarrow Z \longrightarrow X \longrightarrow 0.$$

According to Kalton and Peck [5], twisted sums can be identified with homogeneous maps $\Omega : X \to Y$ satisfying

$$\|\Omega(x_1 + x_2) - \Omega x_1 - \Omega x_2\| \leq C(\|x_1\| + \|x_2\|),$$

which are called quasi-linear maps, and induce an equivalent quasi-norm on X (seen algebraically as $Y \times X$) by

$$\|(y, x)\|_\Omega = \|y - \Omega z\| + \|x\|.$$

This space is usually denoted $Y \oplus_\Omega X$. When Y and X are, for example, Banach spaces of non-trivial type, the quasi-norm above is equivalent to a norm; therefore, the twisted sum obtained is a Banach space. The quasi-linear map is said to be trivial when $Y \oplus_\Omega X$ is isomorphic to the direct sum $Y \oplus X$.

We are mainly interested in the ambient of Köthe functions spaces over a σ-finite measure space (Σ, μ) endowed with their L_∞-module structure. A Köthe function space K is a linear subspace of $L_0(\Sigma, \mu)$, the vector space of all measurable functions, endowed with a quasi-norm such that whenever $|f| \leq g$ and $g \in K$ then $f \in K$ and $\|f\| \leq \|g\|$ and so that for every finite measure subset $A \subset \Sigma$ the characteristic function 1_A belongs to X. A particular case of which is that of Banach spaces with a 1-unconditional basis with their associated ℓ_∞-module structure.

Definition 1.1. *An L_∞-centralizer (resp. an ℓ_∞-centralizer) on a Köthe function (resp. sequence) space $\mathcal{K}$ is a homogeneous map $\Omega : \mathcal{K} \to L_0$ such that there is a constant C satisfying that, for every $f \in L_\infty$ (resp. ℓ_∞) and for every $x \in \mathcal{K}$, the difference $\Omega(fx) - f\Omega(x)$ belongs to $\mathcal{K}$ and*

$$\|\Omega(fx) - f\Omega(x)\|_\mathcal{K} \leq C\|f\|_\infty\|x\|_\mathcal{K}.$$

Observe that a centralizer Ω on $\mathcal{K}$ does not take values in $\mathcal{K}$, but in L_0, and still it induces an exact sequence

$$0 \longrightarrow \mathcal{K} \xrightarrow{\jmath} d_\Omega\mathcal{K} \xrightarrow{Q} \mathcal{K} \longrightarrow 0$$

as follows: $d_\Omega\mathcal{K} = \{(w,x) : w \in L_0, x \in \mathcal{K} : w - \Omega x \in \mathcal{K}\}$ endowed with the quasi-norm

$$\|(w,x)\|_{d_\Omega\mathcal{K}} = \|x\|_\mathcal{K} + \|w - \Omega x\|_\mathcal{K}$$

and with obvious inclusion $\jmath(x) = (x,0)$ and quotient map $Q(w,x) = x$. The reason is that a centralizer "is" quasi-linear, in the sense that for all $x, y \in \mathcal{K}$ one has $\Omega(x+y)-\Omega(x)-\Omega(y) \in \mathcal{K}$ and $\|\Omega(x+y)-\Omega(x)-\Omega(y)\| \leq C(\|x\|+\|y\|)$ for some $C > 0$ and all $x, y \in \mathcal{K}$. Centralizers arise naturally by complex interpolation [1] as can be seen in [4].

In this talk we study the *disjointly supported* versions of the basic (trivial, locally trivial, singular and supersingular) notions in the theory of centralizers and present several examples.

2 Main Results

An operator between Banach spaces is said to be *strictly singular* if no restriction to an infinite dimensional closed subspace is an isomorphism. Analogously, a quasi-linear map (in particular, a centralizer) is said to be *singular* if its restriction to every infinite dimensional closed subspace is never trivial. An exact sequence induced by a singular quasi-linear map is called a *singular sequence.* A quasi-linear map is singular if and only if the associated exact sequence has strictly singular quotient map. Singular ℓ_∞-centralizers exist and the most natural example is the Kalton-Peck map $\mathcal{K}_p : \ell_p \to \ell_p$, $0 < p < +\infty$, defined by $\mathcal{K}_p(x) = x \log \frac{|x|}{\|x\|_p}$.

In [3] where the authors introduced the notion of disjointly singular centralizer on Köthe function spaces, and proved that disjoint singularity coincides with singularity on Banach spaces with unconditional basis and presented a technique to produce disjointly singular centralizers via complex interpolation. An important fact to consider is that the fundamental Kalton-Peck map [5] is disjointly singular on L_p [3, Proposition 5.4], but it is not singular [6]. In fact, as the last stroke one could wish to foster the study of disjoint singularity is the argument of Cabello [2] that no centralizer on L_p can be singular that we extend here by showing that no centralizer can be singular. It is thus obvious that while singularity is an important notion in the domain of Köthe sequence spaces, disjoint singularity is the core notion in Köthe function spaces.

Theorem 2.1. *No singular L_∞-centralizers exist on (admissible) superreflexive Köthe funcion spaces. More precisely, every L_∞-centralizer on an admissible superreflexive Köthe function space is trivial on some copy of ℓ_2.*

The results are part of the work *On disjointly singular centralizers*, https://arxiv.org/pdf/1905.08241.pdf.

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$C(K)$ COM MUITOS QUOCIENTES INDECOMPONÍVEIS

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O primeiro autor teve apoio financeiro da FAPESP (projeto temático 2016/25574-8 e auxílio regular 2018/10254-3) e o segundo, bolsa de doutorado da CAPES.

Abstract

Assumindo o Axioma $\diamondsuit$, mostramos a existência de um espaço de Banach da forma $C(K)$ que contém 2^ω quocientes indecomponíveis não isomorfos e também l_∞ como quociente.

1 Introdução

Para K um espaço topológico compacto e Hausdorff, seja $C(K)$ o espaço de Banach real das funções contínuas de K em $\mathbb{R}$ munido da norma do supremo. Um operador $T : C(K) \longrightarrow C(K)$ linear e contínuo é dito *multiplicador fraco* se, para toda sequência $(f_n)_{n\in\mathbb{N}}$ limitada e duas a duas disjunta (i.e., $f_n \cdot f_m = 0$, se $n \neq m$) em $C(K)$ e para toda sequência $(x_n)_{n\in\mathbb{N}}$ de pontos distintos em K tal que $f_n(x_n) = 0$, para todo $n \in \mathbb{N}$, a sequência $T(f_n)(x_n)$ converge a 0.

Dizemos que $C(K)$ *tem poucos operadores* se todo operador em $C(K)$ é multiplicador fraco. Foi provado em [4] que se $K \setminus F$ é conexo, para todo F finito, e $C(K)$ tem poucos operadores, então $C(K)$ é *indecomponível*, i.e., tem dimensão infinita e não possui subespaço complementado de dimensão e codimensão infinita. Se, além disso, K não contém dois abertos disjuntos V_1 e V_2 tais que $|\overline{V_1} \cap \overline{V_2}| = 1$, a hipótese da conexidade de K é suficiente para garantir que $C(K)$ é indecomponível (consequência de resultados de [4] e [1]).

Apresentamos aqui o seguinte resultado: assumindo $\diamondsuit$, existem $C(K)$ indecomponível e uma família $(L_\xi)_{\xi<2^\omega}$ de subespaços fechados de K tais que $(C(L_\xi))_{\xi<2^\omega}$ são indecomponíveis e dois a dois não isomorfos. Além disso, K contém uma cópia homeomórfica de $\beta\mathbb{N}$, o que diferencia a construção daquela obtida no Corolário 5.4 de [2].

2 Principais resultados

Para um compacto conexo $K \subset [0,1]^{2^\omega}$ e um real $r \in]0,1]$ denotaremos por K_r o conjunto $\overline{\{x \in K : x(0) < r\}}$, visto como subespaço topológico de K. Provamos o seguinte teorema:

Teorema 2.1. *($\diamondsuit$) Existe um compacto $K \subset [0,1]^{2^\omega}$ tal que, para todo $L \in \{K\} \cup \{K_r : r \in]0,1]\}$, temos*

(a) *L é conexo e não contém abertos disjuntos V_1 e V_2 tais que $|\overline{V_1} \cap \overline{V_2}| = 1$;*

(b) *todo operador em $C(L)$ é multiplicador fraco;*

(c) *se $0 < r < s \leq 1$, $C(K_r)$ não é isomorfo a $C(K_s)$;*

(d) *K contém um subespaço homeomorfo a $\beta\mathbb{N}$.*

Do teorema e dos resultados mencionados na introdução segue o seguinte corolário:

Corolário 2.1. *($\diamondsuit$) Sendo K como no Teorema 2.1, $C(K)$ é indecomponível, possui l_∞ como quociente e uma família não enumerável de quocientes indecomponíveis não isomorfos.*

3 Ideia da demonstração

Descreveremos aqui apenas as ideias principais da prova, que se baseiam em [4] e [2].

Se $T : C(K) \longrightarrow C(K)$ não é multiplicador fraco, existem $\varepsilon > 0$, uma sequência $(f_n)_{n\in\mathbb{N}}$ em $C(K)$ limitada e duas a duas disjuntas e uma sequência $(x_n)_{n\in\mathbb{N}}$ de pontos distintos de K tais que, para todo $n \in \mathbb{N}$, $f_n(x_n) = 0$ e $|T(f_n)(x_n)| > \varepsilon$. Usando argumentos combinatórios, podemos assumir que cada f_n tem imagem contida em $[0,1]$ e que $f_n(x_m) = 0$, para todos $m, n \in \mathbb{N}$. Assumimos, ainda, que $(x_n)_{n\in\mathbb{N}}$ pertencem a um conjunto denso fixado.

Construímos K de modo a não ser possível obtermos T com essa propriedade. Para isso, mostramos que, nas condições acima, obtemos funções $(f'_n)_{n\in\mathbb{N}}$ que são “pequenas modificações” de $(f_n)_{n\in\mathbb{N}}$ (em relação à medidas $T^*(\delta_{x_n})$) e um subconjunto infinito e co-infinito b de $\mathbb{N}$ tais que $(f'_n)_{n\in b}$ tem supremo em $C(K)$ e $\overline{\{x_n : n \in b\}} \cap \overline{\{x_n : n \in \mathbb{N} \setminus b\}} \neq \emptyset$. A partir disso provamos a descontinuidade de $T(f)$, onde $f = sup\{f'_n : n \in b\})$. O mesmo argumento se aplica aos subespaços de K que são fechos de abertos (vide [3]).

Fazemos a construção de K usando recursão transfinita. Começamos com $K_2 = [0,1]^2$ e definimos uma sequência $(K_\alpha)_{\alpha \leq 2^\omega}$ de compactos tais que $K_\alpha \subset [0,1]^\alpha$ e $\pi_\beta[K_\alpha] = K_\beta$, para $\beta < \alpha$. Nos ordinais limites definimos K_α como o limite inverso de $(K_\beta)_{\beta<\alpha}$ e tomamos K como K_{2^ω}.

A chave da construção está no passo sucessor da definição recursiva. Em cada passo α “destruímos” um operador não multiplicador fraco ao adicionar o supremo de $(f'_n)_{n\in b}$ descrito acima. Para isso, a partir de uma enumeração pré-fixada, que estabelecemos utilizando o princípio $\diamondsuit$, encontramos $b \subset \mathbb{N}$ e uma sequência $(g_n)_{n\in b}$ de funções contínuas duas a duas disjuntas de K_α em $[0,1]$ e definimos

$$K_{\alpha+1} = \overline{\{(x, \sum_{n\in b} g_n(x)) : \exists U \in \tau_{K_\alpha}(x \in U \wedge |\{n \in b : U \cap supp(g_n) \neq \emptyset\}| < \omega)\}}, \tag{1}$$

onde o fecho está sendo tomado em $K_\alpha \times [0,1]$, τ_{K_α} é o conjunto de abertos de K_α e $supp(g_n)$ é o suporte de g_n, i.e., o fecho do conjunto dos pontos onde a função é não nula.

Seguindo a nomenclatura de Koszmider, em [4], o espaço definido em 1 é chamado de *extensão de* K_α *por* $(g_n)_{n\in b}$. A função $g : K_{\alpha+1} \subset K_\alpha \times [0,1] \longrightarrow [0,1]$ dada por $g(x,t) = t$ é o supremo de $(g_n \circ \pi_{\alpha,\alpha+1})_{n\in b}$ em $K_{\alpha+1}$. A construção irá garantir que, se $f'_n = g_n \circ \pi_{\alpha,2^\omega}$, em $C(K)$, então $g \circ \pi_{\alpha+1,2^\omega}$ é o supremo de $(f'_n)_{n\in b}$ em $C(K)$.

Para garantir a conexidade de K e de cada K_r, precisamos de uma hipótese adicional sobre a extensão, a qual acunhamos de *extensão completa*: para cada ponto $x \in K_\alpha$, $\pi_{\alpha,\alpha+1}[\{x\}]$ ou é unitário ou igual a $\{x\} \times [0,1]$. As modificações das f_n's mencionadas anteriormente servem justamente garantir que a extensão pelas funções correspondentes no passo α da construção seja completa.

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XIII ENAMA - Novembro 2019 *113–114*

ON A FUNCTION MODULE WITH APPROXIMATE HYPERPLANE SERIES PROPERTY

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Abstract

We present a sufficient and necessary condition for a function module space X to have the approximate hyperplane series property (AHSP). As a consequence, we have that the Banach space E has the $AHSP$ if, and only if $\mathcal{C}(K, E)$ has the $AHSP$.

1 Introduction

Let E and F be complex Banach spaces. The Bishop-Phelps theorem states that the set of norm-attaining functionals on E is dense in E^* [2]. After the celebrated Bishop-Phelps theorem, it was a natural question whether the set of norm-attaining linear operators $NA(E, F)$ is dense in $\mathcal{L}(E, F)$ for all Banach spaces E and F. In 1963, J. Lindenstrauss [6] gave a counterexample showing that it does not hold in general. He also proved that the set of all norm-attaining operators is dense in the space of $\mathcal{L}(E, F)$, when E is reflexive. Motivated by the study of numerical range of operators, B. Bollobás proved a refinement of the Bishop-Phelps theorem, nowadays known as the Bishop-Phelps-Bollobás theorem [3, Theorem 1]. In 2008, Acosta, Aron, García and Maestre [1] introduced the notion of Bishop-Phelps-Bollobás theorem for operators (BPBp for operators, in short) [see Definition 2.1]. The BPBp for operators is a stronger property than the denseness of norm-attaining operators. It has been known that the set $NA(\ell_1, F)$ is dense in $\mathcal{L}(E, F)$, because ℓ_1 has a geometric property named α (of Schachermayer) even though the pair (ℓ_1, F) does not have the BPBp, for all F. When the space F has a special property called Approximate Hyperplane Series Property (AHSp, in short), this affirmation is true. This property was introduced in [1], with the purpose of characterizing those Banach spaces F such that (ℓ_1, F) has the BPBp for operators.

In this note we study when a function module space X [see definition 2.3] has the AHSp and we obtain that the space $\mathcal{C}(K, E)$ has the AHSp if, and only if, a Banach space E has AHSp. In this sense, we generalized a result of Choi and Kim [4]. These results are part of the work *On a Function Module with the Approximate Hyperplane Series Property* [5].

2 Main Results

Definition 2.1. *Let E and F Banach spaces. We say that the pair (E, F) has the Bishop-Phelps-Bollobás property for operators (shortly* BPBp for operators*) if given $\varepsilon > 0$, there is $\eta(\varepsilon) > 0$ such that whenever $T \in S_{\mathcal{L}(E,F)}$ and $x_0 \in S_E$ satisfy that $\|Tx_0\| > 1 - \eta(\varepsilon)$, then there exist a point $u_0 \in S_E$ and an operator $S \in S_{\mathcal{L}(E,F)}$ satisfying the following conditions*

$$\|Su_0\| = 1, \quad \|u_0 - x_0\| < \varepsilon, \quad \text{and} \quad \|S - T\| < \varepsilon.$$

Definition 2.2. *A Banach space E has the Approximate Hyperplane Series Property(AHSp) if for all $\epsilon > 0$ there exist $0 < \gamma(\epsilon) < \epsilon$ and $\eta(\epsilon) > 0$ with $\lim_{\epsilon \to 0^+} \gamma(\epsilon) = 0$ such that for every sequence, $(x_k)_{k=1}^{\infty} \subset B_E$ and every convex series $\sum_{k=1}^{\infty} \alpha_k x_k$ satisfying*

$$\left\| \sum_{k=1}^{\infty} \alpha_k x_k \right\| > 1 - \eta(\epsilon),$$

there exist a subset $A \subset \mathbb{N}$, $\{z_k : k \in A\} \subset S_E$ and $x^* \in S_{E^*}$ such that

(i) $\sum_{k\in A} \alpha_k > 1 - \gamma(\epsilon)$,

(ii) $\|z_k - x_k\| < \epsilon$ *for all* $k \in A$,

(iii) $x^*(z_k) = 1$ *for all* $k \in A$.

Definition 2.3. *Function Module is (the third coordinate of) a triple $(K, (X_t)_{t\in K}, X)$, where K is a nonempty compact Hausdorff topological space, $(X_t)_{t\in K}$ a family of Banach spaces, and X a closed $C(K)$-submodule of the $C(K)$-module $\prod_{t\in K}^{\infty} X_t$ (the ℓ_∞-sum of the spaces X_t) such that the following conditions are satisfied:*

1. *For every $x \in X$, the function $t \to \|x(t)\|$ from K to $\mathbb{R}$ is upper semi-continuous.*
2. *For every $t \in K$, we have $X_t = \{x(t) : x \in X\}$.*
3. *The set $\{t \in K : X_t \neq 0\}$ is dense in K.*

Theorem 2.1. *Let $(K, (X_t)_{t\in K}, X)$ be a complex function module and $\epsilon > 0$. Suppose that for all $t \in K$, $(X_t)_{t\in K}$ has the AHSP with the same function $\eta(\epsilon)$ given by Definition 2.2, and for every $x_t \in X_t$ there exists $f \in X$ such that $f(t) = x_t$ and $||f \leq ||x_t$ then X has the AHSP.*

Theorem 2.2. *Let $(K, (X_t)_{t\in K}, X)$ be a complex function module where $X_t = E$, for all $t \in K$ for some Banach space E. Suppose that the mapping $t \in K \mapsto ||x(t)$ is continuous for all $x \in X$. If X has the AHSP, then X_t has the AHSP for all $t \in K$.*

Corollary 2.1. *Let X be a dual complex Banach space such that X can be regarded as a function module space, where $X_t = E$, for all $t \in K$ and E a Banach space. Then, X has the AHSP if and only if X_t has the AHSP for all $t \in K$.*

Corollary 2.2. *Let $K \neq \emptyset$ compact Hausdorff topological space and E be a Banach space. Then E has the AHSP if, and only if $\mathcal{C}(K, E)$ has the AHSP.*

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XIII ENAMA - Novembro 2019 *115–116*

A PROPRIEDADE DE SCHUR POSITIVA É UMA PROPRIEDADE DE 3 RETICULADOS

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Resumo

Seja I um ideal fechado do reticulado de Banach E. Provamos que se dois dos três reticulados de Banach E, I e E/I têm a propriedade de Schur positiva, entÃ£o o terceiro reticulado também tem essa propriedade.

1 Introdução

Em espaços de Banach dizemos que uma propriedade $\mathcal{P}$ é uma *propriedade de 3 espaços (ou 3-SP)* no sentido fraco se o espaço X tiver $\mathcal{P}$ sempre que um subespaço fechado Y de X e o espaço quociente X/Y tiverem $\mathcal{P}$. E $\mathcal{P}$ é uma *propriedade de 3 espaços* no sentido forte se, dado um subespaço fechado Y do espaço de Banach X, se dois dos espaços X, Y e X/Y têm $\mathcal{P}$, entÃ£o o terceiro espaço também tem $\mathcal{P}$.

É conhecido que a propriedade de Schur (sequências fracamente nulas sÃ£o nulas em norma) é uma propriedade de 3 espaços no sentido fraco [3, Theorem 6.1.a], mas nÃ£o no sentido forte (basta notar que ℓ_2 é um quociente de ℓ_1).

No contexto de reticulados de Banach, para que o quociente E/I de um reticulado de Banach E por um subreticulado fechado I seja um reticulado de Banach, é necessário (e suficiente) que I seja um ideal fechado de E. Dessa forma, o conceito análogo Ã propriedade de 3 espaços (no sentido forte) é o seguinte:

Definição 1.1. Dizemos que uma propriedade $\mathcal{P}$ de reticulados de Banach é uma *propriedade de 3 reticulados (3-LP)* se, dado um ideal fechado I do reticulado de Banach E, se dois dos reticulados E, I e E/I têm $\mathcal{P}$, entÃ£o o terceiro reticulado também tem $\mathcal{P}$.

A seguinte propriedade tem sido muito estudada no contexto de reticulados de Banach:

Definição 1.2. Um reticulado de Banach E tem a *propriedade de Schur positiva (PSP)* se toda sequência positiva fracamente nula em E converge em norma para zero.

Exemplo 1.1. Reticulados de Banach com a propriedade de Schur e AL-espaços, em particular espaços $L_1(\mu)$, sÃ£o exemplos de reticulados de Banach com a PSP (veja [5]).

O objetivo deste trabalho é provar que a PSP é uma propriedade de 3 reticulados e apresentar algumas consequências e exemplos.

Seguiremos a notaçÃ£o e terminologia padrÃ£o da teoria de espaços de Riesz e reticulados de Banach (veja [1, 2, 4]).

2 Resultados Principais

Teorema 2.1. *Seja I um ideal fechado de um reticulado de Banach E.*
(a) *Se I e E/I têm a PSP, entÃ£o E tem a PSP.*
(b) *Se E tem a PSP, entÃ£o E/I tem a PSP.*
(c) *A PSP é uma propriedade de 3 reticulados.*

Corolário 2.1. *Se E é um reticulado de Banach com a PSP tal que seu dual topológico E' contém uma cópia reticulada de ℓ_1 entÃ£o o quociente E''/E nÃ£o possui a PSP.*

Exemplo 2.1. *Para cada $n \in \mathbb{N}$ consideremos o reticulado de Banach ℓ_n^∞, onde $\ell_n^\infty = \mathbb{R}^n$ com a norma do máximo e a ordem dada coordenada a coordenada. Consideremos agora o reticulado de Banach obtido a partir da ℓ_1-soma da sequência $(\ell_n^\infty)_n$, isto é, consideremos*

$$E := \left(\bigoplus_{n\in\mathbb{N}} \ell_n^\infty\right)_{\ell_1} := \left\{x = (x_n)_n; x_n \in \ell_n^\infty \text{ para cada } n \in \mathbb{N} \text{ e } \|x\| := \sum_{n=1}^{\infty} \|x_n\| < \infty\right\}$$

com a norma definida acima e a ordem coordenada a coordenada. Como cada ℓ_n^∞ tem a PSP, pois possuem dimensão finita, então E também possui a PSP ([5, p.17]).

Vejamos agora que o dual E' de E contém uma cópia reticulada de ℓ_1. Para isso, relembremos primeiramente que o dual $(\ell_n^\infty)'$ de ℓ_n^∞ é isométrico como reticulado a ℓ_n^1, onde $\ell_n^1 = \mathbb{R}^n$ com a norma da soma. Assim, pelo [1, Theorem 4.6], segue que E' é isométrico como reticulado a

$$E' = \left(\bigoplus_{n\in\mathbb{N}} \ell_n^1\right)_{\ell_\infty} := \left\{x = (x_n)_n; x_n \in \ell_n^1 \text{ para cada } n \in \mathbb{N} \text{ e } \|x\| := \sup_n\{\|x_n\|\} < \infty\right\}$$

Agora basta definirmos a isometria de Riesz entre ℓ_1 e um subreticulado de E',

$$T\colon \ell_1 \longrightarrow \left(\bigoplus_{n\in\mathbb{N}} \ell_n^1\right)_{\ell_\infty} ; \ (x_j)_j \longmapsto ((x_1), (x_1, x_2), (x_1, x_2, x_3), \cdots).$$

Obtemos então que E' possui uma cópia reticulada de ℓ_1 e assim, pelo Corolário 2.1, $\left(\bigoplus_{n\in\mathbb{N}} \ell_n^\infty\right)_{\ell_1}'' \Big/ \left(\bigoplus_{n\in\mathbb{N}} \ell_n^\infty\right)_{\ell_1}$ não possui a PSP. Ainda, Pelo Teorema 2.1, obtemos que qualquer quociente de $\left(\bigoplus_{n\in\mathbb{N}} \ell_n^\infty\right)_{\ell_1}$ por um ideal fechado possui a PSP.

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XIII ENAMA - Novembro 2019 *117–118*

SOBOLEV TRACE THEOREM ON MORREY-TYPE SPACES ON β-HAUSDORFF DIMENSIONAL SURFACES

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Abstract

This paper strengthen to Morrey-Lorentz spaces the principle discover by Adams [1, Theorem 2] and generalised by Xiao and Liu [3] to Hardy spaces, based in Morrey spaces. Precisely, we show that Riesz potential maps

$$I_\delta : \mathcal{M}^{\lambda}_{pl}(\mathbb{R}^n, d\nu) \to \mathcal{M}^{\lambda_\star}_{qs}(M,\, d\mu)$$

if provided the Radon measure μ supported on $\beta-$dimensional surface M satisfies $[\mu]_\beta = \sup_{x\in M,\, r>0} r^{-\beta}\mu(B(x,r)) < \infty$. In particular, the solution of fractional Laplace equation $(-\Delta_x)^{\frac{\delta}{2}} v = f$ satisfies $v \in \mathcal{M}^{\lambda_\star}_{qs}(M,\, d\mu)$, provided that $f \in \mathcal{M}^{\lambda}_{pl}(\mathbb{R}^n, d\nu)$.

1 Introduction

Let μ be a Radon measure supported on β-Hausdorff dimensional surface M of $\mathbb{R}^n$, the symbol $\mathcal{M}^{\ell}_r(M, d\mu)$ here and hereafter expresses the space of real valued $\mu-$measurable functions f on M such that

$$\|f\|_{\mathcal{M}^{\ell}_r(M,d\mu)} = \sup_{Q_R} R^{\frac{\beta}{\ell}-\frac{\beta}{r}} \left(\int_{Q_R} |f(x)|^r d\mu\right)^{\frac{1}{r}} < \infty \tag{1}$$

where the supremum is taken over the balls $Q_R = B(x,R)\cap M$ and $1 \leq r \leq \ell < \infty$. We fix the symbol $\mathcal{M}^{\ell}_r(d\nu) = \mathcal{M}^{\ell}_r(\mathbb{R}^n, d\nu)$ for Morrey space, where $d\nu$ denotes the Lebesgue measure in $\mathbb{R}^n$. The Morrey space $\mathcal{M}^{\ell}_r$ was denoted by $L^{r,\kappa}$ for $\kappa/r = n/\ell$ or denoted by $\mathcal{M}_{r,\kappa}$ with $(n-\kappa)/r = n/\ell$. Let $(-\Delta_x)^{-\frac{\delta}{2}}$ be the so-called Riesz-potential

$$I_\delta f(x) = C(n,\delta)^{-1}\int_{\mathbb{R}^n} |x-y|^{\delta-n} f(y)d\nu(y) \quad \text{as} \quad 0<\delta<n,$$

it is well known from [1, Theorem 2] that Riesz potential has strong trace inequality $I_\delta : L^p(d\nu) \to L^{p_\star}(d\mu)$. In particular, one has the so-called Sobolev trace embedding $\mathcal{D}^{k,p}(\mathbb{R}^n_+) \hookrightarrow L^{p_\star}(\mathbb{R}^{n-1})$ as $1<p<n/k$ and $p<p_\star<\infty$ satisfies $(n-1)/p_\star = n/p - k$. However, in Morrey spaces, as pointed Ruiz and Vega, there is no *Marcinkiewicz interpolation theorem* to make sure that the following weak trace inequality, proved by Adams [2] in 1975,

$$\|I_\delta f\|_{\mathcal{M}^{\lambda_\star}_{q\infty}(d\mu)} \leq C\|f\|_{\mathcal{M}^{\lambda}_p(\mathbb{R}^n)}, \tag{2}$$

implies the strong trace inequality

$$\|I_\delta f\|_{\mathcal{M}^{\lambda_\star}_q(d\mu)} \leq C\|f\|_{\mathcal{M}^{\lambda}_p(\mathbb{R}^n)}. \tag{3}$$

However, employing atomic decomposition of Hardy-Morrey space $\mathfrak{h}^{\lambda}_p = \mathfrak{h}^{\lambda}_p(\mathbb{R}^n, d\nu)$, the space of distributions $f \in \mathcal{S}'(\mathbb{R}^n)$ such that

$$\|f\|_{\mathfrak{h}^{\lambda}_p} = \left\| \sup_{t\in(0,\infty)} |\varphi_t * f| \right\|_{\mathcal{M}^{\lambda}_p} < \infty, \text{ where } \varphi \text{ is a mollifier}$$

the authors [3, Theorem 1.1] have shown that $I_\delta : \mathfrak{h}_p^\lambda(\mathbb{R}^n) \to \mathcal{M}_q^{\lambda_\star}(M, d\mu)$ is continuous if, and only if the Radon measure μ supported on $\beta-$dimensional surface M of $\mathbb{R}^n$ satisfies $[\mu]_\beta < \infty$, provided $\frac{\beta}{\lambda_\star} = \frac{n}{\lambda} - \delta$ and $q\lambda \leq p\lambda_\star$. Since $\mathfrak{h}_{p>1}^\lambda = \mathcal{M}_p^\lambda$, the authors gets the inequality (3). Our first theorem extend the *if-part of [3, Theorem 1.1]* to space of real valued μ-measurable functions f on $M \subset \mathbb{R}^n$ such that

$$\|f\|_{\mathcal{M}_{qs}^{\lambda_\star}(M,d\mu)} = \sup_{Q_R} R^{-\beta\left(\frac{1}{q}-\frac{1}{\lambda_\star}\right)} \|f\|_{L^{qs}(Q_R,d\mu)} < \infty, \tag{4}$$

where $L^{qs}(Q_R, d\mu)$ denotes the Lorentz space

$$\|f\|_{L^{qs}(Q_R)}^s = q\int_0^{\mu Q_R} [t^q \mu_f(t)]^{\frac{s}{q}} \frac{dt}{t} < \infty \tag{5}$$

and μ is a Radon measure with support (sptμ) on $\beta-$dimensional surface $M \subset \mathbb{R}^n$.

Theorem 1.1. *Let $1 < p \leq \lambda < \infty$ and $1 < q \leq \lambda_\star < \infty$ be such that $\lambda/\lambda_\star \leq p/q$ and $1 < p < q < \infty$. If $[\mu]_\beta < \infty$, the map*

$$I_\delta : \mathcal{M}_{pl}^\lambda(\mathbb{R}^n, d\nu) \longrightarrow \mathcal{M}_{qs}^{\lambda_\star}(M,\, d\mu) \ \text{ is continuous}$$

provided $\delta = \frac{n}{\lambda} - \frac{\beta}{\lambda_\star}$, $n - \delta p < \beta \leq n$, $0 < \delta < \frac{n}{\lambda}$ and $1 \leq l < s \leq \infty$.

A few remarks are in order. The solution of fractional Laplace equation $(-\Delta_x)^{\frac{\delta}{2}} v = f$ satisfies $v \in \mathcal{M}_{qs}^{\lambda_\star}(M,\, d\mu)$, provided that $f \in \mathcal{M}_{pl}^\lambda(\mathbb{R}^n, d\nu)$, where $(-\Delta_x)^{\frac{\delta}{2}}$ is given by

$$(-\Delta_x)^{\frac{\delta}{2}} v(x) := C(n,\delta)\, \mathbf{P.V.} \int_{\mathbb{R}^n} \frac{v(x)-v(y)}{|x-y|^{n+\delta}} dy.$$

Indeed, the potential $v = I_\delta f$ solves, in distribution sense, the fractional Laplace equation. If M denotes the n-dimensional Euclidean space $\mathbb{R}^n$ endowed by Lebesgue measure $d\mu = d\nu$, Theorem 1.1 is known as Hardy-Littlewood-Sobolev (HLS) Theorem in Morrey spaces. Now from Stein's Extension of certain regular functions defined on upper half-spaces (also works in Lipschitz domains), we get the following famous Sobolev trace inequality in Morrey spaces.

Corollary 1.1 (Sobolev trace in Morrey)**.** *Let $\frac{1}{p} < \delta < 2$ and $\delta < \frac{n}{\lambda}$ be such that $\frac{n-1}{\lambda_\star} = \frac{n}{\lambda} - \delta$, where $1 < p \leq \lambda < \infty$ and $1 < q \leq \lambda_\star < \infty$ satisfies $\frac{\lambda}{\lambda_\star} \leq \frac{p}{q} < 1$. There is a positive constant $C >$ (independent of f) such that*

$$\|f(x',0)\|_{\mathcal{M}_{qs}^{\lambda_\star}(\partial\mathbb{R}_+^n,d\mu)} \leq C \left\| (-\Delta_x)^{\frac{\delta}{2}} f \right\|_{\mathcal{M}_{pl}^\lambda(\mathbb{R}_+^n)}, \tag{6}$$

where $1 \leq l < s \leq \infty$ and $d\mu = d\sigma$ denotes $(n-1)-$dimensional Lebesgue surface measure on $\partial\mathbb{R}_+^n$.

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UM TEOREMA DE FATORAÇÃO UNIFICADO PARA OPERADORES LIPSCHTIZ SOMANTES

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Abstract

Provaremos uma versão geral do teorema de fatoração para operadores Lipschitz somantes no contexto de espaços métricos. Esta versão fornece e recupera teoremas do tipo fatoração para várias classes de operadores somantes lineares e não-lineares.

1 Introdução

Para $1 \leq p < \infty$, dizemos que um operador linear entre espaços de Banach $u\colon E \longrightarrow F$ é *absolutamente p-somante* (simbolicamente $u \in \Pi_p(E;F)$) se existe uma constante $C \geq 0$ tal que

$$\left(\sum_{j=1}^{m} \|u(x_j)\|^p\right)^{\frac{1}{p}} \leq C \sup_{x^* \in B_{E^*}} \left(\sum_{j=1}^{m} |x^*(x_j)|^p\right)^{\frac{1}{p}},$$

para todos $x_1, \ldots, x_m \in E$ e $m \in \mathbb{N}$ (ver [3]).

Parte do sucesso dessa classe de operadores deve-se às seguintes caracterizações, conhecidas como o Teorema da Dominação de Pietsch e o Teorema da Fatoração de Pietsch: $u \in \Pi_p(E;F)$ se e somente se

• Existem uma constante $C \geq 0$ e uma medida regular de probabilidade de Borel μ sobre B_{E^*} com a topologia fraca-estrela tais que

$$\|u(x)\| \leq C \left(\int_{B_{E^*}} |\varphi(x)|^p \, d\mu\right)^{1/p} \text{ para todo } x \in E.$$

• Existe uma medida regular de probabilidade de Borel μ sobre B_{E^*} com a topologia fraca-estrela e um operador linear limitado $B\colon L_p(B_{E^*}, \mu) \longrightarrow \ell_\infty(B_{F^*})$ tal que o seguinte diagrama é comutativo

$$\begin{array}{ccccc} C(B_{E^*}) & & \xrightarrow{\ j_p\ } & & L_p(B_{E^*}, \mu) \\ \uparrow{\scriptstyle i_E} & & & & \downarrow{\scriptstyle B} \\ E & \xrightarrow[u]{} & F & \xrightarrow[i_F]{} & \ell_\infty(B_{F^*}) \end{array}$$

onde j_p é a inclusão formal e i_E é o mergulho linear canônico, isto é, $i_E(x)(x^*) = x^*(x)$ para $x \in E$ e $x^* \in B_{E^*}$.

Nosso objetivo é mostrar que a tríade

$$\text{Propriedade de Somabilidade} \Leftrightarrow \text{Teorema de Dominação} \Leftrightarrow \text{Teorema de Fatoração}$$

se mantém em um nível muito alto de generalidade.

Em [2, 4] foram apresentadas abordagens num contexto completamente abstrato para a primeira equivalência dessa tríade.

2 Resultados Principais

Em toda esta seção, X é um conjunto arbitrário não vazio, (Y, d_Y) é um espaço métrico, K é um espaço compacto de Hausdorff, $C(K) = C(K; \mathbb{K})$ é o espaço de todas as funções contínuas tomando valores em $\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$ com a norma do sup, $\Psi \colon X \longrightarrow C(K)$ é uma função arbitrária e $p \in [1, \infty)$.

Definição 2.1. *Uma função $u \colon X \longrightarrow Y$ é dita **Ψ-Lipschitz p-somante** se existe uma constante $C \geq 0$ tal que*

$$\left(\sum_{j=1}^{m} d_Y(u(x_j), u(q_j))^p \right)^{\frac{1}{p}} \leq C \sup_{\varphi \in K} \left(\sum_{j=1}^{m} |\Psi(x_j)(\varphi) - \Psi(q_j)(\varphi)|^p \right)^{\frac{1}{p}},$$

para todos $x_1, \ldots, x_m, q_1, \ldots, q_m \in X$ e $m \in \mathbb{N}$.

Dado uma medida regular de probabilidade de Borel μ sobre K, denotamos por $j_p \colon C(K) \longrightarrow L_p(K, \mu)$ a inclusão canonical.

Para o nosso principal resultado, lembramos primeiro o conceito de retração de Lipschitz (ver [1, Proposição 1.2]). Seja Z um subconjunto do espaço métrico W. Uma função lipschitziana $r \colon W \longrightarrow Z$ é chamada de *retração de Lipschitz* se sua restrição a Z for a identidade em Z. Quando tal retração de Lipschitz existe, Z é dito que um *retraimento de Lipschitz de W*. Um espaço métrico Z é chamado de *retraimento absoluto de Lipschitz* se for uma retraimento de Lipschitz de cada espaço métrico que o contém.

Agora podemos enunciar nosso resultado principal:

Teorema 2.1. *As seguintes afirmações são equivalentes para uma função u de X em Y.*

(a) *u é Ψ-Lipschitz p-somante.*

(b) *Existem uma medida regular de probabilidade de Borel μ sobre K e uma constante $C \geq 0$ tais que*

$$d_Y(u(x), u(q)) \leq C \left(\int_K |\Psi(x)(\varphi) - \Psi(q)(\varphi)|^p \, d\mu(\varphi) \right)^{1/p}$$

para todos $x, q \in X$.

(c) *Existe uma medida regular de probabilidade de Borel μ sobre K tal que para algum (ou todo) mergulho isométrico J de Y em um retraimento absoluto de Lipschitz Z, existe um função Lipschitz $B \colon L_p(K, \mu) \longrightarrow Z$ tal que o seguinte diagrama comuta*

$$\begin{array}{ccccc} C(K) & & \xrightarrow{\quad j_p \quad} & & L_p(K,\mu) \\ \uparrow{\scriptstyle \Psi} & & & & \downarrow{\scriptstyle B} \\ X & \xrightarrow[u]{} & Y & \xrightarrow[J]{} & Z \end{array}$$

Observação 1. *Esse teorema recupera os teoremas de fatoração para os operadores absolutamente p-somantes, (D, p)-somantes, Lipschitz p-somantes, Lipschitz p-dominados, Σ-operadores absolutamente p-somantes e fornece um teorema do tipo fatoração para os operadores arbitrários somantes com valores em um espaço métrico.*

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MULTILINEAR MAPPINGS VERSUS HOMOGENEOUS POLYNOMIALS AND A MULTIPOLYNOMIAL POLARIZATION FORMULA

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Abstract

We give an elementary proof that the class of homogeneous polynomials encompasses distinct classes of nonhomogeneous polynomials. In particular, (k, m)-linear mappings introduced in [1], as well as multilinear mappings, are specific cases of polynomials. Applications and contributions to the polarization formula are also provided.

1 Introduction

Let us recall the following definition:

Definition 1.1. *Let $m \in \mathbb{N}$, E and F be vector spaces over $\mathbb{K} = \mathbb{C}$ or $\mathbb{R}$, and let $n_1, \ldots, n_m$ be positive integers. A mapping $P : E^m \to F$ is said to be an $(n_1, \ldots, n_m)$-homogeneous polynomial if, for each j with $1 \leq j \leq m$, the mapping*

$$P\left(x_1, \ldots, x_{j-1}, \cdot, x_{j+1}, \ldots, x_m\right) : E \to F$$

is an n_j-homogeneous polynomial for all fixed $x_i \in E$ with $i \neq j$.

When $m = 1$ we have an n_1-homogeneous polynomial in $\mathcal{P}_a({}^{n_1}E; F)$ and when $n_1 = \cdots = n_m = 1$ then we have an m-linear mapping in $\mathcal{L}_a({}^{m}E; F)$. This kind of map is called a *multipolynomial* and we shall denote by $\mathcal{P}_a({}^{n_1,\ldots,n_m}E; F)$ the vector space of all $(n_1, \ldots, n_m)$-homogeneous polynomials from the cartesian product E^m into F. If $n_1 = \cdots = n_m = n$ we use $\mathcal{P}_a({}^{n,\overset{m}{\ldots},n}E; F)$, whereas we shall denote by $\mathcal{P}_a^s({}^{n,\overset{m}{\ldots},n}E; F)$ the subspace of all symmetric members of $\mathcal{P}_a({}^{n,\overset{m}{\ldots},n}E; F)$.

I. Chernega and A. Zagorodnyuk conceived the concept of multipolynomials in [1, Definition 3.1] (with a different terminology), and it was rediscovered in the current notation/language as an attempt to unify the theories of multilinear mappings and homogeneous polynomials between Banach spaces. An illustration of how it works can be seen in [3].

2 Main Results

From now on, for fixed $m, n_1, \ldots, n_m$ positive integers, we shall write $M := \sum_{j=1}^{m} n_j$.

Theorem 2.1. *Let E and F be vector spaces over $\mathbb{K}$. Let $\{e_i\}_{i\in I}$ be a Hamel basis for E and let ξ_i denote the corresponding coordinate functionals. Then, each $P \in \mathcal{P}_a({}^{n_1,\ldots,n_m}E; F)$ can be uniquely represented as a sum*

$$P\left(x_1, \ldots, x_m\right) = \sum_{i_1,\ldots,i_M \in I} c_{i_1\cdots i_M} \prod_{j=1}^{m} \left(\prod_{r_j=1}^{n_j} \xi_{i_{M-(n_j+\cdots+n_m)+r_j}}\right)(x_j)\,,$$

where $c_{i_1\cdots i_M} \in F$ and where all but finitely many summands are zero.

Proof For simplicity, let us do the proof for $m=2$. The proof of the case $m=2$ makes clear that the other cases are similar. Every $x \in E$ can be uniquely represented as a sum $x=\sum_{i\in I}\xi_i(x)e_i$ where almost all of the scalars $\xi_i(x)$ (i.e., all but a finite set) are zero. So, we can write

$$P(x_1,x_2)=\sum_{i_1,\ldots,i_{n_1}\in I}\left(\xi_{i_1}\cdots\xi_{i_{n_1}}\right)(x_1)\overset{\vee}{P}_{(\cdot,x_2)}\left(e_{i_1},\ldots,e_{i_{n_1}}\right).$$

Since

$$\overset{\vee}{P}_{(\cdot,x_2)}\left(e_{i_1},\ldots,e_{i_{n_1}}\right)=\frac{1}{n_1!2^{n_1}}\sum_{\varepsilon_j=\pm1}\varepsilon_1\cdots\varepsilon_{n_1}\overset{\vee}{P}_{\left(\sum_{k=1}^{n_1}\varepsilon_k e_{i_k},\cdot\right)}x_2^{n_2},$$

repeat the process for $\overset{\vee}{P}_{\left(\sum_{k=1}^{n_1}\varepsilon_k e_{i_k},\cdot\right)}$ and the proof is done with

$$c_{i_1\cdots i_M}=\frac{1}{n_1!n_2!2^M}\sum_{\varepsilon_j=\pm1}\varepsilon_1\cdots\varepsilon_M P\left(\sum_{k=1}^{n_1}\varepsilon_k e_{i_k},\sum_{k=1}^{n_2}\varepsilon_{n_1+k}e_{i_{n_1+k}}\right),$$

for every $i_1,\ldots,i_M\in I$. ■

A suitable choice of an M-linear mapping in $\mathcal{L}_a({}^M E^m;F)$, which is equal to P on the diagonal, leads us straight to the first main result:

Corollary 2.1. *Let E and F be vector spaces over $\mathbb{K}$. Then $\mathcal{P}_a\left({}^{n_1,\ldots,n_m}E;F\right)\subset\mathcal{P}_a\left({}^M E^m;F\right)$.*

It is worth noting that (k,m)-linear mappings, introduced by [1, Definition 3.1], are km-homogeneous polynomials. It suffices to observe that $\mathcal{L}_a({}^k_m E;F)=\mathcal{P}_a({}^{m,\overset{k}{\ldots},m}E;F)$ and apply Corollary 2.1. If $n_1=\cdots=n_m=1$, then Corollary 2.1 also implies the following:

Corollary 2.2. *Let E and F be vector spaces over $\mathbb{K}$. Then every m-linear mapping in $\mathcal{L}_a({}^m E;F)$ is an m-homogeneous polynomial in $\mathcal{P}_a({}^m(E^m);F)$.*

Next, we extend the polarization formula to multipolynomials.

Theorem 2.2. *Let $P\in\mathcal{P}_a^s({}^{n,\overset{m}{\ldots},n}E;F)$. Then for all $x_0,\ldots,x_m\in E$ we have*

$$\begin{aligned}&P(x_1,\ldots,x_m)\\&=\frac{1}{m!(n!2^n)^m}\sum_{\varepsilon_k=\pm1}\varepsilon_1\cdots\varepsilon_{mn}P\left(x_0+\sum_{k=1}^{n}\varepsilon_k x_1+\cdots+\sum_{k=1}^{n}\varepsilon_{(m-1)n+k}x_m\right)^m-\frac{1}{m!2^{mn}}R_n(x_1,\ldots,x_m).\end{aligned}$$

If $n=1$, the *reminder-function* R_n vanishes, then we extract the polarization formula for multilinear mappings.

Corollary 2.3 ([2, Theorem 1.10])**.** *Let $A\in\mathcal{L}_a^s({}^m E;F)$. Then for all $x_0,\ldots,x_m\in E$ we have*

$$A(x_1,\ldots,x_m)=\frac{1}{m!2^m}\sum_{\varepsilon_k=\pm1}\varepsilon_1\cdots\varepsilon_m A\left(x_0+\varepsilon_1x_1+\cdots+\varepsilon_m x_m\right)^m.$$

If $n>1$, the pointwise-polynomial nature of a multipolynomial in $\mathcal{P}_a^s({}^{n,\overset{m}{\ldots},n}E;F)$ is an obstacle to obtain, in general, an *exact* polarization formula, that is, the one with null remainder-function. Indeed, an application of Corollary 2.1 allows us to characterize the class of such mappings as a non-trivial subspace of $\mathcal{P}_a^s({}^{n,\overset{m}{\ldots},n}E;F)$.

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EXPANSIVE OPERATORS ON FRÉCHET SPACES

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Abstract

In this work we study a fundamental notion in the area of dynamical systems, called expansivity, for operators on Fréchet spaces. Some authors have been studied this notion for operators on Banach spaces obtaining, in particular, a characterization for expansive weighted shifts. In this work we extend this characterization for expansive weighted shifts on Fréchet sequence spaces.

1 Introduction

Let (M, d) be a metric space. A homeomorphism $h : M \to M$ is said to be *expansive* (see [2]) if there exists some constant $C > 0$ such that, for any pair x, y of distinct points in M, there exists an integer k with $d(h^k(x), h^k(y)) \geq C$. Hence h is expansive precisely when it is "unstable", in the sense of Utz [4], which is used to study the dynamical behavior saying roughly that every orbit can be accompanied by only one orbit with some certain constant. If in the above definition we replace "a integer k" by "a positive integer k", then h is said to be *positively expansive*. In this case h is not required to be homeomorphism. Many authors have been interested in to explore these notions in the context of linear dynamics (see for instance [1, 3]). More precisely, in [3] Eisenberg and Hedlund investigate the relationship between expansivity and spectrum of operators on Banach spaces. In [1] Bernardes Jr et al give, among other things, a complete characterization of weighted shifts on classical Banach sequence spaces satisfying some notions related with expansivity. A natural question is if it is possible to obtain this characterization in the context of Fréchet spaces. In this work we obtain a characterization of expansive weighted shifts on Fréchet sequence spaces (see Theorem 2.2).

As is usual, the letters $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{K}$ denote the sets of integers, positive integers, and of real or complex scalars, respectively. By $\omega(\mathbb{Z}) := \mathbb{K}^{\mathbb{Z}}$ we denote the Fréchet space of all sequences of scalars equipped with its natural product topology, and $\mathcal{H}(\mathbb{C})$ denotes the Fréchet space of all complex-valued holomorphic mappings on $\mathbb{C}$, equipped with the compact-open topology.

2 Main Results

We start with a characterization of expansive (positively expansive) operators on Fréchet spaces.

Proposition 2.1. *Let X be a Fréchet space and $(\|\cdot\|_n)_{n=1}^{\infty}$ be a fundamental increasing sequence of seminorms defining the topology of X. An invertible operator T on X is expansive (positively expansive) if and only if, there exist $n \in \mathbb{N}$ and $C > 0$ such that, for every nonzero $x \in X$ there exists $k \in \mathbb{Z}$ $(k \in \mathbb{N})$ with $\|T^k x\|_n \geq C$.*

Using this characterization we obtain the following results of expansive (positively expansive) operator in the context of Fréchet space.

Example 2.1. *Consider the non-normable Fréchet space $\mathcal{H}_0(\mathbb{C}) := \{f \in \mathcal{H}(\mathbb{C}) : f(0) = 0\}$ under the induced topology by $\mathcal{H}(\mathbb{C})$. If $\phi(z) = \lambda z$, where $\lambda, z \in \mathbb{C}$ and $0 < |\lambda| \neq 1$ then, the composition operator $C_\phi : \mathcal{H}_0(\mathbb{C}) \to \mathcal{H}_0(\mathbb{C})$,*

$f \mapsto f \circ \phi$, is expansive. In fact, if $|\lambda| > 1$ then C_ϕ is positively expansive, and if $0 < |\lambda| < 1$ then $(C_\phi)^{-1}$ is positively expansive.

Proposition 2.2. *Let X be a Fréchet space, $(\|\cdot\|_n)_{n=1}^\infty$ be a fundamental increasing sequence of seminorms which defines the topology of X, and T an operator on X. Then T is expansive (resp. positively expansive) if and only if there is $N \in \mathbb{N}$ such that for each nonzero $x \in X$*

$$\sup_{n\in\mathbb{Z}} \|T^n x\|_N = \infty \quad (\text{resp. } \sup_{n\in\mathbb{N}} \|T^n x\|_N = \infty).$$

As application of the previous proposition, we obtain a characterization of expansivity for bilateral weighted shifts in terms of the weights. We recall that $F_w : \omega(\mathbb{Z}) \to \omega(\mathbb{Z})$ (resp. $B_w : \omega(\mathbb{Z}) \to \omega(\mathbb{Z})$ denotes the *bilateral weighted forward (resp. backward) shift* on $\omega(\mathbb{Z})$ given by

$$F_w((x_k)_{k\in\mathbb{Z}}) = (w_{k-1}x_{k-1})_{k\in\mathbb{Z}} \quad (\text{resp. } B_w((x_k)_{k\in\mathbb{Z}}) = (w_{k+1}x_{k+1})_{k\in\mathbb{Z}}),$$

where $w = (w_k)_{k\in\mathbb{Z}}$ is a sequence of nonzero scalars, called a *weight sequence*.

The following characterization of expansivity for weighted shifts was obtained in [1].

Theorem 2.1. *Let $X = \ell_p(\mathbb{Z})$ $(1 \le p < \infty)$ or $X = c_0(\mathbb{Z})$, and consider a weight sequence $w = (w_k)_{k\in\mathbb{Z}}$ with $\inf_{k\in\mathbb{Z}} |w_k| > 0$. The following assertions are equivalent:*

(i) *$F_w : X \to X$ is expansive;*

(ii) (a) $\sup_{n\in\mathbb{N}} |w_1 \cdot \ldots \cdot w_n| = \infty$ *or* (b) $\sup_{n\in\mathbb{N}} |w_{-n} \cdot \ldots \cdot w_{-1}|^{-1} = \infty$;

(iii) (a) $F_w : X \to X$ *or* (b) $F_w^{-1} : X \to X$ *is positively expansive.*

Using some ideas of the proof of Theorem 2.1, we extend this result for weighted shifts on Fréchet sequence spaces.

Theorem 2.2. *Let X be a Fréchet sequence space over $\mathbb{Z}$ in which $(e_k)_{k\in\mathbb{Z}}$ is a basis. Suppose that the bilateral weighted forward shift F_w is an invertible operator on X. Then the following assertions are equivalent:*

(i) *$F_w : X \to X$ is expansive;*

(ii) *there exists $N \in \mathbb{N}$ such that*

(a) $\sup_{n\in\mathbb{N}} |w_1 \cdot \ldots \cdot w_n| \|e_{n+1}\|_N = \infty$ *or* (b) $\sup_{n\in\mathbb{N}} |w_{-n+1} \cdot \ldots \cdot w_{-1}w_0|^{-1} \|e_{-n+1}\|_N = \infty$;

(iii) (a) $F_w : X \to X$ *or* (b) $F_w^{-1} : X \to X$ *is positively expansive.*

The study of expansivity for invertible bilateral weighted backward shifts can be reduced to the corresponding case of forward shifts.

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A GENERAL ONE-SIDED COMPACTNESS RESULT FOR INTERPOLATION OF BILINEAR OPERATORS

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Abstract

The behavior of bilinear operators acting on the interpolation of Banach spaces in relation to compactness is analyzed, and an one-sided compactness theorem is obtained for bilinear operators interpolated by the ρ interpolation method.

1 Introduction

Multilinear operators appear naturally in several branches of classical harmonic analysis and functional analysis, including the theory of ideals of operators in Banach spaces. Recently, several singular multilinear operators have been intensively studied and the research on bilinear Hilbert transform (see [6]) has shown the need for new results for bilinear operators. See, for example, the paper by L. Grafakos and N. Kalton [5].

We are interested in this essay in the behavior of compactness for bilinear operators under interpolation by the real method. The study on the behavior of linear compact operators under interpolation has its origin in the classical work of M. A. Krasnoselskii, for L^p spaces. Afterwards, several authors worked on the general question of compactness of operators for interpolation of abstract Banach spaces. The first main authors were J. L. Lions and J. Peetre [1] and A. Calderón [1]. The proof that the real method preserves compactness with only a compact restriction in the extreme spaces was given independently in [2] and [3]. That research continued in the last years not only for more general interpolation methods, and also for the measure of non-compactness, entropy and approximation numbers.

For the real method, if $\mathbf{E} = (E_0, E_1)$, $\mathbf{F} = (F_0, F_1)$ and $\mathbf{G} = (G_0, G_1)$ are Banach couples, a classical result by Lions-Peetre assures that if T is a bounded bilinear operator from $(E_0 + E_1) \times (F_0 + F_1)$ into $G_0 + G_1$, whose restrictions $T|E_k \times F_k$ $(k = 0, 1)$ are also bounded from $E_k \times F_k$ into G_k $(k = 0, 1)$, then T is bounded from $\mathbf{E}_{\theta,p;J} \times \mathbf{F}_{\theta,q;J}$ into $\mathbf{G}_{\theta,r;J}$, where $0 < \theta < 1$ and $1/r = 1/p + 1/q - 1$.

For the multilinear case, the study on the behavior of compact operators in the interpolation spaces goes back to A. P. Calderón [1, p. 119-120]. Under an approximation hypothesis, Calderón established an one-side type general result, but restricted to complex interpolation spaces. On the other hand, the behavior of compact multilinear operators under real interpolation functors until recently had not been investigated. In paper [4], generalizations of Lions-Peetre compactness theorems [1, Theorem V.2.1] (the one with the same departure spaces) and [1, Theorem V.2.2](the one with the same arriving spaces), Hayakawa's (i.e. a two-side result without approximation hypothesis) and a compactness theorem of Persson type were obtained. Here we obtain a theorem of Cwikel type for the compactness of interpolation of bilinear operators by the ρ-method. The results here were published in [8].

2 Main Results

Given X, Y and Z Banach spaces and a bilinear operator $T : X \times Y \to Z$, the norm of T is defined by

$$||T||_{Bil(X\times Y,Z)} = \sup\{||T(x, y)||_Z : (x, y) \in U_{X\times Y}\},$$

where $U_{X\times Y}$ is the unit closed ball and in $X\times Y$ we are considering the norm $||(x,y)|| = \max\{||x||_X, ||y||_Y\}$. We denote by $Bil(X\times Y, Z)$ the space of all bounded bilinear operators from $X\times Y$ into Z.

Given Banach couples $\mathbf{E}=(E_0,E_1)$, $\mathbf{F}=(F_0,F_1)$ and $\mathbf{G}=(G_0,G_1)$, we shall denote by $Bil(\mathbf{E}\times\mathbf{F},\mathbf{G})$ the set of all bounded bilinear mappings from $(E_0+E_1)\times(F_0+F_1)$ to G_0+G_1 such that $T|_{E_K\times F_k}$ is *bounded* from $E_k\times F_k$ into G_k, $k=0,1$.

Given Banach couples $\mathbf{E}=(E_0,E_1)$, $\mathbf{F}=(F_0,\ F_1)$, and $\mathbf{G}=(G_0,G_1)$ and intermediate spaces E, F and G respectively, we shall say that the pair $(E\times F, G)$ is a *bilinear interpolation pair of type* ρ, if for all bilinear operator T from $(E_0+E_1)\times(F_0+F_1)$ into G_0+G_1 such that $T: E\times F\to G$ one has

$$||T||_{Bil(E\times F;G)} \leq C||T||_{Bil(E_0\times F_0,G_0)}\,\overline{\rho}\left(\frac{||T||_{Bil(E_1\times F_1,G_1)}}{||T||_{Bil(E_0\times F_0,G_0)}}\right).$$

The following result characterizes the bilinear interpolation operators which are of our interest. For the classical θ method this property was first established by Lions–Peetre [1, Th.I.4.1]. Here, we use the function parameter version from [4].

Given Banach spaces E, F and G, a bounded bilinear mapping T from $E\times F$ into G is *compact* if the image of the set $M=\{(x,y)\in E\times F : \max\{||x||_E,\ ||y||_F\}\leq 1\}$ is a totally bounded subset of G.

Theorem 2.1. *Let* $\mathbf{E}=(E_0,E_1)$, $\mathbf{F}=(F_0,F_1)$ *and* $\mathbf{G}=(G_0,G_1)$ *be Banach couples. Let* $T\in Bil(\mathbf{E}\times\mathbf{F},\mathbf{G})$ *be given, such that the restriction* $T|_{E_0\times F_0}$ *is compact from* $E_0\times F_0$ *into* G_0*. Then, given* $\rho\in\mathcal{B}^{+-}$*,* T *is compact from* $\mathbf{E}_{\gamma,p}\times\mathbf{F}_{\rho,q}$ *into* $\mathbf{G}_{\rho,r}$ *, where* $\gamma(t)=1/\overline{\rho}(t^{-1})$ *and* $1/r=1/p+1/q-1$.

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APPROXIMATION OF CONTINUOUS FUNCTIONS WITH VALUES IN THE UNIT INTERVAL

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Abstract

We give applications of a Stone-Weierstrass type theorem concerning uniform density of certain subsets with property V in $C(X;[0,1])$ and establish a simultaneous interpolation and approximation result in $C(X;[0,1])$ when X is a compact Hausdorff space.

1 Introduction

Throughout this paper we shall assume that X is a compact Hausdorff space and $\mathbb{R}$ denotes the field of real numbers. We shall denote by $C(X;[0,1])$ the set of all continuous functions from X into the unit interval [0,1] and $C(X;\mathbb{R})$ the vector space over $\mathbb{R}$ of all continuous functions from X into $\mathbb{R}$ endowed with the sup-norm $\|f\| = sup\{|f(x)| : x \in X\}$.

The closure of a set F will be denoted by $\overline{F}$.

Several results related to uniform approximation in $C(X;[0,1])$ have been presented in the literature. See, for instance, Jewett [2], Paltineanu at al. [3], Prolla [4] [5].

In 1990, Prolla obtained a result concerning uniform density of subsets of $C(X;[0,1])$ by using a condition called property V. We give applications of this theorem to certain set of polynomials and semi-algebras of type 0.

We also establish a simultaneous interpolation and approximation result in $C(X;[0,1])$ for sublattices by using a Bonsall's version of Kakutani-Stone Theorem [1].

A subset $A \subset C(X;[0,1])$ is said to have *property V* if

1. $\phi \in A$ implies $1-\phi \in A$;
2. $\phi \in A$ and $\psi \in A$ implies $\phi\psi \in A$.

In 1990, Prolla [4] established the following result concerning the density of a subset $L \subset C(X;[0,1])$ having property V.

Theorem 1.1. *Let X be a compact Hausdorff space and $L \subset C(X;[0,1])$ be a subset with property V. Assume that L separates the points of X and for each $x \in X$, there exists $\phi \in L$ such that $0 < \phi(x) < 1$. Then, L is uniformly dense in $C(X;[0,1])$.*

We give some applications of this theorem.

2 Main results

Theorem 2.1. *The set of polynomials*

$$L = \{p : p = 0 \ \textit{or}\ p = 1,\ \textit{or}\ 0 < p(t) < 1, \forall t \in (0,1) \ \textit{and}\ 0 \le p(0), p(1) \le 1\}$$

is uniformly dense in $C([0,1];[0,1])$.

A non-empty subset Ω of $C(X;\mathbb{R})$ is called a semi-algebra if $f+g$, αf, $fg \in \Omega$ whenever $f, g \in \Omega$ and $\alpha \ge 0$. It is called a semi-algebra with identity if it contains the unit function 1. A semi-algebra Ω is said to be of type 0 if $1/(1+f) \in \Omega$ whenever $f \in \Omega$. Every semi-algebra of type 0 is a semi-algebra with identity.

Theorem 2.2. *Let X be a compact Hausdorff space and Ω be a uniformly closed semi-algebra in $C(X;\mathbb{R})$ of type 0 which separates the points of X. Then,*

$$L := \{f \in \Omega : 0 \leq f \leq 1\} = C(X;[0,1]).$$

Corollary 2.1. *Let X and Y be compact Hausdorff spaces and Ω_1 and Ω_2 uniformly closed semi-algebras of type 0 in $C(X;\mathbb{R})$ and $C(Y;\mathbb{R})$ respectively. If $\Omega_1 \otimes \Omega_2$ separates the points of $X \times Y$, then*

$$L := \{f \in \Omega_1 \otimes \Omega_2 : 0 \leq f \leq 1\} = C(X \times Y;[0,1]).$$

Theorem 2.3. *Let L be a sublattice of $C(X;[0,1])$. If L is an interpolating family for $C(X;[0,1])$, then L has the property of simultaneous approximation and interpolation.*

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A LEIBNIZ RULE FOR POLYNOMIALS IN FRACTIONAL CALCULUS

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Abstract

In this talk we shall introduce and prove a new inequality that involves an important case of Leibniz rule regarding Riemann-Liouville and Caputo fractional derivatives of order $\alpha \in (0,1)$ to polynomial functions. Besides being conjectured by some other authors before, this is the first complete proof of such result. In fact, in this talk we present some technical details of the first part of the proof, which involves just polynomial functions, and some fundamental counter-examples to show that the inequality cannot be improved.

1 Introduction

This talk is dedicated to introduce a new inequality that involves an important case of Leibniz rule regarding Riemann-Liouville and Caputo fractional derivatives of order $\alpha \in (0,1)$ to polynomial functions. More specifically, we prove that for any polynomial function $P(t)$ it holds that

$$D^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[D^{\alpha}_{t_0,t}P(t)\Big]P(t), \quad \text{in } (t_0,t_1],$$

and

$$cD^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}f(t)\Big]P(t), \quad \text{in } [t_0,t_1].$$

where above $D^{\alpha}_{t_0,t}$ denotes the Riemann-Liouville fractional derivative and $cD^{\alpha}_{t_0,t}$ the Caputo fractional derivative.

We also prove that the above inequalities cannot be improved by presenting respective counter examples to any similar inequality; in fact, this is obtained as a consequence of a famous Theorem from Gautschi which discuss properties of the Gamma function.

This is a joint work with Prof. Paulo M. Carvalho Neto.

2 Main Results

We begin by rearranging the inequality

$$D^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[D^{\alpha}_{t_0,t}P(t)\Big]P(t), \quad \text{for every } t > t_0,$$

in order to reinterpret it as

$$0 \leq \Big(v_a(t), \mathcal{B}v_a(t)\Big), \tag{1}$$

where $v_a(t) := \big(a_0, a_1(t-t_0), \ldots, a_n(t-t_0)^n\big)$ and $\mathcal{B} = \big(\psi(i,j)\big)$ is a symmetric matrix of order $n+1$, with $i,j \in \{0,\ldots,n\}$, and $\psi(i,j)$ is the function

$$\psi(i,j) = \frac{\Gamma(i+1)}{\Gamma(i+1-\alpha)} + \frac{\Gamma(j+1)}{\Gamma(j+1-\alpha)} - \frac{\Gamma(i+j+1)}{\Gamma(i+j+1-\alpha)}.$$

Above Γ is the standard Euler gamma function.

Doing the same process with the inequality

$$cD^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}P(t)\Big]P(t), \quad \text{for every } t > t_0,$$

we obtain

$$0 \leq \Big(u_a(t), \mathcal{A}u_a(t)\Big), \tag{2}$$

where $u_a(t) = \big(a_1(t-t_0), \ldots, a_n(t-t_0)^n\big)$, $\mathcal{A} = \big(\psi(i,j)\big)$, with $i, j \in \{1, \ldots, n\}$, is a matrix of order n.

By using Schur complement theory, which can be summarized as

Theorem 2.1. *Assume that $B \in M^{n+1}(\mathbb{R})$ is given by*

$$B = \begin{bmatrix} d & e^T \\ e & A \end{bmatrix},$$

with $d \neq 0$, and define matrix

$$E = \begin{bmatrix} \dfrac{e_1e_1}{d} & \dfrac{e_1e_2}{d} & \cdots & \dfrac{e_1e_n}{d} \\ \dfrac{e_2e_1}{d} & \dfrac{e_2e_2}{d} & \cdots & \dfrac{e_2e_n}{d} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{e_ne_1}{d} & \dfrac{e_ne_2}{d} & \cdots & \dfrac{e_ne_n}{d} \end{bmatrix}.$$

Then B is positive definite if, and only if, $d > 0$ and $D := A - E$ is a positive definite matrix.

we obtain our main result, which is described bellow.

Theorem 2.2. *Consider $\alpha \in (0,1)$, $t_0 \in \mathbb{R}$ and $P : \mathbb{R} \to \mathbb{R}$ a polynomial function with real coefficients. Then we have*

$$D^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[D^{\alpha}_{t_0,t}P(t)\Big]P(t), \quad \textit{for every } t > t_0,$$

and

$$cD^{\alpha}_{t_0,t}\big[P(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}P(t)\Big]P(t), \quad \textit{for every } t > t_0.$$

Finally, the last theorem discuss the sharpness of the above inequalities.

Theorem 2.3. *Assume that $\lambda \in \mathbb{R} \setminus \{2\}$.*

(a) Then, there exists a polynomial function with real coefficients $P_\lambda(t)$ satisfying

$$D^{\alpha}_{t_0,t}\big[P_\lambda(t)\big]^2 > \lambda\Big[D^{\alpha}_{t_0,t}P_\lambda(t)\Big]P_\lambda(t), \quad \textit{for some } t > t_0.$$

(b) Also, there exists a polynomial function with real coefficients $Q_\lambda(t)$ satisfying

$$cD^{\alpha}_{t_0,t}\big[Q_\lambda(t)\big]^2 > \lambda\Big[cD^{\alpha}_{t_0,t}Q_\lambda(t)\Big]Q_\lambda(t), \quad \textit{for some } t > t_0.$$

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SPECTRAL THEOREM FOR BILINEAR COMPACT OPERATORS IN HILBERT SPACES

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Abstract

In this work we define the Schur representation of a bilinear operator $T : H \times H \to H$, where H is a separable Hilbert space. After introducing the concepts of self-adjoint bilinear operator, ordered eigenvalues, and eigenvectors, we show when a compact, self-adjoint bilinear operator has a Schur representation. This corresponds to a spectral theorem for T in Hilbert real spaces.

1 Introduction

Spectral Theorem. Suppose $L \in \mathfrak{L}(H)$ is compact and self-adjoint. Then there exists a system of orthonormal eigenvectors $x_1, x_2, \cdots$ of L and corresponding eigenvalues $\lambda_1, \lambda_2, \cdots$ such that

$$L(x) = \sum_{n=1}^{\infty} \lambda_n < x, x_n > x_n \,,$$

for all $x \in H$. The sequence $\{\lambda_n\}$ is decreasing and, if it is infinite, converges to 0. The series on the right hand side converges in the operator norm of $\mathfrak{L}(H)$.

The representation above is called the Schur Representation of L, see [1] for more details.

Our main goal in the current work is to obtain a similar result for bilinear operators. In order to do that, we will define new concepts and prove some new results, showing the similarities and differences with respect to the linear case. We will write $Bil(H)$ to denote the set of all bilinear operators on H.

2 Main Results

Definition 2.1. *Given $T \in Bil(H)$, a real number λ is an eigenvalue of T if there exists $x \in H$, $x \neq 0$, such that $T(x,x) = \lambda x$. In this case, we say that x is an eigenvector of T associated to the eigenvalue λ.*

Theorem 2.1. *Let $T \in Bil(H)$ be compact and self-adjoint. Then, $\lambda = \|T\|$ is an eigenvalue of T with an associated unitary eigenvector x_0.*

Definition 2.2. *An eigenvalue λ of $T \in Bil(H)$ is a **generalized eigenvalue** of T if the set*

$$O(\lambda) = \{x \in H \,:\, T(x,y) = \lambda \frac{< x, y >}{< x, x >} x \,,\ \text{para todo } y \in H \,, x \neq 0\}$$

*is not empty. If $x \in O(\lambda)$ is unitary, then x is an **ordered eigenvector** associated to the generalized eigenvalue λ of T and, in this case, $T(x,y) = \lambda < x, y > x$, for all $y \in H$. We call λ a **ordered eigenvalue**, and x the ordered eigenvector associated to λ.*

Proposition 2.1. *If Se $\lambda \neq 0$ is a generalized eigenvalue of $T \in Bil(H)$, then all $0 \neq \gamma \in \mathbb{R}$ is a generalized eigenvalue of T.*

Theorem 2.2 (Spectral theorem)**.** *Let $T \in Bil(H)$ be a nonzero, compact and self-adjoint operator. Let us define a sequence of operators (T_k) in $Bil(H)$ in the following way. For all $(x, y) \in H \times H$, we set*

$$T_1(x, y) = T(x, y) \quad , \quad \lambda_1 = ||T|| ,$$

and let x_1 be the unitary eigenvector associated to λ_1. Having defined T_k, λ_k and x_k, for $k \geq 1$, we set

$$T_{k+1}(x, y) = T_k(x, y) - \lambda_k < x, x_k >< y, x_k > x_k \quad , \quad \lambda_{k+1} = ||T_{k+1}||,$$

and let x_{k+1} be the unitary eigenvector associated to λ_{k+1}. Suppose that, for each k, we have

$$T_k(x_k, y) = \lambda_k < x_k, y > x_k, \text{ for all } y \in H. \tag{1}$$

If T_k is nonzero for all k, then $(\lambda_k)_k$ is a decreasing sequence of ordered eigenvalues of T converging to zero, $x_k \in O(\lambda_k)$, (x_k) is an orthonormal sequence of vectors and

$$T(x, y) = \sum_{i=1}^{\infty} \lambda_i < x, x_i >< y, x_i > x_i, \tag{2}$$

for all $x, y \in H$.

Theorem 2.3. *Let H be a separable Hilbert space and $T \in Bil(H)$ with Schur representation according to Theorem 2.2. Then, H has an orthonormal basis formed by eigenvectors of T.*

Theorem 2.4. *Let $T \in Bil(H)$ be nonzero, compact and self-adjoint operator with Schur representation according to Theorem 2.2, that is,*

$$T(x, y) = \sum_{i=1}^{\infty} \lambda_i < x, x_i >< y, x_i > x_i \, , \tag{3}$$

for all $x, y \in H$. If $\gamma \neq 0$ is an ordered eigenvalue of T, then $\gamma = \pm\lambda_n$ for some $n \in \mathbb{N}$.

Proposition 2.2. *Under the conditions in Theorem 2.4, $T \in Bil(H)$ has a unique Schur representation.*

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UNIFORMLY POSITIVE ENTROPY OF INDUCED TRANSFORMATIONS

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Abstract

For a continuous surjective map on a perfect metric space X, we study the concept of uniformly positive entropy (u.p.e) for the induced map on the hyperspace of all nonempty closed subsets of X and for the induced map on the space of all Borel probability measures on X. [1]

1 Introduction

Let X be a perfect (i.e. a compact without isolated points) metric space with metric d. We denote by $\mathcal{K}(X)$ the hyperspace of all nonempty closed subsets of X endowed with the *Vietoris topology*; it is well known that $\mathcal{K}(X)$ is compact and the sets

$$\langle U_1, \dots, U_k\rangle = \{F \subset X; F \text{ is closed}, F \subset \cup_{i=1}^{k} U_i, F \cap U_i \neq \varnothing, i = 1, \dots k\}$$

(U_i is open in X for each $i = 1, \dots, k$, $k \in \mathbb{N}$) form a basis for the Vietoris topology. Moreover, the Vietoris topology is given by the so-called *Hausdorff metric*:

$$d_H(F_1, F_2) = \inf\{\delta > 0 : F_1 \subset F_2^{\delta} \text{ and } F_2 \subset F_1^{\delta}\},$$

where $A^{\delta} := \{x \in X : d(x, A) < \delta\}$ is the δ-neighborhood of A ($A \subset X$).

Also, let $\mathcal{B}_X$ be the set of all Borel subsets of X and denote by $\mathcal{M}(X)$ the space of all Borel probability measures on X endowed with the weak*-topology inherited from $\mathcal{C}(X)^*$. It is well known that $\mathcal{M}(X)$ is compact and that its topology is given by the so-called *Prohorov metric*:

$$d_P(\mu, \nu) = \inf\{\delta > 0 : \mu(A) \leq \nu(A^{\delta}) + \delta \text{ for all } A \in \mathcal{B}_X\}.$$

We denote by $\mathcal{C}(X)$ the set of all continuous maps from X into X.

Given $T \in \mathcal{C}(X)$, the induced maps $\overline{T} : \mathcal{K}(X) \to \mathcal{K}(X)$ and $\widetilde{T} : \mathcal{M}(X) \to \mathcal{M}(X)$ are the continuous maps given by

$$\overline{T}(K) := T(K) \qquad (K \in \mathcal{K}(X))$$

and

$$(\widetilde{T}(\mu))(A) := \mu(T^{-1}(A)) \qquad (\mu \in \mathcal{M}(X), A \in \mathcal{B}_X).$$

If T is a homeomorphism, then so are $\overline{T}$ and $\widetilde{T}$.

For a given $n \in \mathbb{N}$, let us consider

$$\mathcal{K}_n(X) = \{K \in \mathcal{K}(X); \text{card}(K) \leq n\}$$

[1] 2010 *Mathematics Subject Classification:* Primary 37B99, 54H20; Secondary 54E52, 60B10, 28A33.
Keywords: Continuous maps, hyperspaces, probability measures, Prohorov metric, Vietoris topology, dynamics.

and

$$\mathcal{M}_n(X) = \Big\{ \frac{1}{n} \sum_{i=1}^{n} \delta_{x_i} \in \mathcal{M}(X); x_i \in X \text{ not necessarily distinct} \Big\},$$

where δ_x is the Dirac measure on x. It is classical that $\bigcup_{n\geq 1} \mathcal{K}_n(X)$ and $\bigcup_{n\geq 1} \mathcal{M}_n(X)$ are dense in $\mathcal{K}(X)$ and $\mathcal{M}(X)$, respectively. So, sometimes it is useful to consider the restrictions of $\overline{T}$ and $\widetilde{T}$ to $\mathcal{K}_n(X)$ and $\mathcal{M}_n(X)$, respectively.

Finally, an open cover $\mathcal{U} = \{U, V\}$ of X is called a standard cover if both U and V are non-dense in X. The system (X, T) is said to have *uniformly positive entropy* (u.p.e) if the entropy $h(T, \mathcal{U}) > 0$ for every standard cover $\mathcal{U}$ of X.

2 Main Results

Theorem 2.1. *Let X be a perfect metric space and let $T : X \to X$ be a continuous surjective map. The following assertions are equivalent:*

(a) (X, T) has u.p.e.

(b) There exists $n \in \mathbb{N}$ such that $(\mathcal{K}_n(X), \overline{T})$ has u.p.e.

(c) $(\mathcal{K}_n(X), \overline{T})$ has u.p.e for all $1 \leq n \leq \infty$, where $\mathcal{K}_\infty(X) = \mathcal{K}(X)$.

Theorem 2.2. *Let X be a perfect metric space and let $T : X \to X$ be a continuous surjective map. The following assertions are equivalent:*

(a) (X, T) has u.p.e.

(b) There exists $n \in \mathbb{N}$ such that $(\mathcal{M}_n(X), \widetilde{T})$ has u.p.e.

(c) $(\mathcal{M}_n(X), \widetilde{T})$ has u.p.e for all $1 \leq n < \infty$.

Moreover, if (X, T) has u.p.e, then $(\mathcal{M}(X), \widetilde{T})$ has u.p.e.

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A PROBABILISTIC NUMERICAL METHOD FOR A PDE OF CONVECTION-DIFFUSION TYPE WITH NON-SMOOTH COEFFICIENTS

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Abstract

We propose an explicit probabilistic numerical method for the integration of deterministic $d-$dimensional PDE of convention-diffusion type with at most Holder continuous coefficients. The approach is based on the probabilistic representation of this type of PDEs through the solution of an associated stochastic transport equation, which remarkably can be efficiently integrated without considering the standard assumptions that typically are needed by convectional numerical integrators for solving the underlying PDE. Results on the convergence of the proposed method are presented.

1 Introduction

Many initial value problems for Partial Differential Equations (PDEs) that arise in applications usually contain rough, non-smooth coefficients defining the PDE. Consequently the application of conventional numerical integrators for such equations does not make any sense [4]. This is the case of the convection-diffusion equation in $\mathbb{R}^d$

$$\begin{aligned} u_t(t,x) + b(t,x)\cdot\nabla u(t,x) - \frac{1}{2}\Delta u(t,x) = 0 \\ u(0,x) = f(x), \end{aligned} \tag{1}$$

where $b : [0 \quad T] \times \mathbb{R}^d \to \mathbb{R}^d$ is measurable, bounded and $\alpha-$Holder continuous in space uniformly in time, for some $\alpha \in (0,1)$. Since convection-diffusion is an essential constituting part of useful practical models, it has been extensively studied and much research has been carried out concerning the numerical approximation of equation (1). In fact, it is well known that there are different ways to discretize convection-diffusion equations e.g., by using finite element and finite difference methods including fully discrete schemes, Methods of lines, Rothe's method, Exponential Fitting, Meshless methods, IMEX methods, etc (see e.g., [5], [2]). However, when the coefficient b in (1) is rough (for instance not differentiable or even continuous) standard numerical integrators fails to work properly due to the assumptions requisite for convergence are not satisfied [4]. That is why is necessary to resort to alternative methods and mathematical tools for devising new integrators specially tailored for equation (1) when there is a lack of regularity in the coefficient b.

The aim of this work is to construct a numerical integrator for the approximation of the PDE (1) when b is not sufficiently smooth, in particular when b is only $\alpha-$Holder continuous in space uniformly in time, for some $\alpha \in (0,1)$. The approach we follow is based on the probabilistic representation of this PDE through the solution of an associated stochastic transport equation, which can be efficiently integrated via the solution of a suitable Random Differential Equations (RDE) without considering the standard assumptions that typically are needed by conventional numerical integrators for solving the underlying PDE.

2 A Probabilistic Representation for the Equation (1) and the proposed method

Under the assumptions for b in the previous section, F. Flandolli et al., [3] proved that the solution of (1) in (t,x) satisfies $u(t,x) = E\left(\phi_{0,t}^{-1}(x)\right)$, where the value $\phi_{0,t}^{-1}(x) \in \mathbb{R}^d$ is such that the solution of

$$dX(t) = b(t, X(t))dt + d\mathbf{W}(t), \quad X(0) = \phi_{0,t}^{-1}(x),$$

satisfies $X(t) = x$. Here $\mathbf{W}(t) = (W^1(t),,...,W^d(t))$ is a standard Wiener processes.

2.1 The definitive method

Based on results from [1], the numerical integrator for computing the approximation to $u(t,x)$ in (1) can be algorithmically described as follows:

1. Set the step-size $h = \frac{t}{N}$ (with $N \in \mathbb{N}$), set $Z_0 = x$, and set $M \in \mathbb{N}$ (for the Monte Carlo simulations)
2. Repeat from $j = 1$ until $j = M$:
   (a) From $i = 0$ until $i = N - 1$,
      i. generate the Gaussian variable $\eta_i \sim N(0,1)$ and Uniform random variable $R_i \sim Uniform[0\ 1]$
      ii. compute $Z_{i+1} = Z_i - hb\left(t - t_i - hR_i, Z_i - \sqrt{t_i + hR_i}\eta_i\right)\right)$
   (b) Compute $v^{[j]} = f\left(Z_N - \sqrt{t}\eta_N\right)$ with $\eta_N \sim N(0,1)$
3. Then, $u^{[M]}(t,x) = \frac{1}{M}\sum_{j=1}^{M} v^{[j]}$ is the numerical approximation to $u(t,x)$.

2.2 Convergence

Theorem: Let's $u(t,x)$ the solution to the convection-diffusion PDE (1), with b measurable, locally Lipschitz in the second argument and $\alpha-$Holder continuous in space uniformly in time, for some $\alpha \in (0,1)$. Let $h < 1$ and $M \in \mathbb{N}$ with $M \geq \frac{1}{h}$. Then, $u^{[M]}(t,x)$ is almost surely convergent to $u(t,x)$ and we have that

$$\left|u(t,x) - u^{[M]}(t,x)\right| = O(h^{\frac{1}{2}}), \qquad \textit{almost surely}$$

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DISCRETIZAÇÃO POR MÉTODO DE EULER PARA FLUXOS REGULARES LAGRANGEANOS COM CAMPO ONE-SIDED LIPSCHITZ

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Abstract

Esta apresentação visa como principal objetivo estudar aproximação numérica de dos fluxos regulares Lagrangeanos associados a um campo vetorial limitado pertencente a $L^1\left([0,T];W^{1,p}\left(\mathbb{R}^d;\mathbb{R}^d\right)\right)$. Nós provamos a convergencia do método de Euler explícito qunado o campo vetorial satisfaz a condição one-sided Lipschitz.

1 Introdução

Neste trabalho nós consideramos a equação diferencial ordinaria

$$\begin{cases} \gamma'(t) = b(t, \gamma(t)) \\ \gamma(t_0) = x, \end{cases} \tag{1}$$

onde $\gamma : [0,T] \to \mathbb{R}^d$, sob várias hipóteses de regularidade sobre o campo vetorial

$$b(t,x) : [0,T] \times \mathbb{R}^d \to \mathbb{R}^d.$$

Nós provamos a aproximação discreta pelo método de Euler desta quando b satisfaz as hipóteses do Teorema 2.1.

Para desenvolver isto é necessário têr alguns preliminares:

Definição 1.1. *Seja $b \in L^1_{loc}\left([0,T]\times\mathbb{R}^d;\mathbb{R}^d\right)$. Dizemos que uma função $X : [0,T]\times\mathbb{R}^d \to \mathbb{R}^d$ é um fluxo regular Lagrangeano para o campo vetorial b se*

(i) Para m-q.t.p. $x \in \mathbb{R}^d$ a função $t \mapsto X(t,x)$ é uma solução integral absolutamente contínua de (1);

(ii) Existe uma constante L independente de t tal que

$$X(t,\cdot)_{\#}m \le Lm.$$

A constante em (ii) será chamada a constante de compressibilidade de X.

Definição 1.2. *A função $f : [a,b]\times\mathbb{R}^d \to \mathbb{R}^d$ é dita que satisfaz uma condição one-sided Lipschitz se*

$$\langle f(t,y) - f(t,\tilde{y}), y - \tilde{y}\rangle \le \nu(t)\,|y-\tilde{y}|^2 \tag{2}$$

para todo $y, \tilde{y} \in M_t \subset \mathbb{R}^d$ e para $a \le t \le b$. A função $\nu(t)$ é chamada uma constante one-sided Lipschitz.

2 Resultado Principal

Teorema 2.1. *Assumimos que $[\mathrm{div} b]^- \in L^1([0,T];L^\infty(\mathbb{R}))$ e que o campo vetorial b pertence a $L^1\left([0,T];W^{1,p}\left(\mathbb{R}^d;\mathbb{R}^d\right)\right)\cap L^\infty([0,T]\times\mathbb{R}^d)$ para algum $p>1$, e satisfaz a condição one-sided Lipschitz (2) com $|\nu(t)| \le \kappa$ para todo $t\in[0,T]$, onde κ é uma constante positiva. Seja X como na Definição 1.1. Então a solução numérica satisfaz*

$$\|X(t_n,x) - X_n\|_{L^p(B_R(0))} \le C\sqrt{\frac{C_{\exp}-1}{2\kappa}}\,h^{1/2} + C_{\exp}\|X(t_0,x) - X_0\|_{L^p(B_R(0))} \tag{1}$$

Prova: Da Definição 1.1(i) e do método de Euler temos

$$X(t_{n+1},x) = X(t_n,x) + \int_{t_n}^{t_{n+1}} b(s, X(s,x))\, ds \quad \text{e} \quad X_{n+1} = X_n + hb(t_n, X(t_n,x))\, ds,$$

e considerando a relação

$$Y_{n+1} = X(t_n,x) + hb(t_n, X(t_n,x))$$

obtemos que tomando norma $L^{p/2}$ sobre $B_R(0) \subset \mathbb{R}^d$ e pelas hipoteses impostas sobre o campo b

$$\begin{aligned}\|X(t_{n+1},x) - X_{n+1}\|^2_{L^p(B_R(0))} &\leq \|X(t_{n+1},x) - Y_{n+1}\|^2_{L^p(B_R(0))} + 2\|X(t_{n+1},x) - Y_{n+1}\|_{L^p(B_R(0))} \|Y_{n+1} - X_{n+1}\|_{L^p(B_R(0))} \\ &\quad + \|Y_{n+1} - X_{n+1}\|^2_{L^p(B_R(0))} \\ &\leq Ch^2 + (1+2\kappa h)\left(1+Kh^2\right)\|X(t_n,x) - X_n\|^2_{L^p(B_R(0))},\end{aligned}$$

com $C, K \in \mathbb{R}$ constantes. Portanto, com $\alpha = (1+2\kappa h)$, $\beta = \left(1+Kh^2\right)$, e $E_n = \|X(t_n,x) - X_n\|^2_{L^p(B_R(0))}$ obtemos a equação

$$E_{n+1} \leq \alpha\beta E_n + Ch^2.$$

Por indução podemos confirmar que em geral

$$E_n \leq (\alpha\beta)^n E_0 + Ch^2 \sum_{m=0}^{n-1} (\alpha\beta)^m.$$

Assim, como

$$(\alpha\beta)^n \leq \exp\{(T-t_0)(2\kappa + K(T-t_0))\} = C_{\exp}$$

e $\alpha\beta - 1 = h\left(2\kappa + Kh + 2K\kappa h^2\right)$, temos em consequência que

$$E_n \leq Ch\frac{C_{\exp}-1}{2\kappa} + C_{\exp}E_0,$$

ou seja,

$$\|X(t_n,x) - X_n\|_{L^p(B_R(0))} \leq C\sqrt{\frac{C_{\exp}-1}{2\kappa}}h^{1/2} + C_{\exp}\|X(t_0,x) - X_0\|_{L^p(B_R(0))}.$$

■

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ON THE NUMERICAL PARAMETER IDENTIFICATION PROBLEM

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Abstract

The objective of this work is to describes some important aspects related with the reconstruction of parameters in models described with elliptic partial differential equations. Incomplete information about coefficients and source is compensated by an overprescription of Cauchy data at the boundary. The methodology we propose explores concepts as: (i) Lipschitz Boundary Dissection; (ii) Complementary Mixed Problems with trial parameters; (iii)Internal Discrepancy Fields. The main techniques are variational formulation, boundary integral equations and Calderon projector. Various regularization strategies can be adopted.

1 Introduction

Incomplete information about coefficients in partial differential equations is compensated by an overprescription of Cauchy data at the boundary. We analyses this kind of boundary value problems in an elliptic system posed on Lipschitz domains. The main techniques are variational formulation, boundary integral equations and Calderon projector. To estimate those coefficients we propose a variational formulation based on an internal discrepancy field observed in complementary mixed boundary value problems obtained by splitting the overprescrited Cauchy data.

1.1 The engineering problem

Most of the stationary engineering models can be represented as elliptic system of partial differential equations. Those models are mathematically elaborated with continuous thermomechanics and the constitutive theories of materials. Constitutive equations removes ambiguity in the model and frequently presents incomplete information about parameters. To assure uniqueness of model solution we must combine boundary information with correct parameters values. Estimation of missing parameters in diffusion reaction convection like systems of equations are the main problem.

1.2 The inverse problem

In this work we study the problem of reconstruction of coefficients and source parameters in second order strongly elliptic systems [1], [2]. Let Ω be a Lipschitz domain. Its boundary can be locally as the graph of a Lipschitz function, that is, a Holder continuous $C^{0,1}$ function. Let $F_\alpha = [f_\alpha, ..., f_\alpha] \in (L^2(\Omega))^{m\times N_p}$ be the source and $(H, H_\nu) \in (H^{\frac{1}{2}}(\partial\Omega) \times (H^{-\frac{1}{2}}(\partial\Omega)))^{m\times N_p}$ the Cauchy data for N_p problems based on the $m-$fields model.

The inverse boundary value problem for parameter determination investigated here is: To find $(U, \alpha) \in H^1(\Omega)^{m\times N_p} \times \mathbb{R}^{N_a}$ such that

$$P^{\alpha}_{F_\alpha,H,H_\nu} \begin{cases} \mathcal{L}_\alpha U = F_\alpha & \text{if } x \in \Omega; \\ \gamma[U] = H & \text{if } x \in \partial\Omega; \\ \mathcal{B}_\nu[U] = H_\nu & \text{if } x \in \partial\Omega; \end{cases} \tag{1}$$

Here γ is the boundary trace and $\mathcal{B}_\nu$ is the conormal trace. The coefficients of the strongly elliptic operator $\mathcal{L}_\alpha$, self-adjoint, and the source depend on the parameters α.

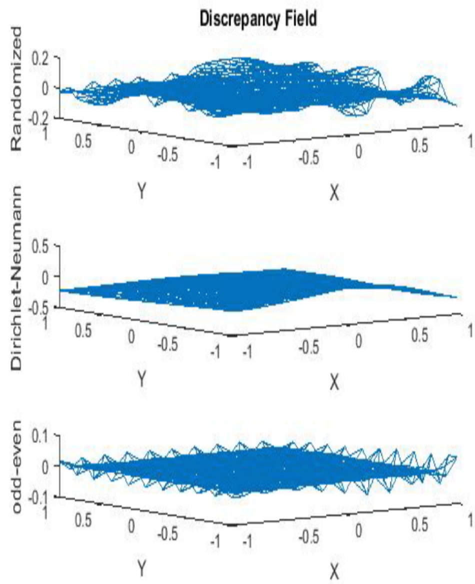

Figure 1: Thye discrepancy field

2 Main Results

The main results in this work are: It is based on over prescription of Cauchy data, Lipschitz Boundary Dissection, a specialized Finite Elements formulation for this class of problems and solutions of Multiple Complementary Direct Mixed Problems with wrong values of trials parameters.

We explore the concept of Complementary Solutions, the existence of Discrepancy Fields for trials with wrong parameters values, the Reciprocity Gap equation for Discrepancy fields parameter determination, the Variational Method for Discrepancy Fields parameter determination and an annihilator set condition for Discrepancy fields parameter determination. Based on Least Squares and L^{∞} norm of Discrepancy Fields, we presents numericals experiments of parameters determination. Finite Elements and Fundamental Solutions based Methods are the main tools.

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UM MÉTODO DO TIPO SPLITTING PARA EQUAÇÕES DE LYAPUNOV

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Abstract

Neste trabalho desenvolvemos um algoritmo para calcular a solução de equações de Lyapunov inspirado em métodos para sistemas lineares construídos a partir da cisão de matrizes, popularmente conhecidos como métodos do tipo "splitting". A construção do nosso método é feita sobre a representação da equação na forma vetorizada de tamanho n^2. No entanto, desenvolvemos uma técnica que permite realizar iterações com um número de operações de ordem n apenas. Além disso, propomos técnicas para contornar situações em que o problema é mal condicionado. Verificamos a eficiência do método com uma breve aplicação, descrita ao final do trabalho.

1 Introdução

Neste trabalho, consideramos uma equação de Lyapunov da forma

$$AP + PA^T = -BB^T, \tag{1}$$

com $A \in \mathbb{R}^{n\times n}$, e $B \in \mathbb{R}^{n\times 1}$. Esta equação é equivalente ao sistema linear $\tilde{A}p = b$, de tamanho n^2, com $\tilde{A} = (I \otimes A + A \otimes I)$, $p = \text{vec}(P)$, e $b = \text{vec}(-BB^T)$. O símbolo "$\otimes$" representa o produto de Kronecker entre matrizes e a expressão "vec(B)" é a representação da matriz B por um vetor coluna.

Note que, dado um $\sigma > 0$, a equação (1) pode ser reescrita como $(A - \sigma I)P + P(A^T + \sigma I) = -BB^T$, que, por sua vez, é equivalente ao sistema $\tilde{A}_\sigma p = b$, com $\tilde{A}_\sigma = [I \otimes (A - \sigma I) + (A + \sigma I) \otimes I]$. A partir disto, definimos a cisão $\tilde{A}_\sigma = M_\sigma - N_\sigma$, com $M_\sigma = I \otimes (A - \sigma I)$ e $N_\sigma = -(A + \sigma I) \otimes I$.

Supondo que a matriz $(A - \sigma I)$ é inversível, a matriz M_σ também é inversível e $M_\sigma^{-1} = I \otimes (A - \sigma I)^{-1}$. Sendo assim, dado um vetor inicial $p_0 \in \mathbb{R}^{n^2}$, para $k = 0, 1, 2, ...,$ definimos a iteração do tipo *splitting* para sistemas lineares [1] por

$$p_{k+1} = M_\sigma^{-1} N_\sigma p_k + M_\sigma^{-1} b. \tag{2}$$

As iterações definidas em (2) são o foco do nosso trabalho e definem um método do tipo *Splitting* Para Equações de Lyapunov (SEL).

2 Resultados Principais

Para simplificar as iterações em (2), vamos escolher $p_0 = 0_{n^2\times 1}$. Assim, $M_\sigma^{-1} N_\sigma = -(A + \sigma I) \otimes (A - \sigma I)^{-1}$. Isso permite demostrar o Teorema 2.1 a seguir, lembrando que $vec(P_k) = p_k$ para todo k inteiro não negativo.

Teorema 2.1. *Os iterados P_k definidos em (2) convergem para a solução P da equação (1) se, e somente se,*

$$\left|\frac{\lambda_i + \sigma}{\lambda_j - \sigma}\right| < 1, \quad \forall\ \lambda_i, \lambda_j \in \lambda(A). \tag{3}$$

Além disso, verificamos que as matrizes iteradas pelo método descrito em (2) podem ser escritas como segue:

$$P_{k+1} = \sum_{i=1}^{k+1} (-1)^{i+1} (A - \sigma I)^{-i} BB^T \left((A + \sigma I)^T\right)^{(i-1)}. \tag{4}$$

Portanto, embora o método seja desenvolvido a partir do sistema de tamanho n^2, cada iteração do método necessitam de um número de operações na ordem n apenas. A garantia de existência de um parâmetro σ que faz com que as matrizes P_k de (4) convirjam para a solução da equação (1) é dada pela proposição a seguir:

Proposição 2.1. *Sejam $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ os autovalores da matriz A da equação (1), com $Re(\lambda_i) < 0$, para $i = 1, ..., n$. Sejam ainda $\tau = \max_{i,j=1,...,n} |Re\lambda_i - Re\lambda_j|$ e $\varsigma = \max_{j=1,...,n} \left|\frac{Im(\lambda_j)^2}{Re(\lambda_j)}\right|$. Se $\alpha > \frac{1}{2}(\tau + \varsigma)$, então, a sequência $(P_k)_{k\in\mathbb{N}}$ definida em (4) converge para a solução P de (1).*

Para evitar a lentidão da convergência nos casos em que a matriz A possui autovalores muito próximos à origem, propomos a estratégia de escolher um $\alpha > 0$, resolver a equação auxiliar $(A - \alpha I)\tilde{P} + \tilde{P}(A - \alpha I)^T = -BB^T$ e reconstruir a solução P de (1) a partir de $\tilde{P}$. Para isso, enunciamos o teorema a seguir, cuja demonstração faz uso da representação da solução P de (1) em termos do espectro de A.

Teorema 2.2. *Dado $\alpha > 0$, seja $\tilde{P}$ uma solução para $(A - \alpha I)\tilde{P} + \tilde{P}(A - \alpha I)^T = -BB^T$. Então, existem uma matriz de Cauchy generalizada C e uma matriz V, construídas a partir de $\tilde{P}$, tais que a solução de (1) é dada por*

$$P = VCV^H \tag{5}$$

Para diminuir o número de operações quando as matrizes do problema são esparsas e de grande porte, desenvolvemos uma técnica de projeção para calcular uma aproximação $P_k = V_k C_k V_k^H$ em (5) baseando-se na análise de componentes principais de $\tilde{P}$. Isso dá origem ao método do tipo Splitting Deslocado Projetado para (1) (SDPEL).

A tabela a seguir contém resultados obtidos considerando o exemplo em que $A = T \otimes I + I \otimes T$ em (1), com T sendo uma matriz tridiagonal cujas entradas da diagonal principal são todas iguais a 2 e as entradas das subdiagonais são iguais a -1. Consideramos ainda $B = (1, 0, ..., 0)^T$. e $n = 900$. Para que haja um comparativo, aplicamos dois métodos baseados em projeção em subespaços de Krylov (KPIK e RKSM) [2] e também o método SLRCF-ADI [3], que é uma variação do método ADI (*Alternating Direction Implicit*), todos muito utilizados atualmente em equações de Lyapunov esparsas. O erro relativo é dado por $\frac{\|AP+PA^T+BB^T\|}{\|A\|\|P_k\|+\|BB^T\|}$ na norma de Frobenius.

	SEL	SDPEL	KPIK	RKSM	SLRCF-ADI
Erro rel.	$1.6 \cdot 10^{-3}$	$4,4 \cdot 10^{-5}$	$4,3 \cdot 10^{-8}$	$8,1 \cdot 10^{-10}$	$1,6 \cdot 10^{-8}$
Nº de it.	20	20	10	10	10

Embora os métodos SEL e SDPEL não tenham a mesma eficiência que os métodos KPIK, RKSM e SLRCF-ADI, eles tornam-se atrativos por exigir apenas uma decomposição LU da matriz A durante o processo todo, além de serem métodos de fácil implementação. O método RKSM, por exemplo, necessita do cálculo das projeções $V_k^T AV_k$, que podem ser caras em sistemas descritores de grande porte. Além disso, não há garantia de convergência para os casos em que a matriz A é não dissipativa. O método SLRCF-ADI, por sua vez, além de necessitar de uma decomposição LU para cada iteração, também depende de um conjunto de parâmetros calculados previamente.

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ELASTICITY SYSTEM ENERGY WITHOUT SIGN

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Abstract

This paper is concerned with the energy of a nonlinear elasticity system with nonlinear boundary condition. We obtain the existence of global weak solutions of this system with small data.

1 Introduction

Consider an open bounded set Ω of $\mathbb{R}^n$ whose boundary Γ of class C^2 is constituted of two disjoint parts Γ_0 and Γ_1 with positive measures and $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset$. Denote by $\nu(x)$ the unit exterior normal at $x \in \Gamma_1$. We analyze the following nonlinear elasticity system:

$$\left| \begin{array}{l} u''(x,t) - \mu\Delta u(x,t) - (\lambda+\mu)\nabla div\ u(x,t) + |u(x,t)|^\rho = 0 \text{ in } \Omega \times (0,\infty); \\ u(x,t) = 0 \text{ on } \Gamma_0 \times (0,\infty); \\ \mu\frac{\partial u}{\partial \nu}(x,t) + (\lambda+\mu) div\ u(x,t)\ \nu(x) + h(u'(x,t)) = 0 \text{ on } \Gamma_1 \times (0,\infty); \\ u(x,0) = u^0(x)\ ,\ u'(x,0) = u^1(x) \text{ in } \Omega. \end{array} \right. \tag{1}$$

Here $u(x,t) = (u_1(x,t), ...u_n(x,t)); \lambda \geq 0, \mu > 0$ are the Lamé's constants of the material; $\rho > 1$ a real number; $|u(x,t)|^\rho = (|u_1(x,t)|^\rho, ..., |u_n(x,t)|^\rho)$ and $h(x,s) = (h_1(x,s), ..., h_n(x,s))$.

2 Main Result

We introduce some spaces. By $\mathbb{L}^2(\Omega)$ and $\mathbb{H}^1_{\Gamma_0}(\Omega)$ are represented, respectively, the Hilbert spaces $\mathbb{L}^2(\Omega) = (L^2(\Omega))^n$ equipped wiht the scalar product

$$(u,v)_{\mathbb{L}^2(\Omega)} = \sum_{i=1}^n \int_\Omega u_i(x)v_i(x)\ dx$$

and $\mathbb{H}^1_{\Gamma_0}(\Omega) = \left(H^1_{\Gamma_0}(\Omega)\right)^n$ provided with the scalar product

$$((u,v))_{\mathbb{H}^1_{\Gamma_0}(\Omega)} = \mu \sum_{i,j=1}^n \int_\Omega \frac{\partial u_i(x)}{\partial x_j}\frac{\partial v_i(x)}{\partial x_j}\ dx + (\lambda+\mu)\int_\Omega (div\ u(x))(div\ v(x))\ dx$$

where $H^1_{\Gamma_0}(\Omega) = \{u \in H^1(\Omega); u = 0 \text{ on } \Gamma_0\}$. Also the spaces $\mathbb{H}^1_0(\Omega) = \left(H^1_0(\Omega)\right)^n, \mathbb{H}^2(\Omega) = \left(H^2(\Omega)\right)^n, \mathbb{L}^{\rho+1}(\Omega) = \left(L^{\rho+1}(\Omega)\right)^n, \mathbb{L}^1(\Gamma_1) = \left(L^1(\Gamma_1)\right)^n, \mathbb{L}^2(\Gamma_1) = \left(L^2(\Gamma_1)\right)^n, \mathbb{H}^{-1/2}(\Gamma_1) = \left(H^{-1/2}(\Gamma_1)\right)^n$ are equipped with its respective product topology.

Let A be the positive self-adjoint of $\mathbb{L}^2(\Omega)$ defined by the triplet $\{\mathbb{H}^1_{\Gamma_0}(\Omega), \mathbb{L}^2(\Omega), ((u,v))_{\mathbb{H}^1_{\Gamma_0}(\Omega)}\}$. Then

$$D(A) = \{u \in \mathbb{H}^1_{\Gamma_0}(\Omega); Au \in \mathbb{L}^2(\Omega), (Au,v)_{\mathbb{L}^2(\Omega)} = ((u,v))_{\mathbb{H}^1_{\Gamma_0}(\Omega)}, \forall v \in \mathbb{H}^1_{\Gamma_0}(\Omega)\}$$

We note that if $u \in D(A)$ then $u \in \mathbb{H}^1_{\Gamma_0}(\Omega) \cap \mathbb{H}^2(\Omega)$ and $\gamma_1 u = 0$ on Γ_1 where $\gamma_1 u = \mu \frac{\partial u}{\partial \nu} + (\lambda + \nu)(div\ u)\nu$. Let W be the Hilbert space defined by

$$W = \{u \in \mathbb{H}^1_{\Gamma_0}(\Omega); Au \in \mathbb{L}^2(\Omega)\}$$

provided wit the scalar product

$$((u,v))_W = ((u,v))_{\mathbb{H}^1_{\Gamma_0}(\Omega)} + (Au, Av)_{\mathbb{L}^2(\Omega)}.$$

We establish the following hypotheses:

$$(H1)\quad \left| \begin{array}{l} \rho > 1 \text{ if } n = 1,2 : \\ \frac{n+1}{n} \le \rho \le \frac{n}{n-2} \text{ if } n \ge 3. \end{array} \right.$$

Introduce the constant

$$\lambda^* = \left[\frac{\rho + 1}{4 k_0^{\rho+1}} \right]^{1/(\rho-1)}.$$

where the embedded constant k_0 satisfies

$$||v||_{\mathbb{L}^{\rho+1}(\Omega)} \le k_0 ||v||_{\mathbb{H}^1_{\Gamma_0}}(\Omega)\ ,\ \forall v \in \mathbb{H}^1_{\Gamma_0}(\Omega).$$

(H2) Consider $u^0 \in D(A)$ and $u^1 \in \mathbb{H}^1_0(\Omega)$ with

$$\left| \begin{array}{l} ||u_0||_{\mathbb{H}^1_{\Gamma_0}(\Omega)} < \lambda^*, \\ \frac{1}{2}|u^1|^2_{\mathbb{L}^2(\Omega)} + \frac{1}{2}\mu ||u^0||_{\mathbb{H}^1_{\Gamma_0}(\Omega)} + \frac{1}{2}(\lambda + \mu)|div\ u^0|^2_{L^2(\Omega)} + \frac{n}{\rho+1} k_0^{\rho+1} ||u^0||^{\rho+1}_{\mathbb{H}^1_{\Gamma_0}(\Omega)} < \frac{1}{4}(\lambda^*)^2. \end{array} \right.$$

(H3) Consider also $h \in C^0(\mathbb{R}; (L^\infty(\Gamma_1)^n)$ with

$$\left| \begin{array}{l} h_i(x,0) = 0\ a.e.\ x \in \Gamma_1,\ i = 1,...,n; \\ \left[h_i(x,s) - h_i(x,r)\right](s-r) \ge d_0(s-r)^2\ , \forall s,r \in \mathbb{R}, a.e.\ x \in \Gamma_1\ , i = 1,...,n\ (d_0 \text{ positive constant}). \end{array} \right.$$

Theorem 2.1. *Assume hypotheses (H1)-(H3). Then there exist a function u in the class*

$$\left| \begin{array}{l} u \in L^\infty(0,\infty; \mathbb{H}^1_{\Gamma_0}(\Omega)) \cap L^\infty_{loc}(0.\infty; W)\ ,\ u' \in L^\infty(0,\infty; \mathbb{L}^2(\Omega)) \cap L^\infty_{loc}(0,\infty; \mathbb{H}^1_{\Gamma_0}(\Omega)) \\ u'' \in L^\infty_{loc}(0,\infty; \mathbb{L}^2(\Omega))\ ,\ div\ u \in L^\infty(0,\infty; L^2(\Omega)) \end{array} \right.$$

such that u satisfies

$$\left| \begin{array}{l} u'' - \mu\Delta u - (\lambda+\mu)\nabla div\ u + |u|^\rho = 0\ in\ L^\infty_{loc}(0,\infty; \mathbb{L}^2(\Omega)); \\ \gamma_1 u + h(u') = 0\ on\ L^1_{loc}(0,\infty; \mathbb{H}^{-1/2}(\Gamma_1) + \mathbb{L}^1(\Gamma_1)), \\ u(0) = u^0\ ,\ u'(0) = u^1 \end{array} \right.$$

In the proof of the theorem we use the Galerkin approach with a special basis, a new method inspirated in an idea of L. Tartar[4] which permits to obtain appropriate a priori estimates, compactness arguments, Strauss'approximations of continuous functions and a trace result for non-smooth functions.

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IMPULSIVE EVOLUTION PROCESSES

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Abstract

In this work, we will present the notions of an impulsive evolution process and its pullback attractors, as well as exhibit conditions under which a given impulsive evolution process has a pullback attractor. We apply our results to a nonautonomous ordinary differential equation describing an integrate-and-fire model of neuron membrane.

1 Introduction

The theory of impulsive dynamical systems describes models on which a continuous evolution is abruptly interrupted by sudden changes of state, which, in applications, can be interpreted as either jumps of state or forced corrections to the evolution law in order to prevent unwanted results. Such topic saw its first light in the early 1970's, when V. Rozko studied a class of periodic motions in pulsed systems - in [11] - and the Lypaunov stability for discontinuous systems - in [12]. Almost twenty years later, S. K. Kaul presented a rigorous mathematical foundation for this theory in [7] and [8], and studied properties of stability and asymptotic stability in [9]. A decade later, K. Ciesielski published very important results in [5, 6]. Since then a vast literature was developed for autonomous impulsive dynamical systems, which are constructed using a semigroup, a fixed set of impulses and a single impulse function.

More recently, some authors turned their attention to nonautonomous impulsive dynamical systems, which are constructed with a cocycle instead of a semigroup, but again considering only a fixed set of impulses and a single impulsive function, and obtained results of existence and semicontinuity of impulsive cocycle attractors. See, for instance, [1, 2].

2 Description of the main results

In this work, we present the theory of impulsive evolution processes, and although it can be seen as a particular case of a nonautonomous impulsive system, here we consider an evolution process, a family of impulsive sets and a family of impulsive functions. We define the impulsive evolution processes, present two concepts of pullback attractors, and obtain conditions to ensure their existence.

This work was inspired by [3], where the authors work with a multivalued autonomous dynamical system, and as an application to their results, they present a multivalued autonomous integrate-and-fire model of nerve membrane, given by

$$u'(t) = -\gamma u + S,$$

where if $u(t) = \theta$, then $u(t)$ resets to either $0 < u_1 < \theta$ or $0 < u_2 < \theta$, and $\gamma, S, \theta > 0$. We consider a slightly more general approach (dropping, however, the multivalued framework), by considering a complete nonautonomous model, where γ, S and θ are now nonnegative functions (not necessarily bounded), and if $u(t) = \theta(t)$, then $u(t)$ resets to $u_r(t)$. The nonautonomous framework, at least for the function S, is consistent with the models presented in [4] and [10]. We study the impulsive evolution process generated by the nonautonomous integrate-and-fire model of nerve membrane and the existence of a pullback attractor for this problem.

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COMPORTAMENTO ASSINTÓTICO PARA UMA CLASSE DE FAMÍLIAS DE EVOLUÇÃO DISCRETAS A UM PARÂMETRO

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Abstract

Estudamos o comportamento assintótico de uma classe de famílias de evolução discretas $n \mapsto S(n)$, associadas a equações de Volterra de tipo convolução em tempo discreto. Mais precisamente, obtemos uma recíproca de uma extensão do Teorema de Katznelson-Tzafriri para essas famílias, bem como a ordem de decaímento de $n \mapsto S(n+1) - S(n)$ via regularidade maximal nos espaços ℓ^p.

1 Introdução

Nessa nota, estamos interessados em estudar o comportamento assintótico de uma classe de famílias de evolução discretas a um parâmetro e explorar suas conexões com propriedades espectrais que surgem naturalmente através do Método da Transformada Z. Um exemplo muito conhecido e clássico de famílias de evolução discretas a um parâmetro é o semigrupo discreto: $n \mapsto T^n$. Operadores limitados em potência (isto é, semigrupos discretos limitados) foram extensivamente estudados nos últimos anos. Podemos citar, por exemplo, [2] para resultados sobre ordem de crescimento $n \mapsto T^n$, [4] para resultados de tipo espectrais de operadores parcialmente limitados em potência e [5] para resultados que exploram a conexão entre a limitação em potência de T e a condição de analiticidade (no sentido de Ritt), a saber, $T^{n+1} - T^n = O\left(\frac{1}{n}\right)$. Em [3], os autores provaram o seguinte resultado:

Teorema 1.1. *Seja $T \in B(X)$ uma contração. Então $\lim_{n\to\infty} \left(T^{n+1} - T^n\right) = 0$ se, e somente se, $\Gamma(T) := \sigma(T) \cap \mathbb{T}$ (o espectro periférico de T) possui no máximo $z = 1$.*

Aqui, $\mathbb{T} = \{z \in \mathbb{C}; |z| = 1\}$. Uma pergunta natural cresce então a partir do Teorema 1.1 (de Katznelson-Tzafriri): é possível obter versões análogas desse resultado para famílias de evoluções mais gerais? Essa questão foi estudada, por exemplo, em [1], para famílias de evolução discretas geradas por uma família de operadores $(A(n))_{n\in\mathbb{Z}^+} \subset B(X)$, isto é, a família $n \mapsto S(n) \in B(X)$ que é a resolvente da equação de Volterra

$$\begin{cases} S(n+1) &=& \displaystyle\sum_{k=0}^{n} A(n-k)S(k), \;\; n \in \mathbb{Z}^+ \\ S(0) &=& I, \end{cases} \tag{1}$$

onde $I \in B(X)$ é o operador identidade. Note que, se $A(n) = 0$ para todo $n \geq 1$, a família S é precisamente o semigrupo discreto gerado por $A(0)$. A partir de técnicas provenientes da teoria espectral de sequências unilaterais, foi provado o seguinte resultado em [1]: se a família de evolução S for limitada e se Γ consistir no máximo de $z = 1$, então $\lim_{n\to\infty} [S(n+1) - S(n)] = 0$. Aqui, Γ é o conjunto periférico dos pontos regulares de $\tilde{S}$ (definido na próxima seção), onde $\tilde{S}$ denota a Transformada Z de S.

O nosso objetivo aqui é obter a recíproca desse resultado provado em [1], pelo menos para uma classe específica da família de operadores $n \mapsto A(n)$. Além disso, obtemos uma ordem de decaímento de tipo polinomial se assumirmos, adicionalmente, que a Equação de Volterra

$$\begin{cases} u(n+1) &=& \displaystyle\sum_{k=0}^{n} A(n-k)u(k) + f(n), \;\; n \in \mathbb{Z}^+ \\ u(0) &=& 0 \end{cases} \tag{2}$$

possua ℓ^p-regularidade maximal, isto é, o mapa $f \mapsto \Delta u \in B(\ell^p(X))$, onde $(\Delta u)(n) = u(n+1) - u(n)$.

2 Resultados Principais

Para os resultados a seguir, assumiremos as seguintes condições:

(H1) a família de operadores $n \mapsto A(n) \in B(X)$ é da forma $A(n) = a_n T$, onde $T \in B(X)$ e $(a_n)_{n\in\mathbb{Z}^+} \in \ell^1(\mathbb{C})$.

(H2) a função $\beta(z) = \dfrac{z}{\tilde{a}(z)}$ é um polinômio. Aqui, $\tilde{a}$ denota a Transformada-Z de $(a_n)_{n\in\mathbb{Z}^+}$.

Definição 2.1. *Seja $n \mapsto S(n) \in B(X)$ uma família de evolução limitada. O conjunto periférico de todos os pontos regulares de $\tilde{S}$ que estão em $\mathbb{T}$ será denotado por Γ, isto é: $\omega \in \Gamma$ se, e somente se, $\omega \in \mathbb{T}$ e se existir $r > 0$ tal que o operador $\tilde{S}(z) = z\left[z - \tilde{A}(z)\right]^{-1} \in B(X)$ existe e é holomorfa para todo $z \in D(\omega, r) := \{\eta \in \mathbb{C}; |\eta - \omega| < r\}$.*

Nosso primeiro resultado é a recíproca da extensão do Teorema de Katznelson-Tzafriri (assumindo, é claro, as condições (H1) e (H2)), provado em [1]:

Teorema 2.1. *Seja $S : \mathbb{Z}^+ \to B(X)$ uma família de evolução gerada pela família de operadores $n \mapsto A(n) \in B(X)$ satisfazendo as condições (H1) e (H2). Assuma que S seja limitada. Se $\lim\limits_{n\to\infty} [S(n+1) - S(n)] = 0$, então $\Gamma \subseteq \{1\}$.*

Portanto, temos a seguinte caracterização: uma família de evolução limitada S gerada pela família de operadores $A(n) = a_n T$ satisfazendo as condições (H1) e (H2) satisfaz $[S(n+1) - S(n)] \to 0$ quando $n \to \infty$ se, e somente se, $\Gamma \subseteq \{1\}$.

Teorema 2.2. *Seja $S : \mathbb{Z}^+ \to B(X)$ uma família de evolução gerada por $n \mapsto A(n) \in B(X)$ satisfazendo as condições (H1) e (H2). Assuma que S seja limitada e que $[S(n+1) - S(n)] \to 0$ quando $n \to \infty$. Se a família $(A(n))_{n\in\mathbb{Z}^+}$ possuir ℓ^p-regularidade maximal, com $p \in (1, \infty)$, então existe $M > 0$ tal que $||S(n+1) - S(n)||_{B(X)} \leq \dfrac{M}{n}$, $n \geq 1$.*

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A NEW APPROACH TO DISCUSS THE UNSTEADY STOKES EQUATIONS WITH CAPUTO FRACTIONAL DERIVATIVE

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Abstract

Motivated by Zhou-Peng paper [2] some researchers started to apply the Faedo-Galerkin method to study partial differential equations with fractional time derivative. However, these papers disregard the fact that absolutely continuous functions are not the broader domain of the Caputo fractional derivative and this can sometimes compromise the prerequisites to apply the Faedo-Galerkin method. In this talk we address the recent work [1] which discuss these questions and proves a new inequality which allows us to completely implement the aforementioned method to study the unsteady Stokes equations with Caputo fractional derivative on bounded domains.

1 Introduction

This talk is dedicated to introduce a new inequality that involves an important case of Leibniz rule regarding Caputo fractional derivative of order $\alpha \in (0,1)$. More specifically, we prove that for suitable functions f, it holds that

$$cD^{\alpha}_{t_0,t}\big[f(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \text{almost everywhere in } [t_0,t_1].$$

In the context of partial differential equations, the aforesaid inequality allows us to address the Faedo-Galerkin method to study the fractional version of the 2D Stokes equation on bounded domains Ω

$$\begin{aligned} cD^{\alpha}_{t}u - \nu\Delta u + \nabla p &= f && \text{in } \Omega,\ t>0,\\ \nabla\cdot u &= 0 && \text{in } \Omega,\ t>0,\\ u(x,t) &= 0 && \text{on } \partial\Omega,\ t>0,\\ u(x,0) &= u_0(x) && \text{in } \Omega. \end{aligned}$$

where cD^{α}_{t} is the Caputo fractional derivative of order $\alpha \in (0,1)$ and f a suitable function.

This is a joint work with Prof. Renato Fehlberg Júnior.

2 Main Results

Our main result concerning this inequality is

Theorem 2.1. *Assume that $f \in C^0([t_0,t_1];\mathbb{R})$ which also satisfies $g_{1-\alpha} * f \in W^{1,1}(t_0,t_1;\mathbb{R})$ and $g_{1-\alpha} * f^2 \in W^{1,1}(t_0,t_1;\mathbb{R})$. Then,*

$$cD^{\alpha}_{t_0,t}\big[f(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \textit{for almost every } t \in [t_0,t_1].$$

This result allowed us to obtain.

Theorem 2.2. *Let V and H be Hilbert spaces that satisfies the hypothesis*

(a) V is dense in H and also is continuously included in H.

(b) If H' represents the dual of H, by the Riesz representation theorem, we consider the identification $H \equiv H'$.

Then if $u \in L^2(t_0,t_1;V)$, $cD^{\alpha}_{t_0,t}u \in L^2(t_0,t_1;V')$ and $g_{1-\alpha} * \|u(t)\|^2_H \in W^{1,1}(t_0,t_1;\mathbb{R})$, then u is almost everywhere equal to a continuous function from $[t_0,t_1]$ into H and

$$cD^{\alpha}_{t_0,t}\left\|u(t)\right\|^2_H \leq 2\left\langle cD^{\alpha}_{t_0,t}u(t), u(t)\right\rangle_{V',V}, \quad \textit{for almost every } t \in [t_0,t_1].$$

This last theorem is enough for us to implement the Faedo-Galerkin method to prove existence and uniqueness of weak solution to the unsteady Stokes equations with Caputo fractional derivative on bounded domains.

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XIII ENAMA - Novembro 2019 *151–152*

ORBITAL STABILITY OF PERIODIC STANDING WAVES FOR THE LOGARITHMIC KLEIN-GORDON EQUATION

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Abstract

The main goal of this work is to present orbital stability results of periodic standing waves for the one-dimensional Logarithmic Klein-Gordon equation. To do so, we first use compactness arguments and a non-standard analysis to obtain the existence and uniqueness of weak solutions for the associated Cauchy problem in the energy space. Second, we show the orbital stability of standing waves using a stablity analysis of conservative systems.

1 Introduction

Consider the Klein-Gordon equation with $p-$power nonlinearity,

$$u_{tt} - u_{xx} + u - \log(|u|^p)u = 0. \tag{1}$$

Here, $u : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ is a complex valued function and p is a positive integer.

By using compactness arguments and ideas treated in [2], [3] and [4], we can prove the existence of global weak solutions in time, uniqueness and existence of conserved quantities to the Cauchy problem

$$\begin{cases} u_{tt} - u_{xx} + u - \log(|u|^p)u = 0, \quad (x,t) \in \mathbb{R} \times [0,T]. \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}. \\ u(x+L,t) = u(x,t) \quad \text{for all } t \in [0,T], \quad x \in \mathbb{R}, \end{cases} \tag{2}$$

where $(u_0, u_1) \in H^1_{per}([0,L]) \times L^2_{per}([0,L])$ and $L > 0$.

Along the last thirty years, the theory of stability of traveling/standing wave solutions for nonlinear evolution equation has increased into a large field that attracts the attention of both mathematicians and physicists. Our purpose is to give a contribution in the stability theory by proving the first result of orbital stability to equation (1) of periodic waves of the form $u(x,t) = e^{ict}\varphi(x)$, $x \in \mathbb{R}$, $t > 0$, where c is called the frequency and φ is a real, even and periodic function. If we substitute this kind of solution in equation (1), one has the following nonlinear ordinary differential equation

$$-\varphi_c'' + (1 - c^2)\varphi_c - \log(|\varphi_c|^p)\varphi_c = 0, \tag{3}$$

where φ_c indicates the dependence of the function φ with respect to the parameter c.

We can use arguments in [5] to obtain a class of smooth periodic positive solutions to the equation (3), depending on c. Based on [1], [5] and [6], we obtain results about its orbital stability.

2 Main Results

Theorem 2.1. *There exists a unique global (weak) solution to the problem (2) in the sense that*

$$u \in L^\infty(0,T; H^1_{per}([0,L])), \quad u_t \in L^\infty(0,T; L^2_{per}([0,L])), \; u_{tt} \in L^\infty(0,T; H^{-1}_{per}([0,L])),$$

and u satisfies

$$\langle u_{tt}(\cdot,t),\zeta\rangle_{H^{-1}_{per},H^1_{per}} + \int_0^L \nabla u(\cdot,t)\cdot\overline{\nabla\zeta}\ dx + \int_0^L u(\cdot,t)\overline{\zeta}\ dx = \int_0^L u(\cdot,t)\log(|u(\cdot,t)|^p)\overline{\zeta}\ dx,$$

a.e. $t\in[0,T]$, *for all* $\zeta\in H^1_{per}([0,L])$. *Furthermore, u must satisfy* $u(\cdot,0)=u_0$ *and* $u_t(\cdot,0)=u_1$. *In addition, the weak solution satisfies the following conserved quantities:*

$$\mathcal{E}(u(\cdot,t),u'(\cdot,t)) = \mathcal{E}(u_0,u_1) \quad \textit{and} \quad \mathcal{F}(u(\cdot,t),u'(\cdot,t)) = \mathcal{F}(u_0,u_1),$$

a.e. $t\in[0,T]$. *Here,* $\mathcal{E}$ *and* $\mathcal{F}$ *are defined by*

$$\mathcal{E}(u(\cdot,t),u_t(\cdot,t)) := \frac{1}{2}\left[\int_0^L |u_x(\cdot,t)|^2 + |u_t(\cdot,t)|^2 + \left(1+\frac{p}{2}-\log(|u(\cdot,t)|^p)\right)|u(\cdot,t)|^2 dx\right]$$

and

$$\mathcal{F}(u(\cdot,t),u_t(\cdot,t)) := Im\int_0^L \overline{u(\cdot,t)}\ u_t(\cdot,t)\ dx.$$

Definition 2.1. *We say that* φ_c *is orbitally stable by the periodic flow of the equation (1), where* φ_c *satisfies (3) if for all* $\epsilon>0$ *there exists* $\delta>0$ *such that if*

$$(u_0,u_1)\in X = H^1_{per}([0,L])\times L^2_{per}([0,L]) \ \textit{satisfies}\ \|(u_0,u_1)-(\varphi,ic\varphi)\|_X<\delta$$

then $\vec{v}=(v,v_t)$ *is a weak solution to equation (1) with* $\vec{v}(\cdot,0)=(u_0,u_1)$ *and*

$$\sup_{t\geq 0}\inf_{\theta\in\mathbb{R},y\in\mathbb{R}}\|\vec{v}(\cdot,t)-e^{i\theta}(\varphi(\cdot+y),ic\varphi(\cdot+y))\|_X<\epsilon.$$

Otherwise, we say that φ_c *is orbitally unstable.*

Theorem 2.2. *Consider* $p=1,2,3$ *and c satisfying* $\frac{\sqrt{p}}{2}<|c|<1$. *Let* φ_c *be a periodic solution for the equation* (3). *The periodic wave* $\widetilde{v}(x,t)=e^{ict}\varphi_c(x)$ *is orbitally stable by the periodic flow of the equation (1).*

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SOBRE OSCILAÇÃO E PERIODICIDADE PARA EQUAÇÕES DIFERENCIAIS IMPULSIVAS

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Abstract

Pretende-se obter condições que garantam a existência de soluções oscilatórias e, em particular, periódicas, de certos problemas impulsivos. Este trabalho foi desenvolvido com apoio financeiro da FAPESP (processo 2018/15183-7).

1 Introdução

As equações diferenciais impulsivas descrevem a evolução de um sistema em que o desenvolvimento contínuo de um processo se alterna com mudanças bruscas do estado. Estas equações se valem das equações diferenciais para descrever os estágios de variação contínua do estado, acrescidas de uma condição para descrever as descontinuidades de primeira espécie da solução ou de suas derivadas nos momentos de impulso. Diversos fenômenos biológicos, naturais, farmacológicos podem apresentar efeitos impulsivos, veja [2].

Neste trabalho, pretendemos introduzir um estudo sobre oscilação e existência de soluções periódicas para certos problemas escalares impulsivos envolvendo equações diferenciais, do tipo:

$$\dot{x} = -p(t)f(x(t-r)), \tag{1}$$

$$x(t) \in M \implies x(t+) = F(x(t)), \tag{2}$$

$$x(t_0) = b, \tag{3}$$

onde $f, p \in C^1$, $r \geq 0$, $M \subset \mathbb{R}$ é fechado e $F : M \to \mathbb{R}$ é contínua. Aqui vamos considerar soluções do problema acima que sejam contínuas à esquerda, isto é, $x(t) = x(t^-)$.

Quando os momentos de impulsos são previamente conhecidos, então a condição (2) é substituída por

$$\Delta(x(t_k)) = x(t_k^+) - x(t_k) = I_k(x(t_k)), \quad k = 1, 2, \ldots \tag{4}$$

onde $I_k : \mathbb{R} \to \mathbb{R}$ são contínuas para cada $k = 1, 2, \ldots$. Trataremos aqui o caso em que $I_k(x(t_k)) = b_k\, x(t_k)$, com $\lim\limits_{k\to\infty} t_k = \infty$.

2 Resultados Principais

Teorema 2.1. *Suponhamos que $t_k - t_{k-1} > r > 0$, $k = 1, 2, \ldots$ e que exista $K > 0$ tal que $k > K$ implica $b_k \neq -1$. Suponhamos também que $|f(x)| \geq \lambda |x|$ para algum $\lambda > 0$ e que*

$$\limsup_{t\to\infty} \frac{\lambda}{1+b_i} \int_{t_i}^{t_i+r} p(s)\, ds > 1,$$

então toda solução do problema (1), (2) é oscilatória.

Pretendemos apresentar também um resultado que garante a existência de soluções periódicas para um problema autônomo, isto é, $x'(t) = f(x(t-r))$ e a condição de impulso dada em (2). Neste caso será importante obter uma relação entre o retardo e os instantes de impulso, que não são conhecidos a priori. A ideia consistirá em construir um determinado conjunto K e um operador de retorno T definido em K e mostrar que T tem ponto fixo não trivial. Por fim, apresentaremos alguns exemplos.

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RESULTADOS DE EXISTÊNCIA DE SOLUÇÕES PARA EQUAÇÕES DINÂMICAS DESCONTÍNUAS EM ESCALAS TEMPORAIS

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Abstract

Neste trabalho, apresentamos dois resultados sobre a existência de soluções para equações dinâmicas descontínuas em escalas temporais. Um dos resultados apresentados aqui diz respeito a existência e unicidade de soluções e pode ser obtido através do Teorema do ponto fixo de Banach. Já o outro resultado apresentado diz respeito a existência de pelo menos uma solução e pode ser obtido através do Teorema do ponto fixo de Schaefer.

1 Introdução

Recentemente, equações dinâmicas descontínuas em escalas temporais foram estudadas de modo independente em [1, 2, 3, 4, 2]. Aqui nós estudamos resultados de existência de soluções para o seguinte problema de valor inicial

$$\begin{cases} x^{\Delta}(t) = f(t, x(t)) \quad \Delta - q.t.p. \;\; t \in [a, b)_{\mathbb{T}} \\ x(a) = x_0 \end{cases} \tag{1}$$

onde $\mathbb{T}$ é uma escala temporal com $a = \min \mathbb{T}$ e $b = \max \mathbb{T}$, $x_0 \in \mathbb{R}^n$, $f : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ e x^{Δ} é derivada delta de x.

Observamos que uma escala temporal é um subconjunto fechado e não-vazio de números reais. Já o conjunto $[a, b)_{\mathbb{T}}$ é dado por $[a, b) \cap \mathbb{T}$. Enquanto que a notação $\Delta - q.t.p.$ $t \in [a, b)_{\mathbb{T}}$ dada na Eq. (1) indica que a equação dinâmica $x^{\Delta}(t) = f(t, x(t))$ é satisfeita para Δ-quase todo ponto $t \in [a, b)_{\mathbb{T}}$. Aqui o campo vetorial f dado na Eq. (1) é possivelmente descontínuo. Dessa forma, a equação dinâmica $x^{\Delta}(t) = f(t, x(t))$ define uma equação dinâmica descontínua na escala temporal $\mathbb{T}$.

Soluções $x : \mathbb{T} \to \mathbb{R}^n$ para a Eq. (1) serão entendidas como funções absolutamente contínuas.

2 Escalas Temporais

Definimos a função $\sigma : \mathbb{T} \to \mathbb{T}$ como

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$$

e a função $\rho : \mathbb{T} \to \mathbb{T}$ como

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\}.$$

Estamos supondo que $\inf \emptyset = \sup \mathbb{T}$ e $\sup \emptyset = \inf \mathbb{T}$. Já a função $\mu : \mathbb{T} \to [0, +\infty)$ é dada por $\mu(t) = \sigma(t) - t$.

Uma função $\beta : \mathbb{T} \to \mathbb{R}$ é rd-contínua se β é contínua em cada ponto $t \in \mathbb{T}$ tal que $\sigma(t) = t$ e $\lim_{s \to t^-} \beta(s)$ existe e é finito em cada ponto $t \in \mathbb{T}$ tal que $\rho(t) = t$. Dizemos que uma função rd-contínua $\beta : \mathbb{T} \to \mathbb{R}$ é positivamente regressiva se $1 + \mu(t)\beta(t) > 0$ para todo $t \in \mathbb{T}$.

Denotaremos por $e_{\beta}(t, a)$ a função exponencial na escala temporal $\mathbb{T}$.

Como em [5], definimos a norma generalizada de Bielecki da função $\beta : \mathbb{T} \to \mathbb{R}$ como

$$\|x\|_{\beta} = \sup_{t \in \mathbb{T}} \frac{\|x(t)\|}{e_{\beta}(t, a)}.$$

3 Resultados Principais

Os resultados principais do trabalho são enunciados no Teorema 3.1 e no Teorema 3.2. A seguir consideramos as hipóteses sobre a função f que são utilizadas nos resultados principais.

H1 A função $f(t,x)$ é contínua em x para Δ-q.t.p. $t \in [a,b]_{\mathbb{T}}$.

H2 A função $f(t,x(t))$ é Δ-mensurável para cada função Δ-mensurável $x : \mathbb{T} \to \mathbb{R}^n$.

H3 Para cada $r > 0$ existe uma função $h_r : \mathbb{T} \to [0,\infty)$ Lebesgue Δ-integrável tal que $\|f(t,x)\| \leq h_r(t)$ para Δ-q.t.p. $t \in [a,b]_{\mathbb{T}}$ e $\|x\| \leq r + \|x_0\|$.

H4 Existe uma função $\beta : \mathbb{T} \to [0,\infty)$ que é rd-contínua e positivamente regressiva de modo que

$$\|f(t,x) - f(t,y)\| \leq \beta(t)\|x - y\|$$

para Δ-q.t.p. $t \in [a,b]_{\mathbb{T}}$ e $x, y \in \mathbb{R}^n$.

H5 Existe uma constante $L > 0$ e uma função $c : \mathbb{T} \to [0,\infty)$ satisfazendo

$$\|f(t,x)\| \leq L\|x\| + c(t)$$

para Δ-q.t.p. $t \in [a,b)_{\mathbb{T}}$ e para todo $x \in \mathbb{R}^n$.

Teorema 3.1. *Suponha que as hipóteses H2, H3 e H4 sejam válidas. Então a Eq.* (1) *tem uma única solução. Além disso, tal solução x satisfaz $\|x\|_\beta \leq \|x_0\| + k\|h_r\|_\beta$, onde $k = (b-a)e_\beta(b,a)$.*

Teorema 3.2. *Suponha que as hipóteses H1, H2 e H5 sejam válidas. Então a Eq.* (1) *tem pelo menos uma solução.*

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O CRITÉRIO DE BÁEZ-DUARTE E ALGUNS ESPAÇOS DE HILBERT DE FUNÇÕES HOLOMORFAS

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Abstract

Neste trabalho estudaremos reformulações da hipótese de Riemann, algumas clássicas como o critério de Nyman-Beurling e outras mais recentes como o respectivo refinamento feito por Báez-Duarte nos espaços $L^2(0,1)$. Estudaremos também o equivalente critério de Báez-Duarte nos espaços de Hardy-Hilbert do disco unitário $H^2(\mathbb{D})$ e do semiplano $H^2(\mathbb{C}_{1/2})$, onde $\mathbb{C}_{1/2} = \{\mathrm{Re}(s) > 1/2\}$. Utilizando ferramentas de um espaço de Hilbert de séries de Dirichlet, o objetivo principal será dar uma aproximação da hipótese de Riemann através desta última reformulação.

1 Introdução

A hipótese de Riemann é considerada o mais importante problema aberto da matemática pura, o qual afirma que os zeros da função zeta de Riemann $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$, depois de uma extensão holomorfa a $\{s \in \mathbb{C} : \mathrm{Re}(s) > 0, s \neq 1\}$, estão localizados sobre a "linha crítica" Re(s)=1/2. Esse problema tem sido estudado por aproximadamente um século e meio, mas ainda não se tem nenhuma prova para ela.

Uma reformulação clássica da hipótese de Riemann feita por Nyman e Beurling (ver [5] e [3]) diz que esta conjectura é verdadeira se, e somente se, a função característica $\chi_{(0,1)}$ pertence ao fecho do espaço gerado por $\{f_\lambda : 0 \leq \lambda \leq 1\}$ em $L^2(0,1)$, onde $f_\lambda(x) = \{\lambda/x\} - \lambda\{1/x\}$ (aqui $\{x\}$ é a parte fracionária de x). Aproximadamente 50 anos mais tarde, Báez-Duarte [2] fez um refinamento deste critério, mostrando que era possível substituir $\lambda \in (0,1)$ por $\lambda = 1/k$, para $k \geq 2$.

Existe uma versão equivalente do critério de Báez-Duarte no espaço de Hardy-Hilbert $H^2(\mathbb{C}_{1/2})$, onde $H^2(\mathbb{C}_\alpha)$ ($\alpha \in \mathbb{R}$) é o espaço das funções analíticas F no semiplano $\mathbb{C}_\alpha = \{s \in \mathbb{C} : \mathrm{Re}(s) > \alpha\}$ tais que

$$\|F\|^2_{H^2(\mathbb{C}_\alpha)} := \sup_{x>\alpha} \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(x+it)|^2 dt < \infty.$$

Para $k \geq 2$, defina

$$G_k(s) = (k^{-s} - k^{-1}) \frac{\zeta(s)}{s} \quad \text{e} \quad E(s) = \frac{1}{s}, \quad s \in \mathbb{C}_{1/2}.$$

Então, o critério de Báez Duarte para $H^2(\mathbb{C}_{1/2})$ (ver [4]) pode ser reformulado da seguinte maneira.

Teorema 1.1. *A hipótese de Riemann é verdadeira se, e somente se, E pertence ao fecho do espaço gerado por $\{G_k : k \geq 2\}$ em $H^2(\mathbb{C}_{1/2})$.*

O propósito principal deste trabalho será mostrar que E pertence ao fecho do espaço gerado por $\{G_k : k \geq 2\}$ em $H^2(\mathbb{C}_{1+\epsilon})$, para cada $\epsilon > 0$. Para isso, precisamos estudar alguns espaços de Hilbert de funções holomorfas tais como os espaços

$$\mathcal{H}^2 = \left\{ f : \mathbb{C}_{1/2} \longrightarrow \mathbb{C} : f(s) = \sum_{n=1}^{\infty} a_n n^{-s}, \|f\|^2_{\mathcal{H}^2} = \sum_{n=1}^{\infty} |a_n|^2 < \infty \right\}$$

e, o espaço de Hardy-Hilbert $H^2(\mathbb{D})$ do disco unitário,

$$H^2(\mathbb{D}) = \left\{ f : \mathbb{D} \longrightarrow \mathbb{C} : f(z) = \sum_{n=0}^{\infty} a_n z^n, \|f\|_{H^2}^2 = \sum_{n=0}^{\infty} |a_n|^2 < \infty \right\}.$$

Generalizando um resultado de [1] e tomando idéias de [2], obteremos os seguintes resultados.

2 Resultados Principais

Para $\tau > 1/2$ e $k \geq 2$, seja

$$g_k^{\tau}(z) = k^{\tau-1} \sum_{n=1}^{\infty} \frac{z^n}{n^{\tau}} - \sum_{n=1}^{\infty} \frac{z^{nk}}{n^{\tau}}.$$

Então

Lema 2.1. *A série* $\sum_{k=2}^{\infty} \dfrac{\mu(k)}{k^{\tau}} g_k^{\tau}$ *coverge a* $-z$ *em* $H^2(\mathbb{D})$*, para cada* $\tau > 1/2$*, onde* μ *é a função de Möbius.*

Teorema 2.1. *A série* $\sum_{k=2}^{\infty} \mu(k) G_k$ *converge a* E *na norma de* $H^2(\mathbb{C}_{1+\epsilon})$*, para cada* $\epsilon > 0$*.*

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NECESSARY AND SUFFICIENT CONDITIONS FOR THE POLYNOMIAL DAUGAVET PROPERTY

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Abstract

In this work, we present some necessary and/or sufficient conditions for the polynomial Daugavet property, generalizing some results that are valid in the linear case.

1 Introduction

A Banach space X is said to have the *polynomial Daugavet property* if every weakly compact polynomial $P : X \to X$ satisfies

$$\|\mathrm{Id} + P\| = 1 + \|P\|, \tag{DE}$$

which is known as the *Daugavet equation*. This property was first studied by Choi et al. [1] on spaces of continuous functions. Since then, several authors have shown that different Banach spaces have the polynomial Daugavet property. The study of the polynomial Daugavet property emerged from the study of the Daugavet property. A Banach space X is said to have the *Daugavet property* if every rank-one operator $T : X \to X$ satisfies

$$\|\mathrm{Id} + T\| = 1 + \|T\|.$$

Note that the polynomial Daugavet property implies the Daugavet property. However, it is not known whether these properties are equivalent or not, and both of them continue to be studied by many authors.

Motivated by recent results presented by Rueda Zoca [3] for the Daugavet property, in this note we will introduce some findings about the Polynomial Daugavet Property.

2 Main Results

From now on we will consider only real Banach spaces. Given a Banach space X, we will denote by X^* the topological dual of X, by $\mathcal{P}(X)$ the normed space of all continuous polynomials from X into $\mathbb{R}$, and by B_X and S_X the closed unit ball and the unit sphere of X, respectively. Given $x \in X$ and $r > 0$, we will denote $B(x, r) = \{y \in X : \|x + y\| \leq r\}$. Moreover, a *polynomial slice of* B_X will be a set of the form

$$S(p, \alpha) = \{x \in B_X : |p(x)| > 1 - \varepsilon\},$$

where $p \in S_{\mathcal{P}(X)}$ and $\alpha > 0$.

Generalizing Lemma 2.1 by Kadets et al. [2], we present the following characterization of the polynomial Daugavet property.

Theorem 2.1. *Let X be a Banach space. The following are equivalent:*

(i) X has the polynomial Daugavet property.

(ii) *For every $x_0 \in S_X$ and every polynomial slice $S(p, \varepsilon_0)$ of B_X there exists another slice $S(q, \varepsilon_1) \subset S(p, \varepsilon_0)$ of B_X such that for every $x \in S(q, \varepsilon_1)$ the inequality*

$$\|x + \operatorname{sgn}(p(x))x_0\| > 2 - \varepsilon_0$$

holds.

(iii) *For every $x_0 \in S_X$, every $\varepsilon > 0$ and every polynomial slice S of B_X there exists $x \in S$ such that*

$$\|x + \operatorname{sgn}(p(x))x_0\| > 2 - \varepsilon_0.$$

Based on the proof of [2, Lemma 2.8] we prove an extension of the last result.

Lemma 2.1. *If X has the polynomial Daugavet property, then for every finite-dimensional subspace Y of X, every $\varepsilon_0 > 0$ and every polynomial slice $S(p, \varepsilon_0)$ of B_X there is a polynomial slice $S(q, \varepsilon)$ of B_X such that*

$$\|y + tx\| \geq (1 - \varepsilon_0)(\|y\| + |t|)$$

for all $y \in Y$, $x \in S(q, \varepsilon)$ and $t \in \mathbb{R}$.

The previous lemma allows us to prove the next proposition.

Proposition 2.1. *Let X be a Banach space. Then X has the polynomial Daugavet property if and only if given a polynomial slice S of B_X, it follows that, whenever there exist $x_1, \ldots, x_n \in X$ such that $S \subset \bigcup_{i=1}^n B(x_i, r_i)$, then there exists $i \in \{1, \ldots, n\}$ such that $r_i \geq 1 + \|x_i\|$.*

Given a Banach space X, the *ball topology*, denoted by b_X, is defined as the coarsest topology on X so that every closed ball is closed in b_X. As a consequence of the characterization given in Proposition 2.1 we obtain:

Proposition 2.2. *Let X be a Banach space. If X has the polynomial Daugavet property then for every nonempty b_X open subset O and for every polynomial slice S of B_X, we have $S \cap O \neq \varnothing$.*

Finally, we present a sufficient condition for the polynomial Daugavet property.

Theorem 2.2. *Let X be a Banach space. If for every polynomial slice $S^* = S(P, \varepsilon)$ of $B_{X^{**}}$, there exists $u \in S^* \cap S_{X^{**}}$ such that*

$$\|u + \operatorname{sgn}(P(u))x\| = 1 + \|x\|$$

for every $x \in X$, then X has the polynomial Daugavet property.

The proof of the last three results made use of the ideas of [3, Lemma 3.1 and Theorem 3.2].

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EXISTENCE AND ASYMPTOTIC PROPERTIES FOR A DISSIPATIVE SEMILINEAR SECOND ORDER EVOLUTION EQUATION WITH FRACTIONAL LAPACIAN OPERATORS

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Abstract

In this work we study asymptotic properties of global solutions for an initial value problem of a second order fractional linear differential equation with super damping.

1 Introduction

In this work we consider the Cauchy problem for a generalized second order linear evolution equation given by

$$\begin{cases} u_{tt} + (-\Delta)^{\delta} u_{tt} + (-\Delta)^{\theta} u_t + (-\Delta)^{\alpha} u = 0, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases} \tag{1}$$

with $u = u(t,x)$, $(t,x) \in (0,\infty) \times \mathbb{R}^n$ and the exponents of the Laplace operators α, δ and θ satisfying $\alpha > 0$ and

$$0 < \delta \leq \alpha, \quad \frac{\alpha+\delta}{2} < \theta < \frac{\alpha+2\delta}{2}.$$

To obtain the decay rates, we employ a method of energy in the Fourier space that has its origin in Umeda-Kawashima-Shizuta [5] combined with the explicit solution of the associated problem in the Fourier space, and an asymptotic profile obtained from the explicit solution. We obtain decay rates for the energy, but our aim is mainly concentrated on proving the optimality of decay rates for the L^2 norm of solutions, although we can also prove the optimality for decay rates of the L^2 norm of the derivatives of solutions by using the same method.

2 Main Results

Theorem 2.1. *Let $n > 2\alpha$, $P_1 \neq 0$, $0 < \delta \leq \alpha$, $\frac{\alpha+\delta}{2} < \theta < \frac{\alpha+2\delta}{2}$, $\kappa \in (0, \min\{1,\delta\})$ and $\epsilon > \frac{2\theta-\alpha}{2\theta}(n-2\alpha)$. Assume*

$$u_0 \in L^1(\mathbb{R}^n) \cap H^{\epsilon}(\mathbb{R}^n), \;\; u_1 \in L^{1,\kappa}(\mathbb{R}^n) \cap H^{\delta+\epsilon-\alpha}(\mathbb{R}^n).$$

Then there exist constants $C_1 > 0$, $C_2 > 0$ and $t_0 >> 1$ such that

$$C_1|P_1| t^{-\frac{n-2\alpha}{4\theta}} \leq \|u(t,\cdot)\| \leq C_2 t^{-\frac{n-2\alpha}{4\theta}},$$

holds for $t \geq t_0$ where $u(t,x)$ is the solution of the problem (1).

Remark 2.1. *Taking the derivatives of the explicit solution and the associated asymptotic profile, we can prove optimal decay estimates of the L^2 norms of u_t, $(-\Delta)^{\alpha/2}u$ and $(-\Delta)^{\delta/2}u_t$:*

$$||u_t(t,\cdot)||^2 \leq Ct^{-\frac{n}{2\theta}}, \quad ||(-\Delta)^{\alpha/2}u(t,\cdot)||^2 \leq Ct^{-\frac{n}{2\theta}}, \quad ||(-\Delta)^{\delta/2}u_t(t,\cdot)||^2 \leq C\, t^{-\frac{n+2\delta}{2\theta}}, \quad t >> 1.$$

Remark 2.2. *We may obtain similar results for the case $\delta = 0$, obtaining the same decay rate, more specifically, we prove the following theorem:*

Theorem 2.2. *Let $n > 2\alpha$, $\kappa > \dfrac{(n-2\alpha)(\theta-\alpha)}{2\theta}$, $\epsilon \in (0, \min\{1,\alpha\})$. If $u_0 \in L^1(\mathbb{R}^n) \cap H^\kappa(\mathbb{R}^n)$, $u_1 \in L^{1,\epsilon}(\mathbb{R}^n) \cap L^2(\mathbb{R}^n) \cap \dot{W}^{\kappa-\alpha,2}(\mathbb{R}^n)$ then*

$$C_1|P_1| t^{-\frac{n-2\alpha}{4\theta}} \leq \|u(t,\cdot)\| \leq C_2 t^{-\frac{n-2\alpha}{4\theta}},$$

for all $t > 0$ large enough, where C_1 and C_2 are positive constants.

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REGULARIDADE GLOBAL PARA UMA VERSÃO β-*PATCH* DE UM MODELO UNIDIMENSIONAL COM DISSIPAÇÃO FRACIONÁRIA CRÍTICA E SUBCRÍTICA

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Abstract

Investigamos uma versão β-*patch* de um modelo unidimensional com dissipação fracionária mostrando existência local de soluções e um critério de *blow-up*. Com base no critério de *blow-up* e na técnica de módulo de continuidade concluímos a existência de soluções globais suaves para a dissipação crítica e subcrítica.

1 Introdução

Neste trabalho, consideraremos o Problema de Valor Inicial (PVI) para a equação de transporte unidimensional com velocidade não local

$$\begin{cases} \theta_t + u\theta_x + \Lambda^\gamma \theta = 0 \ \text{ em } \ \mathbb{T}\times(0,\infty), \\ \theta(x,0) = \theta_0(x) \ \text{ em } \ \mathbb{T}, \\ u = -\Lambda^{-\beta}\mathcal{H}\theta, \end{cases} \tag{1}$$

onde $0<\beta<1$, $0<\gamma<2$, $\Lambda = (-\Delta)^{\frac{1}{2}}$, $\mathbb{T}$ é o toro 1D e $\mathcal{H}$ denota a transformada de Hilbert.

Este modelo surge como um β-*patch* do modelo

$$\theta_t + u\theta_x + \Lambda^\gamma\theta = 0,\ u = \mathcal{H}\theta, \tag{2}$$

proposto por Córdoba, Córdoba e Fontelos em [2]. Resultados de regularidade global foram obtidos para $1 \leqslant \gamma < 2$ e no intervalo $0<\gamma<1$ assumindo uma condição de pequenez, veja [2, 3]. Na parte $0<\gamma<\frac{1}{2}$, foi mostrado formação de singularidades em tempo finito para dados iniciais suaves satisfazendo algumas condições, veja [6, 7]. No intervalo $\frac{1}{2} \leqslant \gamma < 1$, formação de singularidades em tempo finito ou regularidade global é um problema em aberto (apresentado por [6, p. 251]), até mesmo para dados iniciais com condição de sinal. Para uma noção de solução global no caso supercrítico $\frac{1}{2} \leqslant \gamma < 1$, consulte [4].

O estudo do modelo (1) é dividido em três casos: subcrítico $1-\beta<\gamma<2$, crítico $\gamma = 1-\beta$ e supercrítico $0<\gamma<1-\beta$. Em [1], Bae, Granero-Belinchón e Lazar mostraram a existência de soluções fracas globais para (1) no intervalo $1-\beta \leqslant \gamma < 2$ (caso crítico e subcrítico) com dado inicial não negativo $\theta_0 \in L^1\cap L^\infty$.

Nosso objetivo é aplicar a técnica de módulo de continuidade, inspirada por [5], e concluir a existência e unicidade global de soluções suaves para os casos críticos e subcríticos do PVI (1).

2 Resultados Principais

Nosso primeiro resultado é sobre existência e unicidade local de soluções para o PVI (1) que fornece um critério de *blow-up*.

Teorema 2.1. *Sejam $0<\beta<1$ e $0<\gamma<2$. Se $s>\frac{3}{2}$, $s\in\mathbb{R}$, então para cada $\theta_0 \in H^s(\mathbb{T})$ existe um $T = T(||\theta_0||_{H^s(\mathbb{T})})>0$ e uma única solução*

$$\theta(t,x) \in L^\infty([0,T];H^s(\mathbb{T})) \cap L^2([0,T];H^{s+\frac{\gamma}{2}}(\mathbb{T})). \tag{1}$$

Além disso, temos o seguinte critério de blow-up

$$\limsup_{t\to T} ||\theta(t)||_{H^s(\mathbb{T})} = +\infty \iff \int_0^T ||u_x(\tau)||_{L^\infty(\mathbb{T})} + ||\theta_x(\tau)||_{L^\infty(\mathbb{T})} d\tau = +\infty. \tag{2}$$

No nosso principal resultado mostramos que o PVI (1) tem uma única solução suave com dado inicial suave e periódico para a viscosidade crítica e em todos os valores do caso subcrítico.

Teorema 2.2. *Sejam $0 < \beta < 1$ e $1 - \beta \leqslant \gamma < 2$. Se $\theta_0 \in C^\infty(\mathbb{T})$, então existe uma única solução global suave de (1).*

Antes de fornecermos uma ideia da demonstração, vamos definir o que chamamos de módulo de continuidade e o que significa uma função obdecer um módulo de continuidade.

Definição 2.1. *Um módulo de continuidade é uma função crescente, côncava e contínua*

$$\omega : [0, \infty) \to [0, \infty)$$

tal que $\omega(0) = 0$. Diremos que $f : \mathbb{R}^n \to \mathbb{R}^m$ tem módulo de continuidade ω se

$$|f(x) - f(y)| \leqslant \omega(|x - y|)$$

para quaisquer $x, y \in \mathbb{R}^n$.

A ideia da demonstração é construir uma família de módulos de continuidade que são preservados pela evolução e que cada dado inicial $\theta_0 \in C^\infty(\mathbb{T})$ satisfaça um desses módulos de continuidade, fornecendo uma estimativa uniforme no tempo que nos permite extender a solução do Teorema 2.1 globalmente no tempo.

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PROBLEMAS DE VALOR DE FRONTEIRA ELÍPTICOS VIA ANÁLISE DE FOURIER

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Abstract

Neste trabalho estudamos uma classe de problemas de valor de fronteira (PVF) elípticos e não lineares no semiespaço com condições de fronteira que comportam não linearidades e potenciais singulares. Apresentamos resultados de existência e unicidade de soluções para uma formulação integral do problema considerando espaços funcionais cujos elementos são curvas parametrizadas fracamente contínuas de distribuições temperadas, com valores em um espaço com peso na variável de Fourier. A formulação é obtida destacando a variável x_n e aplicando a transformada de Fourier nas outras. Nossa abordagem não é do tipo variacional e cobre uma variedade de PVF elípticos. Em particular, na fronteira podemos considerar o potencial de Kato $V(x') = \lambda/|x'|$ e obter resultados de existência para $|\lambda| < \lambda_* = 2\Gamma^2(n/4)/\Gamma^2((n-2)/4)$ (consequência do Teorema 2.2), sem necessidade de usar a chamada desigualdade de Kato. O valor λ_* é a melhor constante para a desigualdade de Kato no semiespaço (ver[1]) e aparece na literatura como limiar para resultados de existência em abordagens baseadas nessa desigualdade e espaços de funções suaves (ver [4]). Assim, nosso resultado indica que λ_* é intrínseca ao problema e independente da abordagem utilizada no estudo.

1 Introdução

Problemas elípticos com condições de fronteira não lineares são amplamente estudados (ver [3] e suas referências). Neste trabalho consideramos a seguinte classe de problemas no semiespaço, com termos de fronteira contendo potenciais singulares e não linearidades.

$$\begin{cases} -\Delta u = A_1 u^p + V_1 u & \text{em } \mathbb{R}^n_+ \\ B_1 \dfrac{\partial u}{\partial \nu} + B_2 u = g(x') + V_2(x')u + A_2 u^q & \text{em } \partial\mathbb{R}^n_+ = \mathbb{R}^{n-1}, \end{cases} \tag{1}$$

onde $n \geq 3$, $p, q > 1$ são inteiros, $\nu = -e_n$ é a normal exterior a $\partial\mathbb{R}^n_+$ e, $A_i,\ B_i \in \mathbb{R}$ para $i = 1, 2$, de forma que B_1 e B_2 não se anulam simultaneamente ($B_1^2 + B_2^2 \neq 0$) e não possuem sinais opostos ($B_1 B_2 \geq 0$). Para evitar inconsistências impomos $V_2 \equiv 0$, se $B_1 = 0$.

Assumindo regularidade, destacamos a variável x_n, escrevemos $\Delta = \Delta_{x'} + \partial^2_{x_n x_n}$ e aplicamos a transformada de Fourier nas $n-1$ primeiras variáveis obtendo uma EDO na variável x_n cuja solução pode ser expressa como

$$\widehat{u}(\xi', x_n) = \int_0^\infty G(\xi', x_n, t)\Big[A_1 \widehat{u^p}(\xi', t) + \widehat{V_1 u}(\xi', t)\Big]dt + \widetilde{G}(\xi', x_n)\Big[\widehat{g}(\xi') + \widehat{V_2 u}(\xi', 0) + A_2 \widehat{u^q}(\xi', 0)\Big], \tag{2}$$

onde G e $\widetilde{G}$ são definidas como segue

$$G(\xi', x_n, t) = \frac{(2\pi|\xi'|B_1 + B_2)e^{-2\pi|\xi'||x_n - t|} + (2\pi|\xi'|B_1 - B_2)e^{-2\pi|\xi'|(x_n+t)}}{4\pi|\xi'|(2\pi|\xi'|B_1 + B_2)} \quad \text{e} \quad \widetilde{G}(\xi', x_n) = \frac{e^{-2\pi|\xi'|x_n}}{2\pi|\xi'|B_1 + B_2}.$$

Para lidar com (2) e os potencias singulares na fronteira, usamos o espaço de Banach (ver [2])

$$\mathcal{PM}^k(\mathbb{R}^{n-1}) = \left\{ v \in \mathcal{S}'(\mathbb{R}^{n-1}) : \hat{v} \in L^1_{loc}(\mathbb{R}^{n-1}),\ \operatorname*{ess\,sup}_{\xi' \in \mathbb{R}^{n-1}} |\xi'|^k\, |\hat{v}(\xi')| < \infty \right\}$$

com $0 \le k < n-1$, e norma dada por $\|v\|_{\mathcal{PM}^k} = \operatorname{ess\,sup}_{\xi' \in \mathbb{R}^{n-1}} |\xi'|^k |\hat{v}(\xi')|$. Assim, estudamos (2) no espaço $X_k = BC_w([0,\infty), \mathcal{PM}^k)$, formado por todas as funções limitadas, $u : [0,\infty) \to \mathcal{PM}^k$, fracamente contínuas no sentido de $\mathcal{S}'$. X_k munido da norma uniforme $\|u\|_{X_k} = \sup_{x_n > 0} \|u(\cdot, x_n)\|_{\mathcal{PM}^k}$ também é de Banach. Cabe destacar que a norma $\|\cdot\|_{X_k}$ é invariante pelo *scaling* $u_\lambda(x) = \lambda^{\frac{2}{p-1}} u(\lambda x)$, $\lambda > 0$, isto é $\|u_\lambda\|_{X_k} = \|u\|_{X_k}$, se

$$k = n - 1 - \frac{2}{p-1} = n - 1 - \frac{1}{q-1}. \tag{3}$$

Este valor é crítico para (1) e no que segue reservaremos a letra k para ele.

Por fim, expressamos a equação integral (2) através da equação funcional

$$u = \mathcal{H}_1(u) + \mathcal{L}_{V_1}(u) + \mathcal{N}(g) + \mathcal{L}_{V_2}(u) + \mathcal{H}_2(u), \tag{4}$$

onde $u \in X_k$, e os termos do lado direito de (4) são definidos de forma conveniente via transformada de Fourier.

2 Resultados Principais

A seguir enunciaremos dois Teoremas de Existência e Unicidade

Teorema 2.1 (Caso $A_1, A_2, B_1 \neq 0$). *Sejam $n \ge 4$, $p, q \in \mathbb{N}$, $p > (n-1)/(n-3)$ ímpar, $q = (p+1)/2$, $k = n - 1 - 2/(p-1)$, $V_1 \in X_{n-3}$, $V_2 \in \mathcal{PM}^{n-2}$ e $g \in \mathcal{PM}^{k-1}$. Considere*

$$\tau_k = L_1(k)\|V_1\|_{X_{n-3}} + L_2(k)\|V_2\|_{\mathcal{PM}^{n-2}} \;\; e \;\; \epsilon_k = \min\left\{\frac{1-\tau_k}{2}, \frac{(1-\tau_k)^{q/(q-1)}}{2^{q/(q-1)} K_k^{1/(q-1)}}\right\}, \tag{5}$$

para certas constantes positivas $K_K, L_1(k)$ e $L_2(k)$. Se escolhermos V_1, V_2 e g tais que $\tau_k < 1$ e $\|g\|_{\mathcal{PM}^{k-1}} < \epsilon/M$, com $0 < \epsilon < \epsilon_k$ e $M > 0$ apropriado, então a equação funcional (4) possui solução única $u \in X_k$ tal que $\|u\|_{X_k} \le 2\epsilon/(1-\tau_k)$. Mais ainda, $u(\cdot, x_n) \in L^\infty(\mathbb{R}^{n-1}) + L^2(\mathbb{R}^{n-1})$, para todo $x_n \ge 0$.

Teorema 2.2 (Caso $A_1, A_2 = 0$, $B_1 \neq 0$). *Sejam $n \ge 4$ e $k \in \mathbb{R}$ tal que $2 < k < n-1$. Considere também $V_1, V_2, g, \tau_k, L_1(k)$ e $L_2(k)$ como no Teorema 2.1. Se escolhermos V_1 e V_2 tais que $\tau_k < 1$, então a equação funcional (4) tem uma única solução $u \in X_k$. Além disso, se $k > (n-1)/2$ então $u(\cdot, x_n) \in L^\infty(\mathbb{R}^{n-1}) + L^2(\mathbb{R}^{n-1})$, para todo $x_n \ge 0$.*

Observação 2. *Se assumirmos $A_1 = 0$ e $V_1 \equiv 0$ então o Teorema 2.2 continua válido para $n \ge 3$.*

Observação 3. *Diferentemente do Teorema 2.1, no Teorema 2.2 não há restrições sobre o tamanho da solução.*

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UM PROBLEMA DE MINIMIZAÇÃO PARA O $P(X)$-LAPLACIANO ENVOLVENDO ÁREA

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Abstract

No presente trabalho, apresentamos um problema de minimização em $\mathbb{R}^N$ envolvendo o perímetro do conjunto de positividade da solução u e a integral de $|\nabla u|^{p(x)}$, onde $p(x)$ é uma função Lipschitz contínua tal que $1 < p_{min} \leq p(x) \leq p_{max} < \infty$. Provamos que tal função de minimização existe e que ela é uma solução clássica para um problema de fronteira livre. Em particular, a fronteira livre reduzida é uma superfície $\mathcal{C}^2$ e a dimensão do conjunto singular é pelo menos $N-8$. Também, se assumirmos mais regularidade para o expoente $p(x)$ ganhamos mais regularidade para a fronteira livre.

1 Introdução

Seja $E \subset\subset B_R$ satisfazendo uma condição de bola interior. Neste trabalho, analisamos o problema de minimizar o funcional

$$J(u) := \int_{B_R} \frac{|\nabla u|^{p(x)}}{p(x)}\, dx + \operatorname{Per}(\{u > 0\}, B_R)$$

entre todas as funções $0 \leq u \in W_0^{1,p(x)}(B_R)$ tais que $u = 1$ em E. Aqui, para um conjunto $\Omega \subset B_R$,

$$\operatorname{Per}(\Omega, B_R) = \sup\left\{\int_\Omega \operatorname{div}\, \eta\, dx,\ \eta \in C_0^1(B_R; \mathbb{R}^N) \text{ com } \|\eta\|_{L^\infty(B_R)} \leq 1\right\}$$

é o perímetro de Ω em B_R. Além disso, para algum $0 < \alpha < 1$, provamos que $u \in \mathcal{C}^{1,\alpha}((\Omega \cup \partial_{red}\Omega) \setminus \bar{E})$, $\partial_{red}\Omega \in \mathcal{C}^{2,\alpha}$, $\mathcal{H}^s(\partial\Omega \setminus \partial_{red}\Omega) = 0$ se $s > N-8$ e a condição de fronteira livre é satisfeita no sentido clássico.

Um problema similar para $p(x) \equiv 2$ no caso de duas fases foi considerado em [3]. Depois em [5] e [3] os autores consideraram o problema de uma fase para o caso constante $p(x) \equiv p$. Outras variações para este problema no caso linear $p(x) \equiv 2$ foi tratado em [2] e [4]. Por outro lado, para espaços de Orlicz, uma generalização da função t^p para funções convexas $G(t)$ satisfazendo "condição de Lieberman" e com o funcional J incluindo outros termos, foi estudado em [6]. A presença do expoente variável constante $p(x)$ traz certas dificuldades técnicas não presentes nos trabalhos citados anteriormente.

2 Resultados Principais

Começamos provando que o problema de minimização em questão possui uma solução em $\mathcal{A}$.

Teorema 2.1. *Existe um par admissível no conjunto*

$$\mathcal{A} := \big\{(u, \Omega) \,/\, E \subset \Omega \subset B_R,\ 0 \leq u \in W_0^{1,p(x)}(B_R),\ u = 1 \text{ em } E,\ \{u > 0\} \subset \Omega\big\}.$$

que minimiza o funcional

$$\mathcal{J}(u, \Omega) = \int_{B_R} \frac{|\nabla u|^{p(x)}}{p(x)}\, dx + Per(\Omega, B_R). \tag{1}$$

Depois, usando a teoria de superfícies quase mínimas podemos provar certa regularidade para a fronteira reduzida.

Teorema 2.2. *Seja p Lipschitz contínua e seja (u, Ω) um minimizante para o funcional (1). Se $x_0 \in \partial_{red}\Omega \cap B_R$ e $B_r(x_0) \subset B_R$, então*

(1) $\partial_{red}\Omega \cap B_{r/2}(x_0)$ é uma hipersuperfície de classe $C^{1,1/2}$ e existe uma constante $C_0 > 0$ dependendo somente de p_{max}, p_{min},θ_0 e $\|\nabla p\|_{L^\infty}$ tal que

$$|\nu(x) - \nu(y)| \leq C_0|x - y|^{1/2}$$

para todo $x, y \in \partial_{red}\Omega \cap B_{r/2}(x_0)$, onde ν é o vetor unitário normal exterior a $\partial\Omega$.

(2) $\mathcal{H}^s[(\partial\Omega \setminus \partial_{red}\Omega) \cap B_{r/2}(x_0)] = 0$ para todo $s > N - 8$.

Por fim, encontramos a condição de fronteira livre, o que nos leva a concluir a regularidade da fronteira livre.

Teorema 2.3. *Seja p Lipschitz contínua e (u, Ω) um minimizante do funcional (1) em $\mathcal{A}$. Então, $\mathcal{H}^s(\partial\Omega \setminus \partial_{red}\Omega) = 0$ para todo $s > N - 8$. Além disso, seja $x_0 \in \partial_{red}(\Omega)$. Existe $\delta > 0$ e $0 < \alpha < 1$ tais que $u \in C^{1,\alpha}(B_\delta(x_0) \cap \bar{\Omega})$, $\partial\Omega \cap B_\delta(x_0) \in C^{2,\alpha}$ e a condição de fronteira livre $H_\Omega(x) = \Phi(|\nabla u(x)|, x)$ –com $\Phi(t, x) = \big(1 - \frac{1}{p(x)}\big)t^{p(x)}$– é satisfeita no sentido clássico. Mais ainda, se $p \in C^{k,\alpha}$ para algum $k \geq 1$, então $u \in C^{k+1,\alpha}(B_\delta(x_0) \cap \bar{\Omega})$ e $\partial\Omega \cap B_\delta(x_0) \in C^{k+2,\alpha}$.*

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SMALL LIPSCHITZ PERTURBATION OF SCALAR MAPS

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Abstract

In this paper we consider small Lipschitz perturbations of differentiable and Lipschitz maps. We obtain conditions to ensure the permanence of fixed points (sink and source) for scalar Lipschitz maps without requiring differentiability, in a step norm weaker than the C^1-norm and stronger than the C^0-norm.

1 Introduction

Theory of dynamical systems is widely investigated in the literature from the point of view of C^0 and C^1-convergence, that is, usually the maps considered are homomorphisms or diffeomorphisms [1, 2]. In the second situation, the differentiability enables to ensure, under generic assumptions, the permanence of hyperbolic fixed points [3, 4].

In this paper we propose a framework of small Lipschitz perturbation for Lipschitz maps, as well as to show that some of the results which are valid to discrete standard smooth dynamical systems also hold when considering a class of Lipschitz maps instead of considering differentiable maps. Moreover, since a Lipschitz map is not necessarily differentiable, this approach aims to point out some results that lie in the small gap between C^0 and C^1 theory of discrete dynamical systems.

Although the Lipschitz condition does not guarantee differentiability it is known that it guarantees differentiability almost everywhere with respect to the Lebesgue measure. This is the content of the Rademacher's Theorem.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}$ be an open set, and let $f : \Omega \longrightarrow \mathbb{R}$ be a Lipschitz map. Then f is differentiable at almost every point in Ω.*

Thus one Lipschitz map which is not differentiable should produce interesting dynamics even if we start at point of non differentiability or if a fixed point is one point for which the differentially fails. This approach has been proposed in [5] for maps in finite dimension and in [6] for semigroups in infinity dimension.

Our main goal in this paper is to find a class of Lipschitz function whose the dynamics are preserved under small Lipschitz perturbations. We first state precisely what we mean by small Lipschitz perturbation and in the main result we exhibit a class of locally Lipschitz maps that will be unstable under this notion. Our results are in agreement with the works existing in the literature related to permanence of hyperbolic fixed points in the C^1-topology.

2 Results

In the first main result of [5], the authors characterized sink and source for locally Lipschitz and reverse Lipschitz maps, respectively, by means of the Lipschitz constant and reverse Lipschitz constant.

Theorem 2.1. *[5, Thm.3.2] Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a map and $p \in \mathbb{R}$ a fixed point of f.*

1- *If f is strictly locally Lipschitz map at p, with Lipschitz constant $c < 1$, then p is a sink.*

2- *If f is locally reverse Lipschitz map at p, with constant $r > 1$, then p is a source.*

The next result improves Theorem 2.1 by showing that sink and source are isolated fixed points which are stable by small Lipschitz perturbation.

Theorem 2.2. *Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a map and p a fixed point of f.*

(1) If f is locally strictly Lipschitz, with constant $c < 1$ in a neighborhood of p, then p is the unique fixed point in this neighborhood and it is a sink.

(2) If f is reverse Lipschitz with constant $r > 1$, in a neighborhood of p, then p is the unique fixed point in this neighborhood and it is a source.

Proof. To prove Item (1) note that follows by Theorem 2.1 that p is a sink, and then there is a neighborhood $N_\delta(p)$ such that $f(\overline{N_\delta(p)}) \subset \overline{N_\delta(p)}$. Now the result follows from Banach Contraction Theorem. In fact, let $q \in N_\delta(p)$ be a fixed point of f, with $q \neq p$. Take $\varepsilon = \frac{\delta - |p-q|}{2}$, then $N_\varepsilon(q) \subset N_\delta(p)$ and q is a sink, thus, for $x \in N_\varepsilon(q)$, we have $\lim_{k\to\infty} f^k(x) = q$ and $\lim_{k\to\infty} f^k(x) = p$, which is a contradiction.

To show Item (2) we have from Theorem 2.1 that p is a source. Let $q \in N_\delta(p)$, $q \neq p$. Then there will be a positive integer k_0 such that $f^{k_0}(q) \notin N_\delta(p)$, which implies $f(q) \neq q$. Therefore there is no fixed point of f different from p. □

Lemma 2.1. *Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a map and let $p \in \mathbb{R}$. If $g : \mathbb{R} \longrightarrow \mathbb{R}$ is a map such that $\|f - g\|_{N_\delta(p)} < \epsilon$ (that is* (**??**) *is well defined and smaller than ϵ), for some $\delta > 0$, then for ϵ sufficiently small, we have:*

1- *If f is strictly locally Lipschitz with locally Lipschitz constant $c_{f,p} < 1$ in $N_\delta(p)$, then g is locally Lipschitz and the locally Lipschitz constant of g is strictly less than one.*

2- *If f is locally reverse Lipschitz with locally Lipschitz constant $r_{f,p} > 1$ in $N_\delta(p)$, then g is reverse locally Lipschitz and the locally reverse Lipschitz constant of g is strictly greater than one.*

Theorem 2.3. *Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a map and p a fixed point of f such that f is sufficiently differentiable in $\mathbb{R}$ and $|f'(p)| \neq 1$. If g is a locally Lipschitz function such that $\|f - g\|_{N_\delta(p)} < \epsilon$, then for δ and ϵ sufficiently small there is a unique fixed point q of g in $N_\delta(p)$. Moreover, if $|f'(p)| < 1$ then q is a sink and if $|f'(p)| > 1$ then q is a source.*

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LOCAL WELL-POSEDNESS AND REGULARITY THEORY FOR NONLINEAR TIME FRACTIONAL DIFFUSION-WAVE EQUATIONS IN LEBESGUE SPACES

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Abstract

In this work we consider the problem of well-posedness and spatial regularity for nonlinear time fractional diffusion-wave equations in Lebesgue spaces.

1 Introduction

The basic equations of elementary one-dimension theory of linear viscoelasticity are known to be

$$\sigma_x(x,t) = \rho u_{tt}(x,t), \tag{1}$$

$$\epsilon(x,t) = u_x(x,t), \tag{2}$$

$$\epsilon(x,t) = J_0\ \sigma(x,t) + \dot{J}(t) * \sigma(x,t), \tag{3}$$

where ρ is the density, u denotes the displacement, σ the stress and ϵ the strain. Furthermore, $J(t)$ represents the creep compliance, which is assumed to be a non-negative, non-decreasing function with initial value $J_0 = J(0^+) \geq 0$, called the instantaneous compliance. The evolution equation for the the displacement $u(x,t)$ can be derived from (1)-(3) since that

$$u_{xx} = \epsilon_x = (J_0 + \dot{J}*)\sigma_x = (J_0 + \dot{J}*)\rho u_{tt}. \tag{4}$$

If we consider the so called power-law material, for which the creep compliance can be written as

$$J(t) = \frac{1}{\rho a}\frac{t^\beta}{\Gamma(\beta+1)}, \quad 0 < \beta \leq 1, \quad t > 0,$$

where a is a positive constant and Γ is the Gamma function, then from (4) we obtain the evolution equation

$$\partial_t^\alpha u = a u_{xx}, \tag{5}$$

with $1 \leq \alpha = 2 - \beta \leq 2$ and ∂_t^α the time fractional derivative in Caputo's sense. Following [1] and [5], it follows that the above creep law is provided by viscoelastic models whose stress-strain relation verify

$$\sigma = \rho a \partial_t^\beta \epsilon, \quad 0 < \beta \leq 1.$$

For $\beta = 1$ we have the situation of a Newtonian fluid where a represents the kinematic viscosity. In this case (5) becomes the classical diffusion equation. In the limit case $\beta = 0$ we obtain from (5) the classical D'Alembert wave equation with wave-front velocity $c = \sqrt{a}$. When $0 < \beta < 1$, the evolution equation (5) is called the time fractional diffusion-wave equation and has been the subject of many research works, see [2, 3, 4, 6] and references therein. In [6], the authors show that (5) governs the propagation of stress waves in viscoelastic media which are of relevance in acoustics and seismology since their quality factor turns out to be independent of frequency.

2 Main Results

Motivated by the previous discussion, in [2] we consider the following nonlinear time-fractional diffusion-wave equation

$$\begin{cases} \partial_t^\alpha u = \Delta u + |u|^{p-1}u, \text{ in } (0,\infty)\times\Omega, \\ u = 0, \text{ in } [0,\infty)\times\partial\Omega, \\ u(0,x) = u_0(x), \ u_t(0,x) = u_1(x), \text{ in } \ \Omega, \end{cases} \tag{1}$$

where $p > 1$, Ω is an open subset of $\mathbb{R}^N$ with sufficiently smooth boundary $\partial\Omega$, $\alpha \in (1,2)$ and ∂_t^α is the Caputo fractional derivative. Our goal is to analyze the existence and uniqueness of local mild solutions to (1) and their possible continuation to a maximal interval of existence in the $L^q(\Omega)$ setting, $1 < q < \infty$. We also consider the problem of spatial regularity and continuous dependence with respect to initial data.

For this, note that the operator $-\Delta$ with Dirichlet boundary conditions can be seen as a sectorial operator in $E_q^0 = L^q(\Omega)$ with domain $E_q^1 = W^{2,q}(\Omega)\cap W_0^{1,q}(\Omega)$, whose angle is $\phi_q \in (\frac{\pi}{2},\pi)$. Set $X_q^\beta := E_q^{\beta-1}$, $\beta\in\mathbb{R}$, where $\{E_q^\beta\}_{\beta\in\mathbb{R}}$ is the scale of fractional powers spaces associated with $-\Delta$. Specifically, we prove the following result:

Theorem 2.1. *Consider $1 < \alpha < \frac{2\phi_q}{\pi}$, $\max\{1-\frac{N}{2q'},0\} < \beta < 1$ and $1 < p \leq 1 + \frac{2}{N}[q-\beta q]$. If $\alpha(1-\beta) < 1$, then given $v_0 \in L^q(\Omega)$, we can consider $r > 0$ and $\tau > 0$ such that for any $u_0, u_1 \in B_r(v_0) \subset L^q(\Omega)$ there exists a unique mild solution $u \in C([0,\tau], L^q(\Omega))$ of problem (1) which can be continued to a maximal time of existence $t_{max} > 0$ such that $t_{max} = \infty$ or*

$$\limsup_{t\to t_{max}^-} \|u(t)\|_{L^q(\Omega)} = \infty.$$

Furthermore, for all $0 \leq \theta < \beta$ such that $\alpha(1+\theta-\beta) < 1$, it follows that

$$u \in C((0,\tau], X_q^{1+\theta})$$

and if $\theta > 0$ then

$$\lim_{t\to 0} t^{\alpha\theta}\|u(t,u_0,u_1)\|_{X_q^{1+\theta}} = 0.$$

Moreover, if $u_0, w_0, u_1, w_1 \in B_r(v_0) \subset L^q(\Omega)$, then there exists a constant $c > 0$ such that

$$t^{\alpha\theta}\|u(t,u_0,u_1) - u(t,w_0,w_1)\|_{X_q^{1+\theta}} \leq c\left(\|u_0-w_0\|_{L^q(\Omega)} + \|u_1-w_1\|_{L^q(\Omega)}\right),$$

for all $t \in [0,\tau]$.

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TAXA DE DECAIMENTO PARA UM SISTEMA ACOPLADO COM DISSIPAÇÃO DO TIPO MEMÓRIA FRACIONÁRIA

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Abstract

Neste trabalho, estudamos a existência, unicidade, taxa de decaimento ótimo para um sistema abstrato com duas equações. Uma das equações é conservativa e a outra possui propriedade dissipativa. O mecanismo de dissipação é do tipo memória com expoente fracionário, generalizando a dissipação do tipo memória clássica. Utilizando a teoria de semigrupos de operadores, estabelecemos os resultados de existência, unicidade e a taxa de decaimento.

1 Introdução

Equações de evolução descrevem fenômenos em várias áreas da ciência e constantemente é objeto de interesse de pesquisadores. Algumas dessas equações modelam vibrações de materiais elásticos que possuem em sua estrutura algum mecanismo dissipativo. Nesse sentido, estudamos um sistema onde o mecanismo de dissipação é do tipo memória com expoente fracionário. Mais precisamente consideramos o sistema dado por

$$\rho_1 u_{tt} + \beta_1 \mathrm{A}u - \int_0^\infty g(s)\mathrm{A}^\theta u(t-s)ds + \alpha(u-v) = 0, \tag{1}$$

$$\rho_2 v_{tt} + \beta_2 \mathrm{A}v + \alpha(v-u) = 0, \tag{2}$$

com dados iniciais

$$u(0) = u_0,\ v(0) = v_0,\ u_t(0) = u_1,\ v_t(0) = v_1,\ u(-s) = \phi(s),\ s > 0. \tag{3}$$

Os coeficientes $\rho_1,\ \rho_2,\ \beta_1,\ \beta_2$ são positivos, o coeficiente de acoplamento α é positivo e não nulo e $\theta \in [0,1]$. O operador A é auto adjunto com o domínio $\mathcal{D}(A)$ imerso compactamente em $\mathbb{H}$ onde $\mathbb{H}$ é um espaço de Hilbert complexo. Aqui, a norma em $\mathcal{D}(A^\theta)$ é dado por $\|u\|_{\mathcal{D}(A^\theta)} := \|A^\theta u\|$ onde $\|\cdot\|$ denota a norma no espaço de Hilbert $\mathbb{H}$.

O núcleo $g(t)$ da memória satisfaz hipóteses similares as consideradas por [4], são elas:

$$\begin{cases} g \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+), \\ g(0) > 0, g'(s) \le -c_1 g(s), \forall s \in \mathbb{R}^+, \\ 0 < \kappa =: \int_0^{+\infty} g(s)ds < \beta_1 \alpha_1^{1-\theta}, \end{cases} \tag{4}$$

onde α_1 é o primeiro autovalor do operador A. Observe que o operador A generaliza dois operadores diferenciais clássicos, o operador Laplaciano $-\Delta$ e o bi-Laplaciano Δ^2 definidos em subespaços adequados de L^2. Quando o operador A é o operador $-\Delta$, o sistema (1)-(2) descreve um sistema de ondas acopladas e a velocidade de propagação destas equações são dadas por $\sqrt{\frac{\beta_1}{\rho_1}}$ e $\sqrt{\frac{\beta_2}{\rho_2}}$ respectivamente. Deste modo, se denotarmos por $\chi_0 = \frac{\beta_1}{\rho_1} - \frac{\beta_2}{\rho_2}$, implica que as velocidades são iguais quando $\chi_0 = 0$ e diferente caso contrário. Tal informação é importante pois a taxa de decaimento esta diretamente atrelada ao fato das velocidades de propagação serem iguais ou diferentes, fato constatado por exemplo em [3].

Para utilizar a teoria de semigrupos (ver [2]), devemos escrever o sistema (1)-(2) na forma abstrata. Com uma mudança de variável adequada é possível escrever o sistema (1)-(3) como

$$\frac{d}{dt}U(t) = \mathbb{B}U(t), \quad U(0) = U_0, \tag{5}$$

onde $U(t) = (u(t), v(t), u_t(t), v_t(t), \eta)$, $U_0 = (u_0, v_0, u_1, v_1, \eta)$ e o operador $\mathbb{B}$ é dado por

$$\mathbb{B}U = \left(\dot{u},\ \dot{v},\ -\rho_1^{-1}\left\{\beta_1 A_0 u + \mathbb{D}\eta + \alpha(u-v)\right\},\ -\rho_2^{-1}\left\{\beta_2 A v + \alpha(v-u)\right\}, \dot{u} - \partial_s \eta\right),$$

com $\mathbb{D}\eta = \displaystyle\int_0^{+\infty} g(s)A^\theta \eta(s)ds.$

Considerando o espaço de Hilbert $\mathbb{X} = \mathcal{D}(A^{\frac{1}{2}}) \times \mathcal{D}(A^{\frac{1}{2}}) \times \mathbb{H} \times \mathbb{H} \times \mathcal{M}_\theta$, com $\mathcal{M}_\theta = L^2_g(\mathbb{R}^+; \mathcal{D}(A^{\frac{\theta}{2}}))$, definimos o domínio de $\mathbb{B}$ por $D(\mathbb{B}) = \left\{U \in \mathbb{X} : \dot{u}, \dot{v} \in \mathcal{D}(A^{\frac{1}{2}}), A_0 u + \mathbb{D}\eta \in \mathbb{H}, v \in \mathcal{D}(A), \eta \in \mathcal{D}(\partial_s)\right\}$, onde $\mathcal{D}(\partial_s) = \{\eta \in \mathcal{M}_\theta;\ \partial_s \eta \in \mathcal{M}_\theta \text{ e } \eta(0) = 0\}$.

2 Resultados Principais

Usando teoria de semigrupos de operadores lineares prova-se o seguinte resultado de existência e unicidade de solução para o sistema (1)-(2).

Teorema 1: Tomando $U_0 \in D(\mathbb{B})$ existe uma única solução $U(t)$ para o modelo (1)-(2) satisfazendo

$$U \in C([0, +\infty);\ D(\mathbb{B})) \cap C^1([0, +\infty);\ X).$$

Além do teorema 1, usando o teorema de Borichev-Tomilov (ver [1]), demonstra-se que a solução U dada no referido teorema satisfaz a seguinte condição assintótica.

Teorema 2: Consideremos a solução $U(t)$ dada no Teorema 1 , α e $\chi_0 = 0$ conforme definidos anteriormente. O semigrupo $e^{t\mathbb{B}}$ associado ao sistema (1)-(2) possui decaimento polinomial dado por

$$\|e^{t\mathbb{B}}U_0\| \leq Ct^{-\frac{1}{2\theta}} \|U_0\|_{\mathbb{B}} \quad \forall t > 0.$$

Além disso a taxa do decaimento é ótima.

Obs: Esse trabalho ainda esta em desenvolvimento e nosso próximo desafio é encontrar a taxa de decaimento para o caso $\chi_0 \neq 0$ e mostrar que essa taxa é ótima.

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A PROBLEM WITH THE BIHARMONIC OPERATOR

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Abstract

This work tries an eigenvalue problem for the Shrödinger equation that incorporates the bi-harmonic operator. This problem is associated with a single particle of mass $m = \frac{2}{\hbar^2}$ moving under the influence of an electric force field described by the potential ϕ. The problem concerns to find the existence of real numbers ω and real functions u, ϕ satisfying the system

$$\begin{array}{rcll} -\Delta u + \phi u & = & \omega u & \text{in } \Omega \\ \Delta^2 \phi - \Delta \phi & = & u^2 & \text{in } \Omega \end{array} \tag{1}$$

with the boundary and normalizing conditions

$$u = \Delta\phi = \phi = 0 \quad \text{on} \quad \partial\Omega \quad \text{and} \quad \int_\Omega u^2 = 1. \tag{2}$$

1 Introduction

By the classic inspection the function ϕ requires necessarily belong to $H := H^2(\Omega) \cap H_0^1(\Omega)$. H is a Hilbert space with the equivalent norm induced by the inner product

$$(u, v)_H = \int_\Omega (\Delta u \Delta v + \nabla u \nabla v)\, dx.$$

Also, it is not difficult to see that the Euler-Lagrange equations of the functional

$$F(u,\phi) = \frac{1}{2}\int_\Omega |\nabla u|^2\, dx + \frac{1}{2}\int_\Omega \phi u^2 dx - \frac{1}{4}\int_\Omega |\Delta \phi|^2\, dx - \frac{1}{4}\int_\Omega |\nabla\phi|^2\, dx, \tag{3}$$

on the manifold

$$M = \left\{(u,\phi) \in H_0^1(\Omega) \times H; \|u\|_{L^2(\Omega)} = 1\right\},$$

give the solutions of (1). Moreover F is a strongly indefinite functional, this means F is neither bounded from above nor from below. Then, the usual methods of the critical points theory can not be directly used. To deal with this difficulty we shall reduce the functional (3) to suitable functional J of the single variable u, as that was done by Benci and Fortunato in [1], to which we will apply the genus theory, [2].

2 Main Result

Theorem 2.1. *Let Ω be a bounded set in $\mathbb{R}^3$. Then there is a sequence (ω_n, u_n, ϕ_n), with $\{\omega_n\}_{n\in\mathbb{N}} \subset \mathbb{R}$, $\omega_n \to \infty$ and u_n, ω_n are real functions, solving from (1) to (3).*

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NONLINEAR PERTURBATIONS OF A MAGNETIC NONLINEAR CHOQUARD EQUATION WITH HARDY-LITTLEWOOD-SOBOLEV CRITICAL EXPONENT

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Abstract

In this paper, we consider the following magnetic nonlinear Choquard equation

$$-(\nabla + iA(x))^2u + V(x)u = \lambda\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u + \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha-2}u,$$

where $2^*_\alpha = \frac{2N-\alpha}{N-2}$ is the critical exponent in the sense of the Hardy-Littlewood-Sobolev inequality, $\lambda > 0$, $N \geq 3$, $\frac{2N-\alpha}{N} < p < 2^*_\alpha$ for $0 < \alpha < N$, $A : \mathbb{R}^N \to \mathbb{R}^N$ is an C^1, $\mathbb{Z}^N$-periodic vector potential and V is a continuous scalar potential given as a perturbation of a periodic potential. Using variational methods, we prove the existence of a ground state solution for this problem if p belongs to some intervals depending on N and λ.

1 Introduction

In this article we consider, the problem

$$-(\nabla + iA(x))^2u + V(x)u = \lambda\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u + \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha-2}u, \tag{1}$$

where $\nabla + iA(x)$ is the covariant derivative with respect to the C^1, $\mathbb{Z}^N$-periodic vector potential $A\colon \mathbb{R}^N \to \mathbb{R}^N$, i.e,

$$A(x+y) = A(x),\ \forall\, x \in \mathbb{R}^N,\ \forall\, y \in \mathbb{Z}^N. \tag{2}$$

The exponent $2^*_\alpha = \frac{2N-\alpha}{N-2}$ is critical, in the sense of the Hardy-Littlewood-Sobolev inequality, $\lambda > 0$, $N \geq 3$, $\frac{2N-\alpha}{N} < p < 2^*_\alpha$, $0 < \alpha < N$ and $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous scalar potential. Inspired by the papers [2, 5], we assume that there is a continuous potential $V_{\mathcal{P}} : \mathbb{R}^N \to \mathbb{R}$, also $\mathbb{Z}^N$-periodic, constants $V_0, W_0 > 0$ and $W \in L^{\frac{N}{2}}(\mathbb{R}^N)$ with $W(x) \geq 0$ such that

(V_1) $V_{\mathcal{P}}(x) \geq V_0, \quad \forall\, x \in \mathbb{R}^N$;

(V_2) $V(x) = V_{\mathcal{P}}(x) - W(x) \geq W_0, \quad \forall\, x \in \mathbb{R}^N$,

where the last inequality is strict on a subset of positive measure in $\mathbb{R}^N$.

Under these assumptions, we will show the existence of a ground state solution to problem (1).

Initially, we consider the periodic version of (1), that is, we consider the problem

$$-(\nabla + iA(x))^2u + V_{\mathcal{P}}(x)u = \lambda\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u + \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha})\right)|u|^{2^*_\alpha-2}u, \tag{3}$$

where we maintain the notation introduce before and suppose that (V_1) is valid.

As in Gao and Yang in [3], the key step to proof the existence of a ground state solution of problem (3) is the use of cut-off techniques on the extreme function that attains the best constant $S_{H,L}$ naturally attached to the problem. This allows us to estimate the mountain pass value c_λ associated to the energy functional $J_{A,V_{\mathcal{P}}}$ related with (3) in terms of the Sobolev constant $S_{H,L}$. In a demanding proof, this lead us to consider different cases for p, if it belongs to some intervals depending on N and λ, as in the seminal work of BrÃ©zis and Nirenberg [4]. After that, the proof is completed by showing the mountain pass geometry, introducing the Nehari manifold associated with (3) and applying concentration-compactness arguments.

In the sequel, we consider the general case and so we prove that (1) has one nontrivial solution.

2 Main Results

Theorem 2.1. *Under the hypotheses already stated on A and α, suppose that (V_1) is valid. Then problem (3) has at least one ground state solution if either*

(i) $\frac{N+2-\alpha}{N-2} < p < 2^*_\alpha$, $N = 3, 4$ *and* $\lambda > 0$*;*

(ii) $\frac{2N-\alpha}{N} < p \leq \frac{N+2-\alpha}{N-2}$, $N = 3, 4$ *and* λ *sufficiently large;*

(iii) $\frac{2N-\alpha-2}{N-2} < p < 2^*_\alpha$, $N \geq 5$ *and* $\lambda > 0$*;*

(iv) $\frac{2N-\alpha}{N} < p \leq \frac{2N-\alpha-2}{N-2}$, $N \geq 5$ *and* λ *sufficiently large.*

Theorem 2.2. *Under the hypotheses already stated on A, V and α, problem (1) has at least one ground state solution if either*

(i) $\frac{N+2-\alpha}{N-2} < p < 2^*_\alpha$, $N = 3, 4$ *and* $\lambda > 0$*;*

(ii) $\frac{2N-\alpha}{N} < p \leq \frac{N+2-\alpha}{N-2}$, $N = 3, 4$ *and* λ *sufficiently large;*

(iii) $\frac{2N-\alpha-2}{N-2} < p < 2^*_\alpha$, $N \geq 5$ *and* $\lambda > 0$*;*

(iv) $\frac{2N-\alpha}{N} < p \leq \frac{2N-\alpha-2}{N-2}$, $N \geq 5$ *and* λ *sufficiently large.*

Problems (3) and (1) are then related by showing that the minimax value d_λ of the latter satisfies $d_\lambda < c_\lambda$. Once more, concentration-compactness arguments are applied to show the existence of a ground state solution.

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PRINCIPIO DA COMPARAÇÃO PARA OPERADORES ELIPTICOS

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Abstract

Neste trabalho provamos o principio da comparação para certos operadores do tipo elipticos, dentre eles temos o p-laplaciano e o operador curvatura média.

1 Introdução

Seja M uma variedade rimanniana completa e $\Omega \subset M$ um dominio limitado de classe $C^{2,\alpha}$. Consideremos o problema de dirichlet

$$(P.D) = \begin{cases} Q(u) = -F(x,u) \quad em \quad \Omega \\ u = g \quad em \quad \partial\Omega \end{cases}$$

onde $g \in C^{2,\alpha}(\overline{\Omega})$, $Q(u) = div\left(\frac{a(|\nabla u|)}{|\nabla u|}\nabla u\right)$, $a : [0,+\infty) \to \mathbb{R}$ é tal que $a \in C([0,+\infty)) \cap C^1((0,+\infty))$, $a > 0$ e $a' > 0$ em $(0,+\infty)$ e $a(0) = 0$. Para garantir a elipticidade é exigido conforme [1] que

$$\min_{0 \le s \le s_0} \left\{ A(s), 1 + \frac{sA'(s)}{A(s)} \right\} > 0$$

para todo $s_0 > 0$, onde escrevemos $a(s) = sA(s)$.
Além disso supomos que $F : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ é não-crescente em $t \in \mathbb{R}$.
Dizemos que $u \in C^{0,1}(\Omega)$ é solução fraca do (P.D) se

$$\int_\Omega < \frac{a(|\nabla u|)}{|\nabla u|}\nabla u, \nabla\varphi > dx = \int_\Omega f(x,u)\varphi\, dx \quad \forall \varphi \in C^{0,1}(\Omega) \quad tal \quad que \quad \varphi \ge 0 \quad em \quad \Omega \quad e \quad \varphi = 0 \quad em \quad \partial\Omega.$$

Dizemos que $u \in C^{0,1}(\Omega)$ é sub-solução fraca do (P.D) se na igualdade acima tivermos menor ou igual. Analogamente defini-se super-solução fraca do (P.D).
Ao investigarmos existência de solução para um problema do tipo acima é fundamental termos em mãos o principio da comparação, sendo ele o passo inicial na busca de tal existência.
Notemos que quando $a(s) = s^{p-1}$, $p > 1$, temos que $Q(u) = div\left(|\nabla u|^{p-2}\nabla u\right)$ que é o operador do p-laplaciano. No caso particular em que $p = 2$ temos que $Q(u) = \Delta u$. Se $a(s) = \frac{s}{\sqrt{1+s^2}}$ então $Q(u) = div\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)$ que é o operador curvatura média.
O problema de dirichlet acima é uma generalização do caso em que F=0. Os autores em [1] estudam esse caso particular. Muitos resultados se estendem para o caso acima. Dentre eles, temos o principio da comparação.

2 Resultados Principais

Teorema 2.1. *Sejam u e v sub e supersoluções respectivamente, do problema de dirichlet*

$$(P.D) = \begin{cases} Q(u) = -F(x,u) \quad em \quad \Omega \\ u = g \quad em \quad \partial\Omega \end{cases}$$

onde $g \in C^{2,\alpha}(\overline{\Omega})$, $Q(u) = div\left(\frac{a(|\nabla u|)}{|\nabla u|}\nabla u\right)$, $a : [0,+\infty) \to \mathbb{R}$ *é tal que* $a \in C([0,+\infty)) \cap C^1((0,+\infty))$, $a > 0$ *e* $a' > 0$ *em* $(0,+\infty)$ *e* $a(0) = 0$. *Além disso supomos que* $F : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ *é não-crescente em* $t \in \mathbb{R}$.
Supõe que $u \leq v$ *em* $\partial\Omega$ *então* $u \leq v$ *em* Ω.

Prova: Defina $\varphi=$ máx $\{u - v-\varepsilon, 0\}$. Por hipótese, $u \leq v$ em $\partial\Omega$, daí $u - v-\varepsilon \leq 0$ em $\partial\Omega$. Logo, máx $\{u - v-\varepsilon, 0\} = 0$ em $\partial\Omega$. Além disto vale que $\varphi \in C^{0,1}(\Omega)$. Deste modo podemos tomar φ como função teste. Defina

$$\Lambda_\varepsilon = \{x \in \Omega | u(x) - v(x) > \varepsilon\}.$$

Com esta notação temos que

$$\nabla\varphi = \begin{cases} \nabla u - \nabla v & em \quad \Lambda_\varepsilon \\ 0 \quad caso & contrario \end{cases}$$

Como u e v são respectivamente sub e supersoluções fraca, temos por definição que

$$\int_\Omega < \frac{a(|\nabla u|)}{|\nabla u|}\nabla u, \nabla\varphi > \, dx \leq \int_\Omega f(x,u)\varphi \, dx$$

$$\int_\Omega < \frac{a(|\nabla v|)}{|\nabla v|}\nabla v, \nabla\varphi > \, dx \geq \int_\Omega f(x,v)\varphi \, dx$$

Daí,

$$\int_\Omega < \frac{a(|\nabla u|)}{|\nabla u|}\nabla u - \frac{a(|\nabla v|)}{|\nabla v|}\nabla v, \nabla\varphi > \, dx \leq \int_\Omega (f(x,u) - f(x,v))\varphi \, dx$$

Como fora de Λ_ε temos $\varphi = 0$, a desigualdade acima se reduz a

$$\int_{\Lambda_\varepsilon} < \frac{a(|\nabla u|)}{|\nabla u|}\nabla u - \frac{a(|\nabla v|)}{|\nabla v|}\nabla v, \nabla\varphi > \, dx \leq \int_{\Lambda_\varepsilon} (f(x,u) - f(x,v))\varphi \, dx.$$

Mas em Λ_ε temos $u \geq v$ e sendo $f(x,t)$ não-crescente em t, temos que $f(x,u) \leq f(x,v)$ em Λ_ε. Diante disto e da desigualdade acima têm-se que

$$\int_{\Lambda_\varepsilon} < \frac{a(|\nabla u|)}{|\nabla u|}\nabla u - \frac{a(|\nabla v|)}{|\nabla v|}\nabla v, \nabla\varphi > \, dx \leq 0.$$

Após algumas manipulações algébricas conclui-se que $u - v - \varepsilon \leq 0$ em Ω, $\forall\varepsilon > 0$. Segue daí que $u \leq v$ em Ω. □

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CRITICAL ZAKHAROV-KUZNETSOV EQUATION ON RECTANGLES

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Abstract

Initial-boundary value problem for the modified Zakharov-Kuznetsov equation posed on a bounded rectangle is considered. Critical power in nonlinearity is studied. The results on existence, uniqueness and asymptotic behavior of solution are presented.

1 Introduction

We are concerned with initial-boundary value problems (IBVPs) posed on bounded rectangles for the modified Zakharov-Kuznetsov (mZK) equation [5]

$$u_t + u_x + u^2 u_x + u_{xxx} + u_{xyy} = 0. \tag{1}$$

This equation is a generalization [4] of the classical Zakharov-Kuznetsov (ZK) equation [7] which is a two-dimensional analog of the well-known modified Korteweg-de Vries (mKdV) equation [1]. The main difficult here is a critical growth in nonlinear term [2, 3]. Note that both ZK and mZK possess real plasma physics applications [6, 7].

Let L, B, T be finite positive numbers. Define Ω and Q_T to be spatial and time-spatial domains $\Omega = \{(x, y) \in \mathbb{R}^2 : \ x \in (0, L), \ y \in (-B, B)\}$, $Q_T = \Omega \times (0, T)$. In Q_T we consider the following IBVP:

$$u_t + u_x + u^2 u_x + u_{xxx} + u_{xyy} = 0, \quad \text{in } Q_T; \tag{2}$$

$$u(x, -B, t) = u(x, B, t) = 0, \quad x \in (0, L), \ t > 0; \tag{3}$$

$$u(0, y, t) = u(L, y, t) = u_x(L, y, t) = 0, \quad y \in (-B, B), \ t > 0; \tag{4}$$

$$u(x, y, 0) = u_0(x, y), \quad (x, y) \in \Omega, \tag{5}$$

where $u_0 : \Omega \to \mathbb{R}$ is a given function.

2 Main Results

Theorem 2.1. *Let $B, L > 0$ and $u_0(x, y)$ be such that*

$$\frac{2\pi^2}{L^2} - 1 > 0, \quad A^2 := \frac{\pi^2}{2}\left[\frac{3}{L^2} + \frac{1}{4B^2}\right] - 1 > 0 \quad \text{and} \quad \|u_0\|^2 < \frac{A^2}{2\pi^2\left(\frac{1}{L^2} + \frac{1}{4B^2}\right)}.$$

Suppose $u_0 \in L^2(\Omega)$ with $u_{0x} + \Delta u_{0x} \in L^2(\Omega)$ satisfies (3),(4) and $I_0^2 = \|u_{0x} + \Delta u_{0x} + u_0^2 u_{0x}\|^2 < \infty$. If

$$\left[\frac{2(1+L)^2}{1 - 2\|u_0\|^2}\|u_0\|^2\left(I_0^2 + \|u_0\|^2\right)\right]\left[4^2 + \frac{6^3(4!)^2(1+L)^8}{(1 - 2\|u_0\|^2)^2}\left(I_0^2 + \|u_0\|^2\right)^2\right] < \frac{2\pi^2}{L^2} - 1, \tag{1}$$

then for all $T > 0$ there exists a unique solution u to problem (2)-(5) from the following classes:

$$u \in L^\infty\left(0, T; H_0^1(\Omega)\right), \ u_t, \nabla u_y \in L^\infty\left(0, T; L^2\Omega)\right), \ u_{xx}, \nabla u_t \in L^2\left(0, T; L^2(\Omega)\right),$$

Moreover, there exist constants $C > 0$ and $\gamma > 0$ such that

$$\|u\|^2_{H^1(\Omega)}(t) + \|\nabla u_y\|^2(t) + \|u_t\|^2(t) \leq Ce^{-\gamma t}, \quad \forall t \geq 0 \tag{2}$$

and, in addition,

$$u_x(0,y,t),\ u_{xy}(0,y,t),\ u_{xx}(L,y,t) \in L^\infty\left(0,T; L^2(-B,B)\right),$$

$$u_{xx}(0,y,t) \in L^2\left(0,T; L^2(-B,B)\right).$$

Proof We apply the fixed point arguments to prove the local existence and uniqueness of solutions. Then global a priori estimates have been obtained to show the results.

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XIII ENAMA - Novembro 2019 *183–184*

SOLUÇÕES DAS EQUAÇÕES DE NAVIER-STOKES-CORIOLIS PARA TEMPOS GRANDES E DADOS QUASE PERIÓDICOS

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Abstract

Neste trabalho, estuda-se a existência de soluções brandas, para tempos grandes, para as equações de Navier-Stokes-Coriolis com dados iniciais espacialmente quase periódicos. A força de Coriolis aparece em vários modelos de meteorologia, oceanografia e geofísica, pois estes consideram fenômenos em larga escala e a influência do movimento de rotação da terra. Para mostrar a existência de soluções brandas para tempos grandes, usa-se a norma ℓ^1 de amplitudes e considera-se o caso de velocidades de rotação grandes (i.e., força de Coriolis grande). A existência de soluções é provada por meio de técnicas de *limites oscilantes singulares rápidos* e usando um argumento de *bootstrapping*. Mais precisamente, primeiro obtem-se estimativas para o semigrupo associado à parte linear do sistema e para o termo bilinear em um espaço de funções quase periódicas. Posteriormente, de posse destas estimativas, aplica-se os métodos acima para obter os resultados desejados. Esta dissertação é baseada no artigo [3] de T. Yoneda.

1 Introdução

Neste trabalho, considera-se o problema de valor inicial para as equações de Navier-Stokes com a força de Coriolis, o qual tem a seguinte forma

$$\begin{cases} \partial_t u + (u \cdot \nabla)u + \Omega e_3 \times u - \Delta u = -\nabla p & \text{em} \quad \mathbb{R}^3 \times (0,\infty) \\ \nabla \cdot u = 0 & \text{em} \quad \mathbb{R}^3 \times (0,\infty) \\ u|_{t=0} = u_0 & \text{em} \quad \mathbb{R}^3 \end{cases}, \tag{1}$$

onde $u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ e $p = p(x,t)$ denotam as incógnitas o campo velocidade e a pressão escalar do fluido no ponto $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ do espaço e tempo $t > 0$, respectivamente, enquanto $u_0 = u_0(x)$ denota o campo velocidade inicial, satisfazendo a condição de compatibilidade $\nabla \cdot u = 0$. Além disso, $\Omega \in \mathbb{R}$ é o parâmetro de Coriolis que representa a velocidade angular de rotação do fluido em torno do vetor vertical unitário $e_3 = (0,0,1)$; o coeficiente de viscosidade cinemática é normalizado por 1. Estuda-se principalmente o artigo [3] que mostra a existência de soluções de (1) para tempos grandes em um espaço de funções quase periódicas, fazendo uso do resultado de existência local, em $\mathbb{R}^3$, obtido por Giga *et al.* [1] e do resultado de existência, em $\mathbb{R}^2$, devido a Giga *et al.* [2]. Para um dado inicial u_0 quase periódico e tempo de existência T, ambos arbitrários, obtem-se soluções brandas para o parâmetro de Coriolis Ω tendo módulo suficientemente grande.

O método utilizado aqui segue ideias de Babin, Mahalov e Nicolaenko [1] e [2], no entanto, precisa-se introduzir uma classe de espaços funcionais que seja adequada para o contexto quase periódico. Uma aplicação direta da desigualdade de energia é difícil (se não impossível) no caso de dados e soluções quase periódicos. A fim de superar essa dificuldade, utiliza-se a norma ℓ^1 em um conjunto de frequências com soma fechada.

2 Resultados Principais

Inicialmente, precisa-se de dois resultados que descrevemos a seguir. O primeiro é um resultado de existência e unicidade local de soluções brandas em $\mathbb{R}^3$, obtido por Giga *et al.* [1].

Teorema 2.1. *Assuma que $u_0 \in \mathcal{FM}_0$ com $\operatorname{div} u_0 = 0$. Então, existe $T_0 \geq c/\|u_0\|^2_{\mathcal{FM}} > 0$ independente do parâmetro de Coriolis Ω, e uma única solução branda $u = u(t) \in C\left([0, T_0]; \mathcal{FM}_0\right)$ de (1), onde $c > 0$ é uma dada constante.*

O segundo trata-se de um resultado de existência e unicidade de soluções globais brandas para a equação bi-dimensional de Navier-Stokes, proposto por Giga *et al.* em [2].

Teorema 2.2. *Seja $u_0 = \sum_{m \in \mathbb{Z}^2} a_m e^{im \cdot x} \in \mathcal{FM}_0(\mathbb{R}^2)$ com $a_0 = 0$ e $x \in \mathbb{R}^2$. Então, a única solução branda global $u(x,t)$ para a equação (1), com dado inicial $u_0(x)$, pode ser expressa como $u(x,t) = \sum_{m \in \mathbb{Z}^2 \setminus \{0\}} a_m(t) e^{im \cdot x}$. Além disso, a solução u está em $L^\infty_{loc}([0, \infty) : \mathcal{FM}_0(\mathbb{R}^2))$ se u_0 é real. Mais precisamente $\sup_{0 \leq t \leq T} \|u\|_{\mathcal{FM}}(t) \leq C$, onde $C > 0$ depende apenas de $T > 0$ e $\|u_0\|_{\mathcal{FM}}$.*

De posse dos Teoremas 2.1 e 2.2, pode-se provar o resultado principal desta dissertação, o qual enunciamos a seguir.

Teorema 2.3. *Sejam $a(0) = \{a_n(0)\}_{n \in \Lambda} \in \ell^1(\Lambda)$, com $(a_n(0) \cdot n) = 0$ para $n \in \Lambda$, e $a_0(0) = 0$. Então, para qualquer $T > 0$ existe $\Omega_0 > 0$ dependendo apenas de $a(0)$ e T tal que se $|\Omega| > \Omega_0$, existe uma solução branda $a(t)$ de (1) (isto é, de (2)), tal que $a(t) = \{a_n(t)\}_{n \in \Lambda} \in C([0, T] : \ell^1(\Lambda))$, com $a_0(t) = 0$ e $(a_n(t) \cdot n) = 0$, $\forall\ n \in \Lambda$.*

Aqui, $a_n(t)$ são os coeficiente do seguinte sistema equivalente a (1):

$$\partial_t a_n(t) + |n|^2 a_n(t) + \Omega e_3 \times a_n(t) + i\mathbb{P}_n \sum_{n=k+m} (a_k(t) \cdot m) a_m(t) = 0, \qquad (n \cdot a_n(t)) = 0, \qquad a_n(0) = a_{0,n} \ , \tag{2}$$

Ideia da demonstração: *Teorema 2.3* - Primeiro, aplica-se a projeção de Helmholtz no sistema (2) para eliminarmos a parte da pressão, obtendo assim um novo sistema. Em seguida, a ideia é "filtrar" soluções desse novo sistema usando o semigrupo auxiliar $e_n^{-t\Omega\mathbb{S}}$ pelo método dos limites oscilantes singulares rápidos. Para tal, utiliza-se a transformação de Van Der Pol, $a_n(t) := e_n^{-t\Omega\mathbb{S}} c_n(t)$, no novo sistema, obtendo um outro sistema equivalente. Depois, dividi-se o sistema obtido em outros dois. Em seguida, prova-se a existência e unicidade de soluções de um dos sistemas e garante-se que o outro admita solução local. Finalmente, usando um argumento de bootstrapping, consegue-se estender o tempo de exitência de solução, concluindo a demonstração. Para mais detalhes, veja [3].

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WELL-POSEDNESS FOR A NON-ISOTHERMAL FLOW OF TWO VISCOUS INCOMPRESSIBLE FLUIDS WITH TERMO-INDUCED INTERFACIAL THICKNESS

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Abstract

This work develop the study of a thermo-induced diffuse-interface model in dimension two. This model describe the motion of a mixture of two viscous incompressible fluids with viscosity, thermal conductivity and the interfacial thickness being temperature dependent. In previous works, the authors studied this model in two cases: where the viscosity is temperature dependent and the case where the viscosity and the thermal conductivity are temperature dependents. In the last case, a dissipative energy inequality is obtained only for a smallness condition on the initial temperature. The model consists of a modified Navier-Stokes equation coupled with a phase-field equation given by the a convective Allen-Cahn equation and with a temperature equation. It is investigated the existence of local weak solution for the problem.

1 Introduction

In this work we study a general non-isothermal diffuse-interface model when the viscosity, thermal conductivity and the interfacial thickness are temperature dependent. We assume that the fluids have matched densities and the same viscosity and thermal conductivity. More precisely, we want to study the existence of weak solution of the following problem:

$$\begin{aligned} u_t + u\cdot\nabla u - \nabla\cdot(\nu(\theta)Du) + \nabla p &= \lambda\left(-\nabla\cdot(\epsilon(\theta)\nabla\phi) + \frac{1}{\varepsilon(\theta)}F'(\phi)\right)\nabla\phi - \alpha\Delta\theta\nabla\theta, && (1)\\ \nabla\cdot u &= 0, && (2)\\ \phi_t + u\cdot\nabla\phi &= \gamma\left(\nabla\cdot(\epsilon(\theta)\nabla\phi) - \frac{1}{\varepsilon(\theta)}F'(\phi)\right), && (3)\\ \theta_t + u\cdot\nabla\theta &= \nabla\cdot(k(\theta)\nabla\theta), && (4) \end{aligned}$$

with the following initial and boundary conditions:

$$\begin{aligned} &u(x,0) = u_0(x), \quad \phi(x,0) = \phi_0(x), \quad \theta(x,0) = \theta_0(x), \quad x\in\Omega,\\ &u = 0, \quad \frac{\partial\phi}{\partial\eta} = 0, \quad \frac{\partial\theta}{\partial\eta} = 0, \quad (x,t)\in\partial\Omega\times(0,\infty), \end{aligned} \qquad (5)$$

Here, u, p and θ denote the mean velocity of the fluid mixture, the pressure and the temperature, the phase-field variable ϕ represents the volume fraction of the two components. $Du = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$ corresponds to the symmetric part of the velocity gradient. $\nu \geq \nu_0 > 0$ is the viscosity of the mixture, $\lambda > 0$ is the surface tension, $\epsilon \geq \epsilon_0 > 0$ is a small parameter related to the interfacial thickness, $\alpha > 0$ is associated to the interfacial thickness, $F(\phi)$ is the potential energy density, γ is the relaxation time of the interface and $k \geq k_0 > 0$ the thermal conductivity. Here ν, ϵ and k are temperature dependent.

We observe that we do not have an energy inequality for this model. So it is not possible to show the existence of global weak solution, only local in time. In previous works ([4], [5]), we studied the same problems in two cases: when the viscosity is temperature dependent and the case when the viscosity and the thermal conductivity are temperature dependents. In the first case, the existence of global weak solution for dimension 2 and 3, existence and uniqueness of global strong solution for dimension 2, and local strong solution for dimension 3 have been proved. We observe that we do not need to suppose any restriction on the size of the initial data. For the second case, we prove the existence of a global weak solution, the existence and uniqueness of global strong solution in dimension 2, when the initial temperature is suitably small, and the existence and uniqueness of local strong solution in dimensions 2 and 3 for any initial data.

As far as we know, there is no studies about the phase-field equation with interfacial thickness that develop with temperature. A closer study about a variational interfacial thickness is the sharp interface limit and the free boundary problems for phase-field models. In those cases, the thickness of the diffuse interface tends to zero. About the Allen-Cahn equation we can mention [6], for the Stokes-Allen-Cahn system we can mention [2], for the Cahn-Hilliard equation we can mention [3], and for the Navier-Stokes-Cahn-Hilliard system we can mention [1].

2 Mais Result

Now we state our main result about the existence of local weak solution for dimension two.

Theorem 2.1. *Given $u_0 \in H \cap L^4$,$\phi_0, \theta_0 \in H^1 \cap L^\infty$, with $\|\phi_0\|_{L^\infty} \leq 1$, then the problem (1)-(4) with initial and boundary conditions (5), has at least one local weak solution that satisfies*

$$u \in L^\infty(0,T^*;H) \cap L^2(0,T^*;V),$$

$$\phi, \theta \in L^\infty(0,T^*;H^1 \cap L^\infty) \cap L^2(0,T^*;H^2),\ |\phi| \leq 1,\ |\theta| \leq \|\theta_0\|_{L^\infty}\ a.e.\ \Omega \times (0,T^*),$$

for some $T^* \in (0,\infty)$.

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PRINCÍPIO LOCAL-GLOBAL E MEDIDAS DE INFORMAÇÃO

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Os autores agradecem à Faculdade de Tecnologia - FT - Unicamp e à CAPES - (DS), 37 - P - 2774/2019 1 1

Abstract

Expõe-se dois axiomas de completude, equivalentes entre si, que, no sistema de números reais, i.e., num corpo ordenado completo, tornam as demonstrações de teoremas clássicos da Análise Matemática, diga-se, mais simples. Além disso, os clássicos axiomas de completude na literatura, Axioma do Supremo, Completude de Dedekind, Propriedade de Arquimedes etc., podem ser deduzidos diretamente deles. Esses axiomas fundamentam-se no *princípio local-global* e são descritos como segue: **LG (Local-Global)** *Qualquer propriedade local e aditiva é global,* **GL (Global-Limite)** *Qualquer propriedade global e subtrativa tem um ponto limite.* O objetivo é mostrar a relevância desses axiomas ao comparar a demonstração do Teorema do Valor Médio, presente em livros clássicos de Análise Matemática, com aquela obtida com base em LG-GL. Estima-se também analisar as implicações desse princípio na caracterização de medidas de informação.

1 Introdução

As origens do princípio local-global, tais como apresentadas nesse artigo, devem-se ao prof. Olivier Rioul[1]. O artigo [1] expõe os principais resultados com base na relação entre Propriedades $\mathcal{P}$ e Intervalos Fechados Limitados $[u,v]$ de tal forma que $[u,v] \in \mathcal{P}$ se $[u,v]$ satisfaz $\mathcal{P}$. Uma propriedade $\mathcal{P}$ é *aditiva* se, para quaisquer $u < v < w$, $[u,v] \in \mathcal{P} \wedge [v,w] \in \mathcal{P} \Rightarrow [u,w] \in \mathcal{P}$, e é subtrativa se, para quaisquer $u < v < w$, $[u,w] \in \mathcal{P} \Rightarrow [u,v] \in \mathcal{P} \vee [v,w] \in \mathcal{P}$. Uma propriedade é *local* em x se existe uma vizinhança $V(x)$ tal que $\forall [u,v] \subseteq V(x), [u,v] \in \mathcal{P}$, e tem um ponto *limite* x se para toda vizinhança $V(x)$, existe $[u,v] \subseteq V(x)$ com $[u,v] \in \mathcal{P}$. Os axiomas de completude

LG (Local-Global) Qualquer propriedade *local* e *aditiva* em $[a,b]$ é *global*, isto é, satisfeita para $[a,b]$,

GL (Global-Limite) Qualquer propriedade que é *global* e *subtrativa* tem um *ponto limite* em $[a,b]$,

foram observados em artigos acadêmicos, como por exemplo em [2], e com a referência mais antiga sendo o livro francês [3], conforme explicado em [1]. Uma das ideias principais é a de que com os axiomas de completude LG-GL as demonstrações, ao menos dos teoremas clássicos de Análise, fiquem mais simples. Como exemplo, observa-se a demonstração da equivalência entre o axioma conhecido como *Completude de Dedekind* e os axiomas LG-GL.

Definição 1.1. *Um corte de Dedekind é um par (A,B) em que o conjunto A e seu conjunto complemento B em $[a,b]$ são tais que $A < B$, isto é, $u < v$ para qualquer $u \in A$ e $v \in B$, ([1], p.225).*

Teorema 1.1 (Completude de Dedekind). *Qualquer corte (A,B) define um único ponto x tal que $A \leq x \leq B$.*

Proof. Assume-se por hipótese que A e B são conjuntos não vazios. A propriedade $[u,v] \in \mathcal{P}$ com $u \in A$ e $v \in B$ é global e também subtrativa. Pelo axioma GL, $\mathcal{P}$ tem um ponto limite x: qualquer vizinhança $V(x)$ contém $u < x < v$ tal que $u \in A$ e $v \in B$. Com base nisso pode-se deduzir que nenhum ponto $x' \in B$ é menor do que x, caso contrário poderíamos encontrar $u \in A$ tal que $x' < u < x$, o qual contradiz a hipótese $A < B$. Similarmente nenhum ponto em A é maior do que x. Portanto $A \leq x \leq B$, ([1], p.225). □

[1]Télécom ParisTech - LTCI CNRS, https://perso.telecom-paristech.fr/rioul/.

Proposição 1.1. *O axioma LG é equivalente ao teorema de completude de Dedekind.*

Proof. É suficiente demonstrar o axioma GL a partir do teorema de completude de Dedekind. Seja $\mathcal{P}$ uma propriedade global e subtrativa em $[a,b]$ e seja B o conjunto de todos os pontos v para os quais $[a,v'] \in \mathcal{P}$ para todo $v' \geq v$. Claramente $a \notin B$ (uma vez que $[a,a]$ é um intervalo degenerado) e $b \in B$. Já que $v \in B$ implica que todos os $v' \geq v$ estão em B, tem-se então que $A \leq B$, (A,B) é um corte e existe x tal que $A \leq x \leq B$. Em cada vizinhança $V(x)$ pode-se encontrar $[u,v]$ contendo x tal que $[a,v] \in \mathcal{P}$ mas $[a,u] \notin \mathcal{P}$. Já que $\mathcal{P}$ é subtrativa tem-se que $[u,v] \in \mathcal{P}$ e daí $\mathcal{P}$ tem um ponto limite x. ([1], p.225). □

2 Resultados Principais

1. O Teorema do Valor Médio, conforme a demonstração clássica exposta em [4], página 62, leva em conta para sua demonstração 16 teoremas mais o axioma do supremo. Nesta abordagem, em que consideram-se os axiomas LG-GL, tem-se a seguinte justificativa do referido teorema.

   Teorema 2.1. *Para toda função real f contínua em $[a,b]$ e derivável em $]a,b[$, existe um $x \in]a,b[$ tal que* $f'(x) = \dfrac{f(b)-f(a)}{b-a}$.

   Justifcativa: Considera-se que $\lambda = \frac{f(b)-f(a)}{b-a}$. Se $f' \neq \lambda$ em $]a,b[$, tem-se, com base no teorema de Darboux ([1], p.239), que, ou $f' > \lambda$ ou $f' < \lambda$. Tem-se então que $\frac{f(b)-f(a)}{b-a} > \lambda$ ou $\frac{f(b)-f(a)}{b-a} < \lambda$, absurdo. O objetivo, tal como em [4], é indicar a estrutura dessa demonstração, via LG-GL, com vistas a justificar a relevância operacional desse conjunto de axiomas.

2. Após Claude E. Shannon ter lançado as bases para a Teoria da Informação, no artigo *A mathematical theory of communication,* em 1948, e exposto a fórmula $H(X) = -\sum_{i=1}^{n} p_i \log p_i$ para representar *uma medida de informação,* muitos desenvolvimentos matemáticos se sucederam com o intuito de caracterizar a medida de informação $H(X)$ via sistemas axiomáticos de equações funcionais [2, 5]. O objetivo é trabalhar numa caracterização axiomática da fórmula $H(X)$, conforme a proposta do artigo [2], fundamentada nas propriedades local-global de

   $$h(p) = -p \log p - (1-p)\log(1-p)$$

   no intervalo $(0,1)$, ao invés da exigência, [5], de $h(p)$ ser mensurável em $(0,1)$.

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SOBOLEV TYPE INEQUALITY FOR INTRINSIC RIEMANNIAN MANIFOLDS

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Abstract

In this work (in final progress), we present a Riemannian intrinsic version of a Sobolev type inequality for Riemannian varifolds, using a natural extension of the concept of varifold defined in a Riemannian manifold in an intrisic way. We follow the ideas of Simon and Michael in [1] and [5].

1 Introduction

The ordinary Sobolev inequality has been known for many years and its value in the theory of partial differential equations is well known. In [2] Miranda obtained a Sobolev inequality for minimal graphs. A refined version of this new inequality was used by Bombieri, De Giorgi and Miranda to derive gradient bounds for solutions to the minimal surface equation (see [3]). In [1], a general Sobolev type inequality was presented. That inequality is obtained on what might be termed a generalized manifold and as special cases, results in the ordinary Sobolev inequality, a Sobolev inequality on graphs of weak solutions to the mean curvature equation, and a Sobolev inequality on arbitrary C^2 submanifolds of $\mathbb{R}^n$ (of arbitrary co-dimension).
On the other hand, in [4] Allard proves a Sobolev type inequality in a varifold context from a Isoperimetrical inequality for varifolds, for functions with compact support on a varifold V whose first variation δV lies in an appropriate Lebesgue space with respect to $\|\delta V\|$.
We present an intrinsic Riemannian analogue to the Allard result, considering a k-dimensional varifold V defined in an n-dimensional Riemannian manifold (M^n, g) defined intrinsically. This is done by recovering a monotonicity inequality (instead of a monotonicity equality) in this context, which encloses the geometry of M, following the ideas of Simon and Michael in [1] and [5]. The Sobolev type inequality is then obtained by a standard covering argument.

2 Main Results

Definition: Let (M^n, g) a n-dimensional Riemannian manifold, we define an abstract varifold as a Radon measure on $G_k(M)$, where

$$G_k(M) := \bigcup_{x\in M} \{x\} \times Gr(k, T_xM),$$

Let $\mathbf{V}_k(M)$ the space of all k-dimensional varifolds, endowed with the weak topology induced by $C_c^0(G_k(M))$.
We say that the nonnegative Radon measure on M^n, $||V||$, is the weight of V if $||V|| = \pi_\#(V)$. Here, π indicates the natural fiber bundle projection, i.e., for every $A \subseteq G_m(M)$, $x \in M^n$, $S \in G_k(T_xM^n)$, we have $||V||(A) := V(\pi^{-1}(A))$.

Definition: Let (M^n, g) be a n-dimensional Riemannian manifold with Levi-Civita connection ∇, $\mathfrak{X}_c^1(M)$ the set of differentiable vector fields on M and $V \in \mathbf{V}_k(M)$ a k-dimensional varifold $(2 \le k \le n)$. We define the first variation of V along the vector field $X \in \mathfrak{X}_c^1(M)$ as

$$\delta V(X) := \int_{G_k(M)} \operatorname{div}_S X(x) dV(x, S).$$

Let (M^n, g)a Riemannian manifold such that $Sec_g \leq b$, for some $b \in \mathbb{R}$, and $V \in \mathbf{V}_k(M)$. We say that V satisfy **AC** if, for given $X \in \mathfrak{X}_c^1(M)$ such that $\operatorname{spt}\|V\| \subset B_g(\xi, \rho)$, for given $\xi \in M$ and $\rho < inj_{(M,g)}(\xi)$,

$$|\delta V(X)| \leq C \left(\int_{B_g(\xi,\rho)} |X|_g^{\frac{p}{p-1}} \, d\|V\| \right)^{\frac{p-1}{p}}.$$

For a Riemannian manifold (M^n, g), we say that M^n satisfy **GC** if, for $\xi \in M$:
(i) $Sec_g \leq b$ for some $b \in \mathbb{R}$.
(ii) There exists r_0 such that $0 < r_0 < inj_{(M,g)}(\xi)$ and $r_0 b < \pi$.

Theorem (Fundamental Weighted Monotonicity Inequality): If (M, g) is a complete Riemannian manifold satisfying **GC** and $V \in \mathbf{V}_k(M)$ is a varifold satisfying **AC**, then for all $0 < s < r_0$ we have in distributional sense:

$$\frac{d}{ds}\left(\frac{1}{s^k} \int_{B_g(\xi,s)} h(y) d\|V\|(y) \right) \geq \frac{d}{ds} \int_{B_g(\xi,s)} h \frac{\left|\nabla^{\perp} u\right|_g^2}{r_\xi^k} d\|V\| + \frac{1}{s^{k+1}} \left(\int_{B_g(\xi,s)} \langle \nabla h + hH, (u \nabla u) \rangle_g d\|V\| \right) + c^* \frac{k}{s^k} \int_{B_g(\xi,s)} h(y) d\|V\|(y)$$

where

$$c^* = c^*(r_0, b) := \frac{r_0 \sqrt{b} \cot\left(\sqrt{b} r_0\right) - 1}{r_0}, \text{ if } b > 0 \quad \text{and} \quad c^* := \frac{-1}{r_0} \text{ if } b \leq 0$$

Theorem (Sobolev Type Inequality): Let (M^n, g) be a complete manifold satisfying **GC** and $V \in \mathbf{V}_k(M)$ satisfying **AC**. Assume that for $\xi \in M \cap \operatorname{spt}\|V\|$ given, $\Theta^k(x, \|V\|) \geq 1$ for a.e. $x \in B_g(\xi, r_0)$. If $h \in C_c^1(B_g(\xi, r_0))$ is nonnegative, then there exists $C > 0$ such that

$$\left(\int_M h^{\frac{n}{n-1}} \right)^{\frac{n-1}{n}} \leq C \int_M \left(\left|\nabla^M h\right|_g + h \left(|H|_g - c^* k \right) \right) d\|V\|.$$

Proof: The proof follows from the **Fundamental Weighted Monotonicity Inequality** and a standard covering argument, see [6].

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XIII ENAMA - Novembro 2019 *191–192*

ASSOCIATIVIDADE E INCOMPLETUDE NO PRODUTO TENSORIAL PROJETIVO

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Abstract

O objetivo deste trabalho é provar duas propriedades fundamentais do produto tensorial projetivo de espaços de Banach, a saber: (i) o produto tensorial projetivo completado de um número finito de espaços de Banach é associativo, (ii) o produto tensorial projetivo (não completado) de um número finito de espaços de Banach, com pelo menos dois deles de dimensão infinita, não é completo.

1 Introdução

Dados espaços vetoriais $X_1, \ldots, X_n, Z$ sobre o corpo $\mathbb{K} = \{\mathbb{R} \text{ ou } \mathbb{C}\}$, denotamos por $L(X_1, \ldots, X_n; Z)$ o espaço dos operadores n-lineares de $X_1 \times \cdots \times X_n$ em Z. Para todos $x_1 \in X_1, \ldots, x_n \in X_n$, o operador

$$x_1 \otimes \cdots \otimes x_n \colon L(X_1, \ldots, X_n) \to \mathbb{K}\ ,\ (x_1 \otimes \cdots \otimes x_n)(A) = A(x_1, \ldots, x_n),$$

é um funcional linear, isto é, $x_1 \otimes \cdots \otimes x_n \in L(X_1, \ldots, X_n)^*$. Define-se o produto tensorial de $X_1, \ldots, X_n$ por

$$X_1 \otimes \cdots \otimes X_n = \operatorname{span}\{x_1 \otimes \cdots \otimes x_n\ ;\ x_1 \in X_1, \ldots, x_n \in X_n\}.$$

Os elementos de $X_1 \otimes \cdots \otimes X_n$ são chamados de tensores.

Quando $X_1, \ldots, X_n$ forem espaços normados, podemos considerar a norma projetiva π no produto tensorial $X_1 \otimes \cdots \otimes X_n$ definida por:

$$\pi(u) = \inf\left\{\sum_{j=1}^{m} \|x_1^j\| \cdots \|x_n^j\| : m \in \mathbb{N} \text{ e } u = \sum_{j=1}^{m} x_1^j \otimes \cdots \otimes x_n^j\right\}.$$

O espaço normado resultante é denotado por $X_1 \otimes_\pi \cdots \otimes_\pi X_n$, e seu completamento é denotado por $X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n$ e chamado de produto tensorial projetivo de $X_1, \ldots, X_n$.

Neste trabalho provaremos dois resultados. O primeiro diz que o produto tensorial projetivo $X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n$ é associativo no sentido de que é isomorfo isometricamente a

$$(X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_{n_1}) \widehat{\otimes}_\pi (X_{n_1+1} \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_{n_2}) \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi (X_{n_t+1} \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n)$$

para todos $1 \leq n_1 < n_2 < \cdots < n_t < n$. O segundo diz que o espaço normado $X_1 \otimes_\pi \cdots \otimes_\pi X_n$ é incompleto se pelo menos dois dos espaços envolvidos têm dimensão infinita.

Para a teoria bÃ¡sica de espaços de Banach nos referimos a [3], para produtos tensoriais nos referimos a [4] e [2], e para a teoria dos operadores multilineares continuos nos referimos a [4].

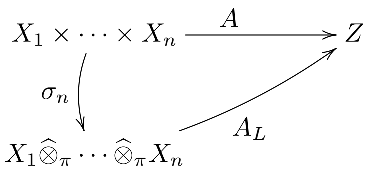
2 Resultados Principais

Usaremos em nossas demonstrações a propriedade universal do produto tensorial projetivo, descrita no teorema a seguir. É fácil ver que

$$\sigma_n \colon X_1 \times \cdots \times X_n \longrightarrow Z\ ,\ \sigma_n(x_1, \ldots, x_n) = x_1 \otimes \cdots \otimes x_n,$$

é um operador n-linear contÃnuo.

Teorema 2.1. *Sejam $X_1, \dots, X_n, Z$ espaços normados, Z espaço de Banach e $A\colon X_1 \times \cdots \times X_n \longrightarrow Z$ um operador n-linear contÃnuo. Então existe um único operador linear contÃnuo $A_L\colon X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n \longrightarrow Z$ tal que $A = A_L \circ \sigma_n$, isto é, o diagrama abaixo é comutativo:*



Mais ainda, a correspondência $A \longleftrightarrow A_L$ é um isomorfismo isométrico entre o espaço de Banach $\mathcal{L}(X_1, \dots, X_n; Z)$ dos operadores n-linares continuos e o espaço de Banach $\mathcal{L}(X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n; Z)$ dos operadores lineares contÃnuos.

Para a demonstração, veja [4] ou [2].

O operador n-linear A_L é chamado de *linearização* do operador n-linear A.

Enunciamos agora os dois resultados a serem demonstrados neste trabalho.

Teorema 2.2. *Sejam $X_1, \dots, X_n$ espaços normados e $n_1, \dots, n_t \in \mathbb{N}$ tais que $1 \leq n_1 < n_2 < \cdots < n_t < n$. Então existe um isomorfismo isométrico*

$$T\colon X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n \longrightarrow (X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_{n_1}) \widehat{\otimes}_\pi (X_{n_1+1} \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_{n_2}) \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi (X_{n_t+1} \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n)$$

tal que

$$T(x_1 \otimes \cdots \otimes x_n) = (x_1 \otimes \cdots \otimes x_{n_1}) \otimes (x_{n_1+1} \otimes \cdots \otimes x_{n_2}) \otimes \cdots \otimes (x_{n_t+1} \otimes \cdots \otimes x_n)$$

para todos $x_1 \in X_1, \dots, x_n \in X_n$.

Teorema 2.3. *Sejam $X_1, \dots, X_n$ espaços de Banach, com pelo menos dois deles de dimensão infinita. Então o espaço normado $X_1 \otimes_\pi \cdots \otimes_\pi X_n$ é incompleto.*

Destacamos que, apesar de serem largamente utilizados, as demonstrações desses dois resultados não são facilmente encontradas na literatura. Na verdade não encontramos nenhuma referência com as demonstrações completas. Por isso nos preocupamos em dar demonstrações detalhadas dos dois resultados.

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O ESPAÇO DOS OPERADORES MULTILINEARES HIPER-$\sigma(P)$-NUCLEARES

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Abstract

Introduzimos neste trabalho a classe dos operadores multilineares hiper-$\sigma(p)$-nucleares entre espaços de Banach e provamos alguns resultados que justificam o estudo desta classe. Em particular, mostramos que essa nova classe é um hiper-ideal p-Banach de operadores multilineares.

1 Introdução

Com o sucesso da teoria de ideais de operadores lineares sistematizada por Pietsch [4], uma consequência natural é sua extensão para o caso não-linear. O próprio Pietsch, em 1983, esquematizou a teoria de ideais de operadores multilineares (multi-ideais) [5]. A partir disso, vários métodos de criar multi-ideais foram desenvolvidos, dentre eles, o mais natural é o de adaptar ideais de operadores lineares já estudados para o caso multilinear. Este método foi empregado (veja [1, 3]) para o ideal dos operadores lineares $\sigma(p)$-nucleares e a classe resultante, chamada de operadores multilineares $\sigma(p)$-nucleares, é definida por:

Definição 1.1. Para $1 \leq p < \infty$, com p' o seu conjugado, e $E_1, \ldots, E_n$ e F espaços de Banach, diz-se que um operador n-linear $A\colon E_1 \times \cdots \times E_n \longrightarrow F$ é *$\sigma(p)$-nuclear* se existem sequências $(\lambda_j)_{j=1}^{\infty} \in \ell_{p'}$, $(\varphi_{ij})_{j=1}^{\infty} \in E_i'$ para $i = 1, \ldots, n$, e $(y_j)_{j=1}^{\infty} \in F$ tais que $A = \sum\limits_{j=1}^{\infty} \lambda_j \varphi_{1j} \otimes \cdots \otimes \varphi_{nj} \otimes y_j$,

$$\sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=1}^{\infty} |\varphi_{1j}(x_1) \cdots \varphi_{nj}(x_n) y'(y_j)|^p \right)^{\frac{1}{p}} < \infty \text{ e } \sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=m}^{\infty} |\varphi_{1j}(x_1) \cdots \varphi_{nj}(x_n) y'(y_j)|^p \right)^{\frac{1}{p}} \overset{m\to\infty}{\longrightarrow} 0.$$

Neste caso dizemos que $A = \sum\limits_{j=1}^{\infty} \lambda_j \varphi_{1j} \otimes \cdots \otimes \varphi_{nj} \otimes y_j$ é uma representação $\sigma(p)$-nuclear de A, definimos

$$||A||_{\sigma(p)} := \inf \left\{ ||(\lambda_j)_{j=1}^{\infty}||_{p'} \cdot \sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=1}^{\infty} |\varphi_{1j}(x_1) \cdots \varphi_{nj}(x_n) y'(y_j)|^p \right)^{\frac{1}{p}} \right\},$$

onde o ínfimo é tomado sobre todas as representações $\sigma(p)$-nucleares de A, e escrevemos $A \in \mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F)$.

Nosso objeto de estudo é a nova classe de operadores multilineares associada à classe dos operadores $\sigma(p)$-nucleares que surge naturalmente no âmbito da teoria de hiper-ideais de operadores multilineares desenvolvida em [6, 2], a qual definimos da forma que segue. Por $\mathcal{L}(E_1, \ldots, E_n; \mathbb{K})$ denotamos o espaço de Banach das formas n-lineares contínuas de $E_1 \times \cdots \times E_n$ no corpo dos escalares $\mathbb{K}$.

Definição 1.2. Para $1 \leq p < \infty$, com p' seu conjugado, e $E_1, \ldots, E_n$ e F espaços de Banach, dizemos que um operador n-linear $A\colon E_1 \times \cdots \times E_n \longrightarrow F$ é *hiper-$\sigma(p)$-nuclear* se existem sequências $(\lambda_j)_{j=1}^{\infty} \in \ell_{p'}$, $(B_j)_{j=1}^{\infty} \in \mathcal{L}(E_1, \ldots, E_n; \mathbb{K})$ e $(y_j)_{j=1}^{\infty} \in F$ tais que $A = \sum\limits_{j=1}^{\infty} \lambda_j B_j \otimes y_j$,

$$\sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=1}^{\infty} |B_j(x_1, \ldots, x_n) y'(y_j)|^p \right)^{\frac{1}{p}} < \infty \quad \text{e} \quad \lim_{m\to\infty} \sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=m}^{\infty} |B_j(x_1, \ldots, x_n) y'(y_j)|^p \right)^{\frac{1}{p}} = 0.$$

Neste caso, dizemos que $A = \sum\limits_{j=1}^{\infty} \lambda_j B_j \otimes y_j$ é uma representação hiper-$\sigma(p)$-nuclear de A, definimos

$$||A||_{\mathcal{H}\sigma(p)} := \inf \left\{ ||(\lambda_j)_{j=1}^{\infty}||_{p'} \cdot \sup_{x_i \in B_{E_i}, y' \in B_{F'}} \left(\sum_{j=1}^{\infty} |B_j(x_1, \ldots, x_n) y'(y_j)|^p \right)^{\frac{1}{p}} \right\},$$

onde o ínfimo é tomado sobre todas as representações hiper-$\sigma(p)$-nucleares de A, e denotamos por $\mathcal{L}_{\mathcal{H}\sigma(p)}(E_1, \ldots, E_n; F)$ o espaço dos operadores multilineares hiper-$\sigma(p)$-nucleares de $E_1 \times \cdots \times E_n$ em F.

2 Resultados Principais

Neste primeiro trabalho sobre essa nova classe, provamos os seguintes resultados, que: (i) relacionam a nova classe com a anterior, (ii) mostram que a nova classe não é nem muito pequena e nem muito grande, (iii) a nova classe satisfaz uma propriedade de ideal mais forte que a classe anterior. A nosso ver, essas propriedades justificam o estudo dos operadores hiper-$\sigma(p)$-nucleares.

Teorema 2.1. *Sejam $1 \leq p < \infty$, $E_1, \ldots, E_n$ e F espaços de Banach. Então:*

(a) $\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F) \subseteq \mathcal{L}_{\mathcal{H}\sigma(p)}(E_1, \ldots, E_n; F)$ *e, em geral,* $\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F) \neq \mathcal{L}_{\mathcal{H}\sigma(p)}(E_1, \ldots, E_n; F)$.

(b) *Denotando por $\mathcal{L}_{\mathcal{F}}$ a classe dos operadores multilineares de posto finito, temos*

$$\mathcal{L}_{\mathcal{F}}(E_1, \ldots, E_n; F) \subseteq \mathcal{L}_{\mathcal{H}\sigma(p)}(E_1, \ldots, E_n; F) \subseteq \overline{\mathcal{L}_{\mathcal{F}}(E_1, \ldots, E_n; F)}.$$

(c) *A classe $(\mathcal{L}_{\mathcal{H}\sigma(p)}, ||\ .\ ||_{\mathcal{H}\sigma(p)})$ dos operadores multilineares hiper-$\sigma(p)$-nucleares é um hiper-ideal p-Banach de operadores multilineares.*

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SIMULAÇÃO DE UM MODELO MATEMÁTICO DE CRESCIMENTO TUMORAL UTILIZANDO MÉTODO MULTIESTÁGIO

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Abstract

Neste trabalho considera-se um modelo matemático não linear de crescimento tumoral descrito por quatro equações diferenciais que representam a evolução da densidade de células cancerígenas, densidade da matriz extracelular (MEC), concentração das metaloproteinases da matriz (MMP) e concentração dos inibidores teciduais de metaloproteinases (TIMP). Para fins de simulações numéricas, utiliza-se a técnica de diferenças finitas com a finalidade de discretizar o modelo. Os termos temporais das equações são discretizados utilizando método multiestágio de segunda ordem, introduzindo um nível de tempo intermediário entre os níveis k e $k+1$. Quanto aos termos espaciais utiliza- se diferenças finitas centrais. Como uma alternativa para evitar a necessidade da resolução de um sistema não linear em cada passo de tempo, aplica-se a expansão da série de Taylor linearizando os termos quadráticos do modelo. Por fim, apresenta-se um estudo de convergência para o esquema numérico proposto utilizando soluções analíticas conhecidas.

1 Introdução

Para compreender o crescimento desordenado das células, muitos pesquisadores usam a modelagem matemática, que por meio das simulações do modelo descrito por equações diferenciais torna-se possível obter resultados que possam representar as alterações que o tumor pode causar. Neste contexto, este trabalho expõe um modelo de equações diferenciais parciais bidimensionais que descreve a invasão e crescimento do tumor, considerando a evolução do espaço (x, y) e do tempo t, dado em Kolev e Zubik-Kowal (2011) [3].

O modelo é apresentado pelas variáveis $n(x, y, t)$, $f(x, y, t)$, $m(x, y, t)$ e $u(x, y, t)$, que representam, densidade das células cancerígenas, MEC, MMP e TIMP, respectivamente. Para o estudo de convergência com solução analítica conhecida [2], considera-se os termos f_n, f_f, f_m e f_u, no modelo, resultando em

$$\frac{\partial n}{\partial t} - d_n \Delta n + \gamma \nabla (n \nabla f) - \mu_1 n(1 - n - f) = f_n \tag{1}$$

$$\frac{\partial f}{\partial t} + \eta m f - \mu_2 f(1 - n - f) = f_f \tag{2}$$

$$\frac{\partial m}{\partial t} - d_m \Delta m - \alpha n + \theta u m + \beta m = f_m \tag{3}$$

$$\frac{\partial u}{\partial t} - d_u \Delta u - \xi f + \theta u m + \rho u = f_u, \tag{4}$$

onde, d_n, d_m e d_u são as constantes de difusão da densidade das células cancerígenas, de MMP e de TIMP, respectivamente. As constantes μ_1 e μ_2 representam as taxas de proliferação das células tumorais e de crescimento do MEC, enquanto que γ, η, α, θ, β e ρ são constantes positivas.

Os termos f_n, f_f, f_m e f_u são obtidos de forma que as equações (1)-(4) satisfaçam as soluções analíticas, $n = m = u = e^t \sin(2\pi x) \sin(2\pi y)$ e $f = e^{-t} \sin(2\pi x) \sin(2\pi y)$.

2 Resultados Principais

O método de diferenças finitas é empregado para discretizar os termos das equações do modelo. No termo temporal das equações (1)-(4), utiliza-se o método multiestágio [1], resultando nos estágios explícito e implícito. O termo espacial de primeira e segunda ordem discretiza-se utilizando diferenças finitas centrais.

Considerando as discretizações no nível de tempo k em um ponto (i, j), tem-se o estágio explícito, descrito por

$$n\Big|_{i,j}^{k+\frac{1}{2}} = n\Big|_{ij}^{k} + \frac{\Delta t}{2}\left(d_n \Delta n\Big|_{ij}^{k} - \gamma\nabla\left(n\Big|_{ij}^{k}\nabla f\Big|_{ij}^{k}\right) + \mu_1 n\Big|_{ij}^{k}\left(1 - n\Big|_{ij}^{k} - f\Big|_{ij}^{k}\right)\right) \quad (5)$$

$$f\Big|_{ij}^{k+\frac{1}{2}} = f\Big|_{ij}^{k} + \frac{\Delta t}{2}\left(-\eta m\Big|_{ij}^{k} f\Big|_{ij}^{k} + \mu_2 f\Big|_{ij}^{k}(1 - n\Big|_{ij}^{k} - f\Big|_{ij}^{k})\right) \quad (6)$$

$$m\Big|_{ij}^{k+\frac{1}{2}} = m\Big|_{ij}^{k} + \frac{\Delta t}{2}\left(d_m \Delta m\Big|_{ij}^{k} + \alpha n\Big|_{ij}^{k} - \theta u\Big|_{ij}^{k} m\Big|_{ij}^{k} - \beta m\Big|_{ij}^{k}\right) \quad (7)$$

$$u\Big|_{ij}^{k+\frac{1}{2}} = u\Big|_{ij}^{k} + \frac{\Delta t}{2}\left(d_u \Delta u\Big|_{ij}^{k} + \xi f\Big|_{ij}^{k} - \theta u\Big|_{ij}^{k} m\Big|_{ij}^{k} - \rho u\Big|_{ij}^{k}\right). \quad (8)$$

Agora, considerando o nível de tempo $k+1$ em um ponto (i, j), tem-se o estágio implícito do método com os termos linearizados por expansão da série de Taylor [4], dado por

$$n\Big|_{ij}^{k+1} = n\Big|_{ij}^{k+\frac{1}{2}} + \frac{\Delta t}{2}\left(d_n \Delta n\Big|_{ij}^{k+1} - \gamma\nabla(n\Big|_{ij}^{k+1}\nabla f\Big|_{ij}^{k+1}) + \mu_1\left(n\Big|_{ij}^{k+1}(1 - f\Big|_{ij}^{k+1}) - 2n\Big|_{ij}^{k+\frac{1}{2}} n\Big|_{ij}^{k+1} + n^2\Big|_{ij}^{k+\frac{1}{2}}\right)\right) \quad (9)$$

$$f\Big|_{ij}^{k+1} = f\Big|_{ij}^{k+\frac{1}{2}} + \frac{\Delta t}{2}\left(-\eta m\Big|_{ij}^{k+1} f\Big|_{ij}^{k+1} + \mu_2\left(f\Big|_{ij}^{k+1}(1 - n\Big|_{ij}^{k+1}) - 2f\Big|_{ij}^{k+\frac{1}{2}} f\Big|_{ij}^{k+1} + f^2\Big|_{ij}^{k+\frac{1}{2}}\right)\right) \quad (10)$$

$$m\Big|_{ij}^{k+1} = m\Big|_{ij}^{k+\frac{1}{2}} + \frac{\Delta t}{2}\left(d_m \Delta m\Big|_{ij}^{k+1} + \alpha n\Big|_{ij}^{k+1} - \theta u\Big|_{ij}^{k+1} m\Big|_{ij}^{k+1} - \beta m\Big|_{ij}^{k+1}\right) \quad (11)$$

$$u\Big|_{ij}^{k+1} = u\Big|_{ij}^{k+\frac{1}{2}} + \frac{\Delta t}{2}\left(d_u \Delta u\Big|_{ij}^{k+1} + \xi f\Big|_{ij}^{k+1} - \theta u\Big|_{ij}^{k+1} m\Big|_{ij}^{k+1} - \rho u\Big|_{ij}^{k+1}\right). \quad (12)$$

As simulações são realizadas no domínio do quadrado unitário $\Omega = [0, 1] \times [0, 1]$, usando malha com tamanho $\Delta_x = \Delta_y = 0,05$, em diferentes intervalos de tempo Δt. O esquema numérico é implementado em GNU Octave (versão 5.1.0) e o sistema de equações algébricas é resolvido em Maple 18. As computações são realizadas usando a CPU Intel (R) Core (TM) i7-7500U com 3.5 GHZ e 8 GB de RAM.

Através de uma análise da norma L2 [2], considerando uma malha fixa com $M_x = M_y = 20$ e variando $M_t = 2^n \times 20$, $n = 0, 1, 2, 3, 4$, obteve-se erros de ordem entre 10^{-3} e 10^{-5}. Verificando que o esquema linearizado, quando comparado com soluções analíticas e análise de erros, mostra-se satisfatório.

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PERIODIC SOLUTIONS OF GENERALIZED ODE

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Abstract

In [3], the author presents results of the existence of periodic solutions to ordinary differential equations, in which the functions involved are continuous. In this work we define periodicity of solutions of generalized ordinary differential equations that assume values in $\mathbb{R}^n$ and in a Banach space any. We present conditions necessary and sufficient for a solution to be periodic. We deal with the integral forms of the differential equations using the Kurzweil integral. Thus the functions involved can have many discontinuities and be of unbounded variation and yet we obtain good results which encompass those in the literature.

1 Introduction

In order to generalize certain results in the continuous dependence of the solution of ordinary differential equations (ODEs) in relation to the initial data, J. Kurzweil introduced in 1957 the notion of generalized ordinary differential equations for functions that take values in Euclidian Banach spaces. We refer to these equations as generalized EDOs or simply EDOGs. See [5, 6, 7]. This concept proved to be useful for dealing with differential equations in measure, equations with impulses, among others.

This generalization of the notion of ODE includes the notion of Perron integral generalized or integral of Kurzweil as it is called nowadays. This integral is much more general than the Riemann and Lebesgue integrals, for example.

Let X be a Banach space. Denote by $L(X)$ the Banach space of linear and bounded operators in X. In this work, we demonstrate the existence of periodic solutions of the following generalized ODE

$$\frac{dx}{d\tau} = D[A(t)x + g(t)], \tag{1}$$

where $A : [0,\infty) \to L(X)$ and $g : [0,\infty) \to L(X)$ are functions of locally bounded variation and periodic.

In addition to the result of the existence of a periodic solution of equation (1), we present the Floquet's Theorem, so well known for the case of EDOs, for generalized EDOs.

2 Main Results

The main results to be proved in this work are based on those presented in [3] and are described below.

Theorem 2.1. *Let A and g be T-periodic. Suppose that g is Perron integrable. Then the equation (1) has a T-periodic solution $x(t)$ if and only if $x(0) = x(T)$.*

Consider the linear generalized ODE

$$\frac{dx}{d\tau} = D[A(t)x], \tag{2}$$

where $A : [0,\infty] \to L(\mathbb{R}^n)$ is Kurzweil integrable and T-periodic, that is, exists $T > 0$ such that $A(t+T) = A(t)$, for all $t \in [0,\infty)$. Under these conditions, it is possible to prove the following result.

Theorem 2.2. **(Floquet's theorem generalized)** *Every fundamental matrix $X(t)$ of (2) has the form*

$$X(t) = P(t)\, e^{Bt},$$

where $P(t)$ and B are matrices $n \times n$, with $P(t+T) = P(t)$, $t \in [0, \infty)$.

Theorem 2.3. *If the number one is not a characteristic multiplier of the generalized ODE T-periodic (2), then (1) has at least one T-periodic solution.*

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XIII ENAMA - Novembro 2019 *199–200*

EXISTENCE, STABILITY AND CRITICAL EXPONENT TO A SECOND ORDER EQUATION WITH FRACTIONAL LAPLACIAN OPERATORS

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Abstract

In this work, we consider a nonlinear fractional evolution equation, we study the critical exponent, prove global existence of small data solutions to the Cauchy problem and also study the stability of this problem.

1 Introduction

We prove global existence (in time) of small data solutions to the Cauchy problem

$$\begin{cases} u_{tt} + (-\Delta)^{\delta} u_{tt} + (-\Delta)^{\alpha} u + (-\Delta)^{\theta} u_t = |u_t|^p, \quad t \geq 0, x \in \mathbb{R}^n, \\ (u, u_t)(0, x) = (u_0, u_1)(x), \end{cases} \tag{1}$$

with $p > p_c$ (p_c the critical exponent), $2\theta \leq \alpha$ and $\delta \in (0, \alpha]$. Here we denote the fractional Laplacian operator by $(-\Delta)^b f = \mathfrak{F}^{-1}(|\xi|^{2b} \hat{f})$, with $b > 0$, , where $\mathfrak{F}$ is the Fourier transform with respect to the space variable, and $\hat{f} = \mathfrak{F} f$.

The term $(-\Delta)^{\theta} u_t$ represents a damping term. The assumption $2\theta \leq \alpha$ means that the damping is *effective*, according to the classification introduced in [5]. In particular, when the damping is effective, we may derive low-frequencies estimates to the linear problem

$$\begin{cases} u_{tt} + (-\Delta)^{\delta} u_{tt} + (-\Delta)^{\alpha} u + (-\Delta)^{\theta} u_t = 0, \quad t \geq 0, x \in \mathbb{R}^n, \\ (u, u_t)(0, x) = (u_0, u_1)(x), \end{cases} \tag{2}$$

which are proved to be sharp, thanks to the diffusion phenomenon. This latter means that the asymptotic profile of the solution to (2) may be described by the solution to an anomalous diffusion problem [7].

If $\delta \leq \theta$, the presence of the structural damping generates a strong smoothing effect on the solution to (2), and it guarantees the exponential decay in time of the high-frequencies part of the solution to (2). Therefore, the decay rate for (2) is only determined by the low-frequencies part of the solution to (2), which behaves like the solution to the corresponding anomalous diffusion problem [3, 4]. However, if $\delta > \theta$, the rotational inertia term $(-\Delta)^{\delta} u_{tt}$ ([8]) creates a structure of regularity-loss type decay in the linear problem. In the case of the plate equation with exterior damping ($\alpha = 2$, $\theta = 0$ and $\delta = 1$), we address the reader to [1] for a detailed investigation of properties like existence, uniqueness, energy estimates for the solution to (2), and a global existence (in time) of small data solutions to (1) for $p > 2$.

The energy for (2) dissipates and it is

$$E(t) = \frac{1}{2} \|u_t(t, \cdot)\|_{L^2}^2 + \frac{1}{2} \|(-\Delta)^{\frac{\delta}{2}} u_t(t, \cdot)\|_{L^2}^2 + \frac{1}{2} \|(-\Delta)^{\frac{\alpha}{2}} u(t, \cdot)\|_{L^2}^2.$$

2 Main Results

The results have different hypotesis involving n, θ, α, and δ. One of them are:

Theorem 2.1. *Let $1 \leq n \leq 4\theta \leq 2\alpha$, and $\delta \in (\theta, \alpha)$. Let $p > p_c = 1 + \frac{4\theta}{n}$. Then there exists a sufficiently small $\varepsilon > 0$ such that for any data $(u_0, u_1) \in \mathcal{A} \doteq H^{\alpha+\delta}(\mathbb{R}^n) \times H^{2\delta}(\mathbb{R}^n)$, $\|(u_0, u_1)\|_{\mathcal{A}} \leq \varepsilon$, there exits a global (in time) energy solution $u \in \mathcal{C}([0,\infty), H^\alpha \cap L^\infty) \cap C^1([0,\infty), L^2 \cap L^\infty)$ to (1). Also, the solution to (1) satisfies the estimates*

$$\||D|^{k\alpha}\partial_t^j u(t,\cdot)\|_{L^2} \lesssim (1+t)^{1-j-k\left(1+\frac{\alpha-2\theta}{2(\alpha-\theta)}\right)}\|(u_0,u_1)\|_{\mathcal{A}}, \qquad j+k=0,1. \tag{1}$$

We define the space $Y(T) \doteq \mathcal{C}([0,T), H^\alpha \cap L^\infty) \cap C^1([0,T], L^2 \cap L^\infty)$, equipped with the norm

$$\|u\|_{Y(T)} \doteq \sup_{t\in[0,T]} \Big\{(1+t)^{-1}\|u(t,\cdot)\|_{L^2} + (1+t)^{\frac{\alpha-2\theta}{2(\alpha-\theta)}}\||D|^\alpha u(t,\cdot)\|_{L^2} + (1+t)^{\frac{n}{4\theta}-1}\|u(t,\cdot)\|_{L^\infty} + \|u_t(t,\cdot)\|_{L^2} + (1+t)^{\frac{n}{4\theta}}\|u_t(t,\cdot)\|_{L^\infty}\Big\}.$$

By Duhamel's principle, a function $u \in Y(T)$ is a solution to (1) if, and only if, it satisfies the equality $u(t,x) = u^{\mathrm{lin}}(t,x) + \int_0^t E_1(t-s,\cdot) *_{(x)} f(u_t(s,x))\,ds$ in $Y(T)$, with $f(u_t(s,x)) = |u_t|^p$ and $u^{\mathrm{lin}} \doteq K_0(t,x) *_{(x)} u_0(x) + K_1(t,x) *_{(x)} u_1(x)$, is the solution to the linear Cauchy problem (2). The proof of our global existence results is based on the following scheme. Thanks to estimates for the linear problem (2) obteined in [2], we have $u^{\mathrm{lin}} \in Y(T)$ and it satisfies $\|u^{\mathrm{lin}}\|_{Y(T)} \leq C\,\|(u_0,u_1)\|_{\mathcal{A}}$. We define the operator F such that, for any $u \in Y(T)$, by $Fu(t,x) \doteq \int_0^t E_1(t-s,x) * f(u_t(s,x))\,ds$, then we prove the estimates

$$\|Fu\|_{Y(T)} \leq C\|u\|_{Y(T)}^p, \qquad \|Fu - Fv\|_{Y(T)} \leq C\|u-v\|_{Y(T)}\big(\|u\|_{Y(T)}^{p-1} + \|v\|_{Y(T)}^{p-1}\big). \tag{2}$$

By standard arguments, it follows that $F + u^{\mathrm{lin}}$ maps balls of $Y(T)$ into balls of $Y(T)$, for small data in $\mathcal{A}$, and that estimates (2) lead to the existence of a unique solution $u = u^{\mathrm{lin}} + Fu$.

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XIII ENAMA - Novembro 2019 *201–202*

ANALYTICITY IN POROUS-ELASTIC SYSTEM WITH KELVIN-VOIGT DAMPING

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Abstract

In this work we study the porous elastic system with two viscoelastic dissipative mechanism od Kelvin-Voigt type. We prove that the model is analytical if and only if the viscoelastic damping is present in both equations, of the displacement of the solid elastic material and the volume fraction. Otherwise, the corresponding semigroup is not exponentially stable independently of any relationship between the coefficients of wave propagation speed, that is , we show that the resolvent operator is not limited uniformly along the imaginary axis. However, it decays polynomially with optimal rate.

1 Introduction

In this work we present a porous elastic system with two dissipative mechanisms is considered. Thus, the system of equations considered here is

$$\begin{cases} \rho u_{tt} - \mu u_{xx} - b\phi_x - \gamma_1(u_x+\phi)_{xt} = 0 \quad \text{in} \quad (0,L)\times(0,\infty), \\ \phi_{tt} - \delta\phi_{xx} + bu_x + \xi\phi + +\gamma_1(u_x+\phi)_t - \gamma_2\phi_{xxt} = 0 \quad \text{in} \quad (0,L)\times(0,\infty). \end{cases} \tag{1}$$

We added to system (1) the initial conditions given by

$$\begin{cases} (u(x,0),\phi(x,0)) = (u_0(x),\phi_0(x)) \quad \text{in} \quad (0,L), \\ \\ (u_t(x,0),\phi_t(x,0)) = (u_1(x),\phi_1(x)) \quad \text{in} \quad (0,L), \end{cases} \tag{2}$$

and Dirichlet-Neumann boundary conditions given by

$$u(0,t) = u(L,t) = \phi_x(0,t) = \phi_x(L,t) = 0 \quad \forall t>0. \tag{3}$$

We considered two dampings, with viscoelasticity Kelvin-Voigt type, each acting in one of two equations of our model (1)-(3). As coupling is considered, b must be different from 0, but its sign does not matter in the analysis. It is worth noting that γ_1,γ_2 are nonnegative, $\gamma_1>0$ and $\gamma_2>0$, we say that the system (1)-(3) is fully viscoelastic and in case $\gamma_1>0$, $\gamma_2=0$ or $\gamma_1=0$, $\gamma_2>0$, that the system is partial viscoelastic.
The constitutive coefficients, in one-dimensional case (see [2, 6]), satisfy

$$\xi>0,\ \delta>0,\ \mu>0,\ \rho>0,\ J>0,\ \text{and}\ \mu\xi\geq b^2. \tag{4}$$

Let us consider the Hilbert space and inner product given by

$$\mathcal{H} \quad = \quad \mathcal{H} = H_0^1(0,L)\times L^2(0,L)\times H_*^1(0,L)\times L_*^2(0,L), \tag{5}$$

$$\langle U,V\rangle_{\mathcal{H}} \quad := \quad \int_0^L \left(\rho\varphi\overline{\Phi} + \mu u_x\overline{v}_x + J\psi\overline{\Psi} + \delta\phi_x\overline{\zeta}_x + \xi\phi\overline{\zeta} + b(u_x\overline{\zeta} + \overline{v}_x\phi)\right)\, dx, \tag{6}$$

with $U=(u,\varphi,\phi,\psi)'\in\mathcal{D}(\mathcal{A})$, where the operator $\mathcal{A}$ is given by

$$\mathcal{A}\left(u,\varphi,\phi,\psi\right) = \left(\varphi, \frac{\mu}{\rho}u_{xx} + \frac{b}{\rho}\phi_x + \frac{\gamma_1}{\rho}(u_x+\phi)_{xt}, \psi, \frac{\delta}{J}\phi_{xx} - \frac{b}{J}u_x - \frac{\xi}{J}\phi - \frac{\gamma_1}{J}(u_x+\phi)_t - \frac{\gamma_2}{J}\phi_{xxt}\right),$$

2 Main Results

Theorem 2.1. *The operator $\mathcal{A}$ is the infinitesimal generater of a C_0 semigroup of contraction $S(t) = e^{\mathcal{A}t}$ over $\mathcal{H}$.*

Theorem 2.2. *Let $S(t) = e^{\mathcal{A}}t$ the C_0-semigroup of contraction on Hilbert space $\mathcal{H}$ associated with the system (1)-(3) . Then, case $\gamma_1 > 0$ and $\gamma_2 = 0$, or $\gamma_1 = 0$ and $\gamma_2 > 0$, $S(t)$ is not exponentially stable, independently of any relationship between the coefficients μ, ρ, δ e J.*

Lemma 2.1. *For the system (1)-(3), we have $i\mathbb{R} \equiv \{i\lambda :\ \lambda \in \mathbb{R}\} \subset \rho(\mathcal{A})$ provided one of the coefficients γ_1, γ_2, is positive.*

Theorem 2.3. *Under the conditions of Theorem 3.1 (see [3]), the corresponding semigroup $S(t) = e^{\mathcal{A}t}$ of the system (1)-(3) is analytic and exponentially stable C_0-semigroup on $\mathcal{H}$, if and only if, $\gamma_1 > 0$ and $\gamma_2 > 0$.*

Theorem 2.4. *The semigroup $S(t) = e^{\mathcal{A}t}$ associated with the system (1)-(3), satisfies*

$$\|S(t)U_0\|_{\mathcal{H}} \leq \frac{C}{\sqrt{t}}\|U_0\|_{\mathcal{D}(\mathcal{A})}, \forall U_0 \in \mathcal{D}(\mathcal{A}).$$

Moreover, this rate is optimal.

The above results were obtained using multiplicative techniques and the well-known Theorem due to A. *Gearhart-Herbst-Prüss-Huang* for dissipative systems, from semigroup theory (see [2, 4]), as *Borichev and Y. Tomilov (see [1], Theorem 2.4)*

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XIII ENAMA - Novembro 2019 *203–204*

SOLUÇÃO GLOBAL FORTE PARA AS EQUAÇÕES DE FLUIDOS MAGNETO-MICROPOLARES EM $\mathbb{R}^3$

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Abstract

Estudamos o problema de Cauchy para as equações de um fluido magneto-micropolar incompreensível em todo o espaço $\mathbb{R}^3$. Primeiramente, baseado em estimativas de energia, mostramos a existência e unicidade de solução local forte para o problema em questão. Após isso, impondo uma condição de pequenez nos dados iniciais, demonstramos a unicidade da solução global forte.

1 Introdução

Consideramos o problema de valor inicial (PVI)

$$\begin{cases} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\mu+\chi)\Delta\boldsymbol{u} + \nabla\left(p + \frac{1}{2}|\boldsymbol{b}|^2\right) = \chi\,\mathrm{rot}\,\boldsymbol{w} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{b}, \\ \mathrm{div}\,\boldsymbol{u} = \mathrm{div}\,\boldsymbol{b} = 0, \\ \boldsymbol{b}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{b} - \nu\Delta\boldsymbol{b} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{u}, \\ \boldsymbol{w}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{w} - \gamma\Delta\boldsymbol{w} - \kappa\nabla(\mathrm{div}\,\boldsymbol{w}) + 2\chi\boldsymbol{w} = \chi\,\mathrm{rot}\,\boldsymbol{u}, \\ \boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x}), \quad \boldsymbol{b}(\boldsymbol{x},0) = \boldsymbol{b}_0(\boldsymbol{x}), \quad \boldsymbol{w}(\boldsymbol{x},0) = \boldsymbol{w}_0(\boldsymbol{x}), \end{cases} \tag{1}$$

em $\mathbb{R}^3 \times (0,T)$, onde $\boldsymbol{u}_0$, $\boldsymbol{b}_0$ e $\boldsymbol{w}_0$ são funções dadas e $0 < T \leq \infty$.

O sistema acima descreve o fluxo de um fluido magneto-micropolar incompressível (veja [2]). Aqui, as incógnitas são as funções $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^3$, $p(\boldsymbol{x},t) \in \mathbb{R}$, $\boldsymbol{b}(\boldsymbol{x},t) \in \mathbb{R}^3$ e $\boldsymbol{w}(\boldsymbol{x},t) \in \mathbb{R}^3$, as quais representam, respectivamente, o campo de velocidade incompressível, a pressão hidrostática, o campo magnético e a velocidade micro-rotacional do fluido em um ponto $\boldsymbol{x} \in \mathbb{R}^3$ no tempo $t > 0$. As constantes positivas μ, χ, ν, γ e κ estão associadas a propriedades específicas do fluido. Vale ressaltar que o sistema (1) inclui, como caso particular, as clássicas equações de Navier-Stokes ($\boldsymbol{b} = \boldsymbol{w} = \boldsymbol{0}$ e $\chi = 0$), as equações MHD ($\boldsymbol{w} = \boldsymbol{0}$ e $\chi = 0$) e as equações micropolares ($\boldsymbol{b} = \boldsymbol{0}$).

2 Resultados Principais

Os resultados que demonstramos são análogos ao de *F. W. Cruz* para as equações micropolares 3D (veja [1]) e ao de *X. Zhong* para as equações de Navier-Stokes com amortecimento (veja [3]). Por simplicidade, assumimos $\mu = \chi = 1/2$ e $\nu = \gamma = \kappa = 1$.

Antes de enunciarmos o principal resultado obtido, apresentaremos a definição de solução forte para o PVI (1).

Definição 2.1. *Suponha que* $(\boldsymbol{u}_0, \boldsymbol{b}_0, \boldsymbol{w}_0) \in \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3)$ *com* $\mathrm{div}\,\boldsymbol{u}_0 = \mathrm{div}\,\boldsymbol{b}_0 = 0$*. Por uma* ***solução forte*** *do problema* (1)*, entendemos funções*

$$\boldsymbol{u},\, \boldsymbol{b},\, \boldsymbol{w} \in L^\infty\left(0,T;\boldsymbol{H}^1(\mathbb{R}^3)\right) \cap L^2\left(0,T;\boldsymbol{H}^2(\mathbb{R}^3)\right),$$

com $(\boldsymbol{u},\boldsymbol{b},\boldsymbol{w})$ *satisfazendo as equações* $(1)_1$*,* $(1)_2$*,* $(1)_3$*,* $(1)_4$ *q.s. em* $\mathbb{R}^3 \times (0,T)$*, e as condições iniciais* $(1)_5$ *em* $\boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3)$.

Teorema 2.1. *Suponha que os dados iniciais* $(\boldsymbol{u}_0, \boldsymbol{b}_0, \boldsymbol{w}_0)$ *satisfazem*

$$(\boldsymbol{u}_0, \boldsymbol{b}_0, \boldsymbol{w}_0) \in \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3), \quad \operatorname{div} \boldsymbol{u}_0 = \operatorname{div} \boldsymbol{b}_0 = 0.$$

Então, existe uma constante $\varepsilon_0 > 0$*, independente de* $\boldsymbol{u}_0$*,* $\boldsymbol{b}_0$ *e* $\boldsymbol{w}_0$ *tal que se*

$$\left(\|\boldsymbol{u}_0\|^2 + \|\boldsymbol{b}_0\|^2 + \|\boldsymbol{w}_0\|^2\right)\left(\|\nabla \boldsymbol{u}_0\|^2 + \|\nabla \boldsymbol{b}_0\|^2 + \|\nabla \boldsymbol{w}_0\|^2\right) \leq \varepsilon_0,$$

o problema de Cauchy (1) *possui uma única solução global forte.*

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XIII ENAMA - Novembro 2019 *205–206*

EXISTÊNCIA E COMPORTAMENTO ASSINTÓTICO DA SOLUÇÃO FRACA PARA A EQUAÇÃO DE VIGA NÃO LINEAR ENVOLVENDO O $P(X)$-LAPLACIANO

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Abstract

Neste trabalho estudamos a existência de soluções fracas para uma Equação Diferencial Parcial de quarta ordem não linear envolvendo o operador $p(x)$-Laplaciano sobre um domínio limitado. Para demonstrar a existência de soluções fracas utilizamos o método de Faedo-Galerkin acoplado com resultados de Análise Funcional, espaços de Lebesgue e Sobolev com expoente variável que podem ser encontrados [1] e [2]. Utilizamos uma técnica introduzida por Nakao em [3] e obtivemos o decaimento exponencial e polinomial das soluções. Está em fase de conclusão a análise numérica e simulação da solução. Ressaltamos que a unicidade da solução fraca é um problema em aberto, entretanto estamos trabalhando para solucionar a aludida conjectura.

1 Introdução

Estudamos o seguinte problema

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} + \Delta\left(|\Delta u|^{p(x)-2}\Delta u\right) - \Delta\dfrac{\partial u}{\partial t} + f\left(x,t,\dfrac{\partial u}{\partial t}\right) = g\left(x,t\right) \text{ em } Q_T, \\ u = \Delta u = 0 \text{ em } \partial\Omega \times (0,T), \\ u\left(x,0\right) = u_0(x), \ \dfrac{\partial u(x,0)}{\partial t} = u_1(x) \text{ em } \Omega. \end{cases} \tag{1}$$

Onde $\Omega \subset \mathbb{R}^N$, $N \geq 3$, é um domínio limitado com fronteira $\partial\Omega$ suave, $0 < T < \infty$, $Q_T = \Omega \times (0,T)$ e as funções p, f, g, u_0 e u_1 satisfazem as seguintes hipóteses:

(H.1) $p : \overline{\Omega} \to (1,\infty)$ é log-Hölder contínua, isto é, existe uma constante $c > 0$ tal que

$$|p(x) - p(y)| \log|x-y| \leq c \quad \forall\, x, y \in \overline{\Omega} \tag{2}$$

e satisfaz

$$1 < p^- = \inf_{\overline{\Omega}} p(x) \leq p^+ = \sup_{\overline{\Omega}} p(x) < \frac{N}{2} \quad \forall\, x \in \overline{\Omega} \tag{3}$$

onde $\overline{\Omega}$ denota o fecho de Ω;

(H.2) $f\left(x,t,s\right) \in C\left(\Omega \times [0,\infty) \times \mathbb{R}\right)$ e existem constantes positivas c_1, c_2 e c_3 tais que

$$\begin{cases} f\left(x,t,s\right) s \geq c_1 \left|s\right|^{q(x)} - c_3 \\ \left|f\left(x,t,s\right)\right| \leq c_2 \left(\left|s\right|^{q(x)-1} + 1\right) \end{cases} \tag{4}$$

para todo $(x,t,s) \in \Omega \times [0,\infty) \times \mathbb{R}$, onde $q : \overline{\Omega} \to (1,\infty)$ é log-Hölder contínua satisfazendo

$$1 < q^- = \inf_{\overline{\Omega}} q(x) \leq q(x) < \frac{Np(x)}{N-2p(x)} \quad \forall\, x \in \overline{\Omega}; \tag{5}$$

(H.3) $u_0 \in W^{2,p(x)}\left(\Omega\right) \cap W_0^{1,2}\left(\Omega\right)$, $u_1 \in L^2\left(\Omega\right)$ e $g \in L^{q'(x)}\left(Q_T\right)$ onde

$$\frac{1}{q(x)} + \frac{1}{q'(x)} = 1 \quad \forall\, x \in \overline{\Omega}. \tag{6}$$

2 Resultados Principais

Teorema 2.1. *Sob as hipóteses **(H.1)**, **(H.2)** e **(H.3)** o Problema (1) tem solução fraca.*

(H.4) $f(x,t,s) \in C(\Omega \times [0,\infty) \times \mathbb{R})$ e existem constantes c_7 e c_8 tais que

$$\begin{cases} f(x,t,s)\, s \geq c_7 |s|^{q(x)} \\ |f(x,t,s)| \leq c_8 |s|^{q(x)-1} \end{cases} \tag{1}$$

para todo $(x,t,s) \in \Omega \times [0,\infty) \times \mathbb{R}$.

Teorema 2.2. *Sejam **(H.1)**, **(H.3)**, **(H.4)**, $p^- > \max\left\{1, \dfrac{2N}{N+2}\right\}$ e $g(x,t) \equiv 0$. Então existem constantes $C > 0$ e $\gamma > 0$ tais que as soluções fracas satisfazem:*

Se $q^- \geq 2$, então

$$\int_\Omega \left|\frac{\partial u(x,t)}{\partial t}\right|^2 + |\Delta u(x,t)|^{p(x)}\, dx \leq \begin{cases} Ce^{-\gamma t}, & \forall t \geq 0, \ \text{se } p^+ = 2, \\ C(t+1)^{-\frac{p^+}{p^+-2}}, & \forall t \geq 0, \ \text{se } p^+ > 2. \end{cases} \tag{2}$$

Se $1 < q^- < 2$, então

$$\int_\Omega \left|\frac{\partial u(x,t)}{\partial t}\right|^2 + |\Delta u(x,t)|^{p(x)}\, dx \leq \begin{cases} C(t+1)^{-\frac{p^+\left(q^- -1\right)}{p^+-q^-}}, & \forall t \geq 0, \ \text{se } p^+ < q^-, \\ Ce^{-\gamma t}, & \forall t \geq 0, \ \text{se } p^+ \geq q^-. \end{cases} \tag{3}$$

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GLOBAL ANALYTIC HYPOELLIPTICITY FOR A CLASS OF LEFT-INVARIANT OPERATORS ON $\mathbb{T}^1 \times \mathbb{S}^3$

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Abstract

We present necessary and sufficient conditions to have global analytic hypoellipticity for a class of first-order operators defined on $\mathbb{T}^1 \times \mathbb{S}^3$. In the case of real-valued coefficients, we prove that the operator is conjugated to a constant coefficient operator that satisfies a Diophantine condition, and that such conjugation preserves the global analytic hypoellipticity. If the imaginary part of the coefficients is nonzero, we show that the operator is globally analytic hypoelliptic if the Nirenberg-Treves condition (P) holds in addition to a Diophantine condition.

1 Introduction

Since the 70's the property called Global Hypoellipticity is being studied for a great variety of (pseudo)differential operators acting on many different manifolds. One of the pioneering works in the area is [2], which related the concept of Global Hypoellipticity (GH) of an operator on a torus with the kind of growth of the symbol of this operator at infinity. In particular, when the operator is a vector field with constant coefficients on a 2-torus, the condition (GH) translates into a Diophantine condition, that is, the global hypoellipticity becomes a problem about approximation of real number using rational numbers.

This lead to one of the major problems in this area, the Greenfield-Wallach conjecture, which claims the following: if a vector field defined on a closed connected orientable manifold is (GH), then the manifold is diffeomorphic to a torus and the vector field is conjugated to a Diophantine constant vector field. There are positive partial answers for this conjecture, for example, it is true in dimensions 2 and 3.

Then, a lot of different directions are natural to consider. The direction we are interested here is to ask the similar questions about global analytic hypoellipticity for a class of operators defined on compact Lie Groups. This is natural since the standard approach used since Greenfield and Wallach was the analysis of Fourier, and on compact Lie Groups there is a very well established Fourier theory, see [4].

In this work we present results about global analytic hypoellipticity of a class of operators defined on $\mathbb{T}^1 \times \mathbb{S}^3$. This is joint work with Alexandre Kirilov (UFPR) and Wagner A. A de Moraes (UFPR).

2 Main Results

A continuous linear operator $P : \mathcal{D}'(\mathbb{T}^1 \times \mathbb{S}^3) \to \mathcal{D}'(\mathbb{T}^1 \times \mathbb{S}^3)$ is called globally analytic hypoelliptic (GAH) if the following condition is true:

$$u \in \mathcal{D}'(\mathbb{T}^1 \times \mathbb{S}^3) \text{ and } P(u) \in C^\omega(\mathbb{T}^1 \times \mathbb{S}^3) \Rightarrow u \in C^\omega(\mathbb{T}^1 \times \mathbb{S}^3).$$

In this work we are interested in the following class of operators

$$L = \partial_t + c(t)\partial_0 + \lambda(t, x),$$

where $c = a + ib \in C^\omega(\mathbb{T}^1)$, $\lambda \in C^\omega(\mathbb{T}^1 \times \mathbb{S}^3)$, and ∂_0 the neutral operator on $\mathbb{S}^3$.

In the study of the global analytic hypoellipticity of L we are lead to study the same property of the constant coefficient operator

$$L_0 = \partial_t + c_0\partial_0 + \lambda_0.$$

where $c_0 = a_0 + ib_0$,

$$a_0 = (2\pi)^{-1}\int_{\mathbb{T}^1} a(t)dt,\ b_0 = (2\pi)^{-1}\int_{\mathbb{T}^1} b(t)dt \ \text{ and } \ \lambda_0 = (2\pi)^{-2}\int_{\mathbb{S}^3}\int_{\mathbb{T}^1} \lambda(t,x)dtdx.$$

Theorem 2.1. *If $b \equiv 0$ then L is (GAH) if and only if L_0 is (GAH).*

The study of global analytic hypoellipcity for constant coefficients operators is done following the Greenfield's and Wallach's ideas for the torus.

Theorem 2.2. *Assume that $b \not\equiv 0$ and does not change sign; that L_0 is (GAH); and that λ_0 satisfies any of the conditions following conditions*

C1) $\mathfrak{Re}(\lambda_0) = 0$ *and* $\mathfrak{Im}(\lambda_0) \notin \mathbb{Z}$*;*

C2) $\mathfrak{Re}(\lambda_0) \neq 0$ *and* $\dfrac{\mathfrak{Re}(\lambda_0)}{b_0} \notin \frac{1}{2}\mathbb{Z}$*;*

C3) $\dfrac{\mathfrak{Re}(\lambda_0)}{b_0} \in \frac{1}{2}\mathbb{Z}$ *but* $\mathfrak{Re}(\lambda_0)\dfrac{a_0}{b_0} + \mathfrak{Im}(\lambda_0) \notin \mathbb{Z}$*.*

then L is (GAH).

Theorem 2.3. *If b changes sign, then L_λ is not (GAH) for all $\lambda \in C^\omega(\mathbb{T}^1 \times \mathbb{S}^3)$.*

The proof of the first theorem is done by constructing a conjugation, and the proofs of the second and third theorems are adaptations of the ideias implemented on [1, 3].

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SMOOTHING EFFECT FOR THE 2D NAVIER-STOKES EQUATIONS

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Abstract

In this work we present a “smoothing” effect for initial-boundary value problems of the 2D Navier-Stokes equations posed on Lipschitz and smooth bounded and unbounded domains.

1 Introduction

In [1], Kato has observed "smoothing" effect for the initial value problem of the KdV equation. This means that solutions of the KdV equations are more regular for $t > 0$ than the initial data. In this work, we establish this effect for solutions of initial-boundary value problems for the 2D Navier-Stokes system.

In $\Omega \subset \mathbb{R}^2$, consider the initial-boundary value problem for the two-dimensional Navier-Stokes equations.

$$u_t + (u \cdot \nabla)u = \nu \Delta u - \nabla p \ \text{ in } \ \Omega \times (0,t), \tag{1}$$

$$\nabla \cdot u = 0 \ \text{ in } \ \Omega, \quad u|_{\partial\Omega} = 0, \qquad u(x,0) = u_0(x), \ \ x = (x_1, x_2). \tag{2}$$

The problem of the energy decay for weak solutions had been stated by J. Leray [3]. Regularity and exponential decay of solutions to (1.1),(1.2) when initial data $u_0 \in V \cap H^2(\Omega)$, where V is the closure of $\mathcal{V} = \{v \in C_0^\infty(\Omega);\ \nabla \cdot u = 0\}$, for various bounded and unbounded Lipschitz and smooth domains have been considered in [2].

2 Main Results

Theorem 2.1. *Consider a rectangle* $\Omega = \{(x_1,x_2) \in \mathbb{R}^2; 0 < x_1 < L_1,\ \ 0 < x_2 < L_2\}$. *Given* $u_0 \in H^2(\Omega) \cap V$, *the unique regular solution* (u,p) *of the problem (1)-(2)*

$$u \in L^\infty(0,\infty; H^2(\Omega)), \qquad u_t \in L^\infty(0,\infty; L^2(\Omega)) \cap L^2(0,\infty; V) \tag{1}$$

is such that

$$u \in L^2(0,\infty; H^3(\Omega_0)), \qquad \nabla p \in L^2(0,\infty; H^1(\Omega_0)). \tag{2}$$

for any subdomain $\Omega_0 \subset \Omega$ *such that* $dist(\partial\Omega_0, \Omega) \geq \delta > 0$.

Theorem 2.2. *Consider the half-strip* $\Omega = \{(x_1,x_2) \in \mathbb{R}^2; 0 < x_1,\ 0 < x_2 < L_2\}$. *Given* $u_0 \in H^2(\Omega) \cap V$, *the unique strong solution* (u,p) *of the problem (1)-(2) with the condition* $\lim_{x_1 \to \infty} |u(x_1,x_2,t)| = 0$,

$$u \in L^\infty(0,\infty; V), \qquad u_t \in L^\infty(0,\infty; L^2(\Omega)) \cap L^2(0,\infty; V) \tag{3}$$

is such that

$$u \in L^\infty(0,\infty; H^2_{loc}(\Omega)) \cap L^2(0,\infty; H^3(\Omega_0)), \qquad \nabla p \in L^\infty(0,\infty; H^1_{loc}(\Omega)) \cap L^2(0,\infty; H^1(\Omega_0)). \tag{4}$$

where Ω_0 *is a bounded subdomain of* Ω *such that* $dist(\partial\Omega_0, \Omega) \geq \delta > 0$.

Theorem 2.3. *Let $\Omega \subset \mathbb{R}^2$ be a bounded domain of class C^3. The unique regular solution of the problem (1)-(2) is such that*

$$u \in L^\infty(0,\infty; H^2(\Omega)) \cap L^2(0,\infty; H^3(\Omega)),\ \nabla p \in L^\infty(0,\infty; L^2(\Omega)) \cap L^2(0,\infty; H^1(\Omega)). \tag{5}$$

For the next result consider the half-strip $D = \{(x_1, x_2) \in \mathbb{R}^2; 0 < x_1; 0 < x_2 < L_2\}$, where $L_2 > 0$ is the minimal value since $\Omega \subset D$.

Theorem 2.4. *Let $\Omega \subset D \subset \mathbb{R}^3$ be an unbounded domain of class C^3. The unique strong solution (u, p) of the problem (1)-(2), with the condition $\lim_{x_1 \to \infty} |u(x_1, x_2, t)| = 0$, is such that*

$$u \in L^\infty(0,\infty; H^2_{loc}(\Omega)) \cap L^2(0,\infty, H^3_{loc}(\Omega)), \tag{6}$$

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EXISTENCE AND UNIQUENESS TO A SEMILINEAR EQUATION GENERALIZED WITH FRACTIONAL DAMPING

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Abstract

In this work we study the existence and uniqueness of solutions to a second order semilinear evolution equation with fractional damping term which includes some partial differential equations as plate equation under effects of rotational inertia and Boussinesq type equations that model hydrodynamics problems.

1 Introduction

In this work we consider the Cauchy problem o a generalized second order evolution equation in $\mathbb{R}^n$ given by

$$\begin{cases} \partial_t^2 u + (-\Delta)^\delta \partial_t^2 u + b(-\Delta)^\beta u + a(-\Delta)^\alpha u + (-\Delta)^\theta \partial_t u = \omega\,(-\Delta)^\gamma\, u^p, \\ u(0,\,x) = u_0(x), \quad \partial_t u(0,\,x) = u_1(x), \end{cases} \tag{1}$$

where $u = u(t,\,x)$ with $(t,\,x) \in\,]0,\,\infty[\times\mathbb{R}^n$, $n \geq 1$, $b,\ a > 0$, $\omega \in \mathbb{R}$ constants, $p > 1$ an integer and $u_0,\ u_1$ are the initial data. The exponents $\delta,\ \beta,\ \alpha,\ \theta$ and γ of the Laplacian operator are such that $0 \leq \delta \leq 2$, $\beta > \alpha$, $0 \leq \alpha \leq 2$, $0 \leq \theta \leq (2+\delta)/2$ and $\max\{0,\,\alpha/2 - n/4\} \leq \gamma \leq (\alpha+\delta)/2$.

2 Existence and Uniqueness: Linear Problem

In this section using the well known theory of semigroups we show existence and uniqueness of solutions to the linear Cauchy problem associated to the system (1) given by

$$\begin{cases} \partial_t^2 u + (-\Delta)^\delta \partial_t^2 u + (-\Delta)^\beta u + a(-\Delta)^\alpha u + (-\Delta)^\theta \partial_t u = 0 \\ u(0,\,x) = u_0(x), \quad \partial_t u(0,\,x) = u_1(x), \end{cases} \tag{1}$$

where $u = u(t,\,x)$, with $(t,\,x) \in\,]0,\,\infty[\times\mathbb{R}^n$, $n \geq 1$, $a > 0$ is a constant. The exponents of Laplacian $\delta,\ \beta,\ \alpha,\ \theta$ are such that $0 \leq \delta \leq 2$, $\alpha < \beta$, $0 \leq \alpha \leq 2$ and $0 \leq \theta \leq (2+\delta)/2$.

Taking formally the inner product in $L^2(\mathbb{R}^n)$ of the differential equation in (1) by $\partial_t u$ we obtain that

$$\frac{d}{dt}E(t) + \|(-\Delta)^{\theta/2}\partial_t u\|^2 = 0, \qquad \text{for all } \ t > 0, \tag{2}$$

where $E(t)$ represents the total energy of the system (1) and is given by

$$E(t) = \frac{1}{2}\Big(\|\partial_t u\|^2 + \|(-\Delta)^{\delta/2}\partial_t u\|^2 + \|(-\Delta)^{\beta/2}u\|^2 + a\|(-\Delta)^{\alpha/2}u\|^2\Big). \tag{3}$$

Then is natural consider the associated space of energy as

$$X = H^\beta(\mathbb{R}^n) \times H^\delta(\mathbb{R}^n) \tag{4}$$

since we imposed that $\alpha \leq \beta$.

Theorem 2.1. *Let $n \geq 1$, $\alpha < \beta$, $0 \leq \delta \leq 2$ and $0 \leq \theta \leq (2+\delta)/2$. If $u_0 \in H^{4-\delta}(\mathbb{R}^n)$ e $u_1 \in H^{\beta}(\mathbb{R}^n)$ then the Cauchy Problem* (1) *have only one solution u in following class*

$$u \in C^2\big([0,\, \infty[; H^{\delta}(\mathbb{R}^n)\big) \cap C^1\big([0,\, \infty[;\; H^{\beta}(\mathbb{R}^n)\big) \cap C\left([0,\, \infty[; H^{4-\delta}(\mathbb{R}^n)\right).$$

Before proving the existence and uniqueness of the solution we need the definition of two important operators A_2 and A_θ. These operators are essential for the definition of the operator B. For the case $0 \leq \theta < \delta$ we use the operator A_2 to define B, while in the case $0 \leq \delta \leq \theta$ we use the two operators A_2 and A_θ, to define the operator B.

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